A Study of Cosmic Accelerating Behavior with Interacting Dark Sectors

By

Muhammad Jameel Imtiaz

CIIT/FA17-RMT-031/LHR

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Name	Registration Number
Muhammad Jameel Imtiaz	CIIT/FA17-RMT-031/LHR

Supervisor

____

Dr. Rabia Saleem

Assistant Professor

Department of Mathematics,

COMSATS University Islamabad (CUI)

Lahore Campus

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Final Approval

This thesis titled

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Muhammad Jameel Imtiaz

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Has been approved

For the COMSATS University Islamabad, Lahore Campus

External Examiner : ______________________

Supervisor : ______________________

(Dr. Rabia Saleem)

HoD/Incharge : ______________________

(Dr. Sarfraz Ahmad)

(Mathematics Department, CUI, Lahore Campus)

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I, **Muhammad Jameel Imtiaz** with registration number **CIIT/FA17-RMT-031/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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(CIIT/FA17-RMT-031/LHR)

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It is certified that Muhammad Jameel Imtiaz with CIIT/FA17-RMT-031/LHR has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore campus and the work fulfills the requirement for award of MS degree.

Dated :____________________

Supervisor:____________________

(Dr.Rabia Saleem)

Assistant Professor

Head/Incharge Of Department

____

(Dr. Sarfraz Ahmad)

Associate Professor

Department of Mathematics

DEDICATED TO

My loving Parents, Teachers, My Family,
and Friends who always encouraged
me to work hard and guided me towards
the right way and destination.

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ABSTRACT

The main focus of this thesis is to investigate an interacting phenomena between dark matter (DM)-dark energy (DE) within a non-flat Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime geometry bounded by a horizon with specific Ricci cut-off. The Ricci model of DE has been under taken as a cosmic fluid ingredient along with DM. To check consistency of the considered model with astrophysical data coming from recent observations, we assume a positive interaction term Q between two components of fluid along with "Chevallier-Polarsky-Linder type parametrization" of the coincidence parameter $r(z)$. By calculating all the physical quantities like energy density and pressure, we constraint the model parameters and found specific region of validity for these parameters. It is observed that our model possess a future singularity of Type III. After finding the singular behavior, we examine the nature of our model via cosmological parameters like $q,\ r,\ s$. It is noted that our model is very close to Λ cold dark matter (ΛCDM).

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Chapter 1

Introduction

All through the twentieth century, “Einstien’s general theory of relativity (GR)" has successes in experimentation as experimental scrutiny starting from millimeter to cosmological scale. It solely expected “*gravitational red-shift* (RS), *precession of mercury*" and “*bending of light*". This is, therefore wealthy in its applications that “cosmology" and “astrophysics" could not exit without it. The commonplace cosmological model supported GR attracted an entire image of cosmos. In addition, few other basic theories like “*quantum gravity* (QG)", “*particle physics*" and “*thermodynamics*" exist in literature also that are considered to be very helpful in understanding of the cosmic nature.

“Cosmology" is an ancient science, dealing with origin, dynamics, large scale structure (LSS) and evolution of the universe. Human nature has always been curious in disclosing the mysteries of the universe. Latest advancements in this branch of science have many fascinating discoveries concerning the cosmic evolution via distance detecting means like satellites and telescopes. The remarkable event that have brought several challenges in this subject is the “accelerating expansion" of the universe. Scientists have endeavored to collect the evidences through observations regarding the matter of a new type, which would end up over cosmic expansion. These experiments reminded Aristotle’s idea: “the global motion, which tend to observe in distinct parts of our cosmos is unable to explain by standard matter". The whole amount of matter, which surrounded us, our solar system and galaxy, contain a little pressure which gathered under gravitational effects. Thus, this type of matter acts its part in cosmic deceleration.

