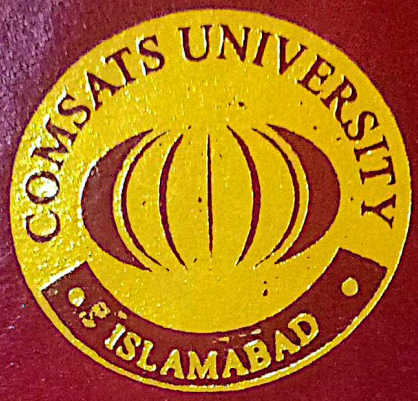
Effect of Modified Entropy on Black Hole Thermodynamics in Bumblebee Gravity



By

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CIIT/SP24-RMT-019/LHR

MS Thesis

In

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by
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Abstract

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This thesis is devoted to the thermodynamic characteristics and stability analysis of anti-de Sitter black hole in the framework of bumblebee gravity where a vector field spontaneously breaks Lorentz symmetry. In either the standard and extended phase space, the effects of the Lorentz-violating or bumblebee parameter on the event horizon structure and thermodynamic quantities are examined. We investigate the local and global thermal stability parameters by using the heat capacity and Helmholtz free energy, respectively. We demonstrate that large bumblebee anti-de Sitter black hole can concurrently meet both stability requirements. It is explicitly shown that the stable regions rely on the Lorentz-violating value. We also investigate the heat capacity at constant pressure by extending the study to the extended phase space, where the cosmological constant is understood as thermodynamic pressure. We discover that the stability conditions cannot be met. Our results demonstrate how important Lorentz symmetry violation is to black hole thermodynamics. We also study the sparsity parameter and the rate at which Hawking emission occurs in a Bumblebee gravity BH solution, within the framework of generalized entropies. Using the Sharma-Mittal as well as the three-parameter entropy indices, the effect of the sign change of the cosmological constant has been observed. The sign change shows that the negative value of the cosmological constant results in a decrease in the thermodynamic sparsity along with an increase in the entropy level, thereby marking the regime of a denser thermodynamic system, whereas the positive value maintains a higher level of thermodynamic sparsity. The emission spectrum takes a sharp form when the cosmological constant is negative, thereby marking the increased emission at low energies within a confining AdS space. The presence of a positive cosmological constant universally suppresses the emission rate.