Latest observational data of “*Type 1a Supernovae*" and “*Willkinson Microwave Anisotropy Prob* (WMAP)" indicate that the current universe is transforming from decelerating stage to an accelerating one. Initially, it is supposed that an enigmatic kind of energy dubbed as DE, having large negative pressure effecting the evolution of the universe. Later on, it is confirmed by “cosmic microwave background radiation (CMBR)". The most adopted and simplest DE component is the "cosmological constant (Λ)", with constant "*equation of state* (EoS)", *i.e.*, "$\omega = -1$", is well fitted with recent observational data. Yet, it suffers with two major issues: "*fine-tuning*; *cosmic coincidence*". The first puzzle is generated due to larger gaps between observed and anticipated value of Λ. The second problem can be explained as: "In present time, our cosmos is experiencing an expansion when the energy densities of DM and DE are comparable. How this is possible?" To overcome such problems, a large number of DE models have been proposed in literature such as "quintessence", "k-essence", "tachyon", "phantom", "holographic DE (HDE)", "Ricci DE (RDE)" and "Chaplygin gas (CG)" etc. [1]. However our knowledge about the past and future fate of the universe is quite limited till now. Still there is no certain unified scenario for such evolution of the universe. This motivates us to explore the DE problem by using some other procedure like, "holographic principle (HP)".

This principle is an attribute of QG, which states that "the information contents of a space volume lie on the boundary surface that bound this volume". This principle also suggests that the "ultraviolet (UV) cutoff scale Λ" is connected to its "infrared (IR) cutoff scale L". *Cohen et al.* [2] pointed out that for a system with size L and UV cutoff scale Λ without decaying into a black hole (BH), the quantum vacuum energy (ρ_Λ) of the system should not exceed the mass of BH with the same size. Mathematically, it is written as "$L^3\rho_\Lambda \leq LM_p^2$", where "$M_p^2 = 8\pi G$" is the "reduced Planck mass". Applying this idea to cosmology, one can choose the largest L, which satisfies this inequality by assuming ρ_Λ as DE.

Fischler and *Susskind* [3] proposed that HP could be replaced by the "generalized second law of thermodynamics (GSLT)" for an isotropic open and flat universe with fixed parameter ω. The HDE in an extra dimension, known to be modified HDE (MHDE) is studied by evaluating mass of a BH in $N+1$ dimensional spacetime. Holographic type DE model seems to be a reasonable choice as it solved some DE related issues but unfortunately this model attached with "causality issue", *i.e.*, future event horizon is supposed in this model. *Gao et al.* [4] put forward an idea of proportionality relation between DE density (ρ_D) and Ricci scalar (R), *i.e.*, RDE. This model is physically viable as it provides consistent results with latest astrophysical data. In addition, also eliminate the causality and cosmic coincidence issues. The scalar R for flat FRW universe is calculated as $6(\dot{H} + 2H^2)$, which leads to $\rho_{RDE} = 3c^2(\dot{H} + 2H^2)$, where H is the "Hubble parameter". *Chattopadhyay* [5] noticed that when we worked with generalized RDE (GRDE) within the background of "Horava-Lifshitz gravity (HLG)", its behavior is the same as of quintessence taking $c^2 >, =, < \frac{1}{2}$.

Most of the investigation has been done by considering DE and DM as separate candidates, but there is no logic to neglect associations in the dark sector. It is found that cold DM (CDM) is decaying into DE, which favors the interaction between these two components. Many models with interacting DE have been investigated in literature. The possibility of DE-DM interaction is the usual phenomena and gives a richer dynamic. *Aydiner* [6] observed that the interaction between DE and DM can be modeled with the help of few types of "non-linear Lotka-Volterra equations" suitable for cosmology. *Arevalo et al.* [7] studied the interaction phenomena between DE-DM by considering a "holographic Ricci-like DE model" in flat FRW spacetime. They noted that during cosmic evolution a change of sign involves in the interaction function. They obtained results that are well-fitted with the current observable universe.

To explain and vindicate the nature of DE, there are two methods to explain this unknown matter. The first method is for the argument of current expansion of the universe after the modification of geometric part of the Einstein-Hilbert action. The second one is to take different methods of DE, which are recognized as dynamical DE models. Also there are many cosmological parameters, that are very helpful to understand the spiritual behavior of DM and DE. A basic parameter, which is known as EoS is identified as "$\omega = \frac{p}{\rho}$", where ρ and p are known to be the energy density and pressure of a fluid, respectively. The deceleration parameter is denoted by q and utilized to discover the nature of cosmos whether it is accelerating, decelerating or static. The Hubble parameter (H) and q can effectively describe the behavior of cosmos but are not the best indicators to discriminate among various DE models. This motivates *Sahni et al.* [8] to introduce a statefinder pair (r, s), dimensionless as well as static and can be discriminated among a large number of DE models and might be a good identifier for cosmic models and has ability to check how far a model from the Λ-CDM model. In the $s - r$ plane for Λ-CDM model, the above parameters corresponds a fixed point, *i.e.*, $(r, s) = (1, 0)$. The (r, s) pair is said to be a geometrical diagnostic pair because it is formed from spacetime metric as it involves scale factor $a(t)$ and its time derivatives.

Zhang [9] proposed the idea of boundedness of our universe by an event horizon for which ρ_Λ is shifted to HDE density. Using FRW background, the accelerating behavior of RDE is being verified by *Kim et al.* [10]. *Kamenshchik et al.* [11] studied the behavior of q and (r, s) pair for GHDE/GRDE models using the framework of FRW. *Sharif* and *Saleem* [12] investigated the accelerating expanding nature of BI universe composed of GHDE/GRDE via statefinder diagnostic pair, the deceleration as well as EoS parameters. *Setare et al.* [13] studied the evolution of the non-flat cosmic model with HDE through (r, s) for specific values of model parameters. For non-flat FRW model, *Malekjani et al.* [14] plotted the $r - s$ and $q - r$ diagrams using "new HDE (NHDE)" model.

del Campo et al. [15] studied different HDE models from a unique point of view. They compared models for which the parameter H, the future event horizon or a quantity proportional to R are considered as an IR cutoff. They formulated the relationship between the energy density ratio of both components (HDE and DM) and the EoS parameter along with the possibility of non-interacting and scaling solutions. Estimation of parameter for all the three cut-offs are performed with the help of a “Bayesian statistical analysis", using data from *supernovae type Ia* and the history of the H. The ΛCDM model is the significant triumph of the analysis. *Sharif* and *Jawad* [16] explored interacting HDE model with a new IR cutoff in the non-flat FRW background. They constraint an Eos parameter, which evolutes from vacuum DE to quintessence era. Against the small perturbations, it is noted that the considered model possess unstable behavior. Finally, by corresponding their model with different DE models, like tachyon, quintessence, K-essence *etc*. checked the holdness of GSLT. *Huang et al.* [17] proposed new boundary conditions to reconstruct $f(T)$ gravity using “holographic Ricci DE (HRDE) model". They showed that the new conditions are much better than the previous one.

In a spatially non-flat universe, *Pasqua et al.* [18] analyzed the “logarithmic entropy corrected" and “power law" versions of the RDE model within HLG. For non-interacting and interacting RDE and DM, they get exact differential equation that admits the evolutionary form of the RDE density. *Som et al.* [19] studied DE models encouraged by the HP in homogeneous isotropic spacetime along with a DE density $\rho_D E = 3(\alpha H^2 + \beta \dot{H})$ where $\alpha,\ \beta$ are constants. They worked by introducing different general types of interaction terms among three HDE models including HRDE and DM. Taking HRDE, they examined the phantom divide (PD) crossing can be escaped for $\beta > 0.5$ regardless of the existence of interaction. *Jawad* [20] studied the behavior of “pilgrim DE (PDE) conjecture", which can explain that phantom-like DE model holds the sufficient resistive force to prevent the formation of a BH. He considered the non-flat geometry, comprised of interacting CDM and “ghost PDE (GPDE)". In this scenario, he discussed the nature of DE model through famous cosmological parameters, statefinder pair, evolution parameter (ω_Λ), $\omega_\Lambda - \omega'_\Lambda$ planes and squared speed of sound.

Jimenez et al. [21] explored interacting DE and DM models and provided new physics using current observational data using Q-interaction. They analyzed these models in detail and determined their potential relation with cosmological future singularities. They explored an interaction singularity named Q-singularity that can be mapped into every future singularity existed in the literature "(Big-Rip (BR), Sudden, Big-Freeze, Generalized Sudden and ω singularities)". Additionally, they identified a new type of future singularity coming from the divergence of first derivative of DE EoS parameter. Ali and Amir [22] reconstructed different scalar field models including K-essence, tachyon, quintessence and dilaton using MHRDE model within "Chern-Simon modified gravity (CSMG)". In this scenario, they observed the accelerating expanding cosmic behavior via q. Further, it is noted that the tachyon model admits non-increasing behavior while K-essence and quintessence models follow exponentially increasing behaviors and the dilaton model has no solution.

Zadeh et al. [23] explored the cosmological effects of the sign-changeable interacting HDE model in the context of FRW universe by considering the "Brans-Dicke (BD)" theory of gravity. They choose three different cut-offs, *i.e.*, the Granda-Oliveros, the future event horizon and the Ricci cut-offs. For every cut-off, they obtained the parameters such as ω, the density parameter (ω_D) and q. They observed that ω_D can cross the PD line only for future event horizon. Moreover, the stability of the sign-changeable interacting HDE model against perturbations in BD theory has been checked. *Feng et al.* [24] considered interacting HDE model in the framework of a perturbed universe. It is the first work in literature in which generalized post-Friedmann approach is being used to avoid the large scale instability issue. They constrained the model by employing red-shift (RS) space distortions measurements. They found that for both of the cases: $Q = \beta H_0 \rho_c$ and $Q = \beta H \rho_c$, the interacting HDE model is more preferred over non-interacting model by the recent observational data.

Cruz and *Lepe* [25] considered a generalized function for c^2 term, which seems in the conventional expression for HDE model in curved FRW universe bounded by apparent horizon. They explored the slowly varying condition for c^2 term and obtained a range of validity for the HDE model. They found that the holographic cut-off is satisfactory to define late time cosmic evolution. Furthermore, it is showed that the HDE model remains close to the ΛCDM model for some cosmological parameters like ω and q at observational constraints. The same authors [26] discussed the DE model by considering future event and particle horizons in flat FRW universe within a holographic background. They found that the model experienced genuine BR singularity, when the DE density is drew by future horizon resulting parameter state cross the PD. Also, they investigated that the cosmological fluid emulate usual matter for particle horizon and for the future horizon the coincidence parameter $r(z)$ has a decreasing behavior. Further, they analyzed that in the DE-DM interaction, the SLT cannot be satisfied because of the production of negative entropy.

Ahmed [27] studied the behavior of Ricci-Gauss-Bonnet HDE model in CSMG. He explored a well-defined solution using two physical forms of the scale factor: “the logamediate inflation form" and a specific “hyperbolic empirical form", which permits the cosmic transition from deceleration to acceleration. He analyzed the density, pressure, jerk parameter and EoS parameter and explained that a cosmic pressure predicted by both models, is negative throughout the late-time acceleration phase and positive throughout the early-time deceleration era. In both cases, the EoS parameter indicated quintessence dominated universe in far future. *Cruz* and *Lepe* [28] formulated the DE-DM interaction under a holographic approach within curved FLRW spacetime. According to the observational results, they considered a positive interaction coefficient with a “Chevallier-Polarsky-Linder (CPL)" type parametrization of the RS parameter and realized that the considered model admits a future singularity of Type III. Also, obtained the crossing of PD with the aid of some cosmological parameters constrained with astrophysical data.

The purpose of this thesis is to study the evolution of the universe by constructing the non-flat FRW background equations. The cosmic ingredients are considered to be RDE and DM with an interaction. Using a CPL type parametrization for the RS parameter, we calculate the analytical solutions for the field equations. Further, by constraining the model parameters, we plot the trajectories of cosmological parameters verses z to check the accelerating cosmic nature. The arrangement of this thesis is as under:

- In chapter **2**, we will discuss some fundamental concepts and general definitions related to our research work.

- In chapter **3**, behavior of the universe (accelerating or decelerating) will be explored by using a Ricci cutoff in a curved FLRW universe model comprised of RDE-DM interaction. Via CPL parametrization, we will restrict the region of model parameters and will plot the trajectories of cosmological parameters using the domain values. We will also check that what type of future singularity is possessed by our model?

- The results of this thesis are summarized in chapter **4**.

Chapter 2

Preliminaries

Here, we provide some fundamental definitions and basic ideas, which are necessary to understand the thesis.

2.1 Einstein Field Equations:

Field equations offer an interaction between fundamental forces with matter. In GR, the EFE's describe how gravity geometrizes the spacetime due to existence of matter or energy. Mathematically, "the EFE's are written as follows

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{2.1}$$

where $T\mu\nu$ (the *energy-momentum tensor*), R (the *Ricci scalar*), $g_{\mu\nu}$ (the *metric tensor*) and $R_{\mu\nu}$ (the *Ricci tensor*)". *Einstein* used these field equations to describe a model of the universe. Therefore, an expanding/contracting universe can be attained, depending on the energy/matter density. Afterwards, for a static universe, he added a constant Λ in these equations. After modification, EFE's becomes

$$\text{``}R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}\text{''}. \tag{2.2}$$

2.2 Friedmann-Robertson-Walker Metric:

Friedman was the first who presented the solution of EFE's, which represent an expanding cosmic behavior. The FRW model is developed on the basis of isotropic and homogenous universe model on large scales. The FRW model is identified by the following line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{2.3}$$

where $a(t)$ is known as the cosmic scale factor, k is the constant parameter, representing spatial curvature having three different values of k given as under:

- $k = 0$: no curvature, a flat universe, Euclidean geometry;
- $k = -1$: negative curvature, an open universe, hyperbolic geometry;
- $k = +1$: positive curvature, a closed universe, spherical geometry.

2.3 Cosmology:

Cosmology is the study of the LSS, dynamics and the cosmic evolution from the Big Bang (BB). It can also predict the origin and ultimate fate of our universe as a whole. In 1920s, the initial hypothetical works of some physicists, such as *Friedmann*, *Lemaître* and *Einstein*, discussed expansion of the universe alongside *Hubble* observations and also verified that EFE's are capable to model the universe not just on local gravitational fields but as a whole.

Cosmology may be characterized as follows: In "*physical cosmology*", famous laws of physics are used to understand these observational facts about the cosmos. Physical cosmology also deals with the early accelerating cosmic behavior. "*Observational cosmology*" is related with the chemical composition, density and expansion cosmic rate. In "*theoretical cosmology*", the cosmological models are constructed in order to get the mathematical formulation of the cosmic properties.

2.4 Dark Components:

"Dark energy" has become the origin of attraction among all cosmic ingredients. The word "dark" is attached with this energy because of its mysterious features. It is a theoretical type of energy having large negative pressure, which is distributed uniformly in our universe. This is said to be main factor for current accelerating evolution of our universe. Though, the existence of DE is confirmed by the recent observational data, but still it is a biggest mystery in physics and astronomy. The nature of DE is still unknown to us. Dark matter is invisible as it has no interaction with photons and is detectable by its gravitational influence on astronomical bodies. "Baryonic matter" is composed of *electrons*, *protons* and *neutrons* on the other hand, "non-baryonic matter (DM)" made up of axions (a hypothetical particle having small mass, no charge and zero spin) and neutrinos. According to the observations it is admitted that our universe is made up of invisible components, DE, DM components and of visible matter. Approximately, our universe constitute 68.3% of DE, 26.8% of DM and 4.9% visible matter.

2.5 Theory of Big Bang:

About the creation of the initial universe the BB theory gives an extraordinary explanation. Around 13.7 billion years ago, our present cosmos had been concentrated at a unique point (dubbed as an initial singularity), then a powerful explosion occurred and universe started to expand gradually. In this way our universe came into existence. Its a remarkable discovery to brief cosmic origin in detail. It is interpreted by a "primeval atom" (known as cosmic egg). Unfortunately, there is no physics about any event happened before BB. The expansion of the universe and discovery of CMBR are some of the scientific evidences strengthen the grounds of BB theory.

2.6 Cosmological Parameters:

This section is devoted to understand some essential cosmological parameters to investigate the puzzling DE nature and the associated cosmological models. These cosmological parameters are very useful to distinguish among variety of

DE models.

2.6.1 Scale Factor:

This dimensionless cosmological parameter explains the cosmic evolution by measuring its size. Depending upon this parameter, we can clarify the nature of the model,whether it is isotropic (if same scale factor is all spatial directions like FRW) or anisotropic (different scale factors in different directions like Bianchi models). It is always an increasing function of time as universe is expanding with respect to time and narrates distance between galaxies through cosmological RS.

2.6.2 Hubble Parameter:

Edwin Hubble introduced a law named after his name, which states that "the recessional velocity of two astronomical objects is directly proportional to the distances between them". Mathematically, it can be expressed as

$$v = H(t)r, \tag{2.4}$$

here v (*recessional velocity*)and r (*distance between galaxies*), which is the time dependent proportionality constant. It is associated with the rate of cosmic expansion that is expressed as follows

$$H = \frac{\dot{a}(t)}{a(t)}, \tag{2.5}$$

here dot represents the derivative with respect to time and H describes how slow or fast the universe is expanding? Also, the current value of this parameter is denoted by H_0 at time t_0. According to current observations, H_0 is constraint to

$$H_0 = 65 - 75kms^{-1}Mpc^{-1} \quad \text{and} \quad 1Mpc \cong 3 \times 10^{24}cm.$$

2.6.3 Cosmological Red-shift:

"The relative difference between two wavelengths of observed and emitted objects is characterized by red-shift", denoted by z. It is defined as

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{\triangle\lambda}{\lambda_{em}}, \tag{2.6}$$

where λ_{em} and λ_{obs} are emitted and observed wavelengths, respectively. The cosmological RS is related to $a(t)$ and happens because of universe's expansion. In terms of $a(t)$, the parameter is given as follows

$$1 + z = \frac{1}{a(t)}. \tag{2.7}$$

2.6.4 Equation of State Parameter:

A formula which provides a relation between physical quantities like energy density (ρ), pressure (P), temperature (T) *etc.* of a system is called EoS. To probe the physical state of substances (solids, liquids and gases), this parameter is most effective. It is found to be important for physical description of fluids, mixture of fluids and interior configuration of stars. Under local thermodynamic equilibrium, a mathematical relation of the type

$$P = P(\rho, T), \tag{2.8}$$

exits. In cosmological context, EoS is the relation between ρ and p, is given by

$$\omega = \frac{P}{\rho}. \tag{2.9}$$

To define the nature of any cosmic model, the dimensionless EoS parameter ω is used *i.e.,* in which region (phantom, cosmological constant, quintessence *etc.*) our model lies. For phantom region, $\omega < -1$ while opposite inequality $(\omega > -1)$ shows the quintessence region and for quintom $\omega = -1$.

2.6.5 Deceleration Parameter:

This parameter is denoted by q and is used to discover the cosmic nature whether the universe is static, decelerating or accelerating. The case q is equal to zero preserves a static cosmic behavior, $q < 0$ leads to an accelerating cosmos whereas $q > 0$ shows decelerating universe. Deceleration parameter in the form of $a(t)$ and further in $H(t)$ is given as respectively

$$q = -\frac{a\ddot{a}}{\dot{a}^2}, \quad q = -1 - \frac{\dot{H}}{H^2}. \tag{2.10}$$

2.6.6 Statefinder Parameters:

These parameters r and s are defined as under, respectively

$$r = \frac{\dddot{a}}{aH^3} = \frac{\ddot{H}}{H^3} + 3\frac{\dot{H}}{H^2} + 1, \quad s = \frac{r-1}{3(q-\frac{1}{2})}. \tag{2.11}$$

The Statefinder diagnostic pair is an effective geometrical tool that is used to compare the properties of a DE model with other DE models and then check how far this model from the Λ-CDM. As the universe has an accelerating expansion, so the rates of $a(t)$ are important to investigate the cosmic behavior. In more general sense, these can be used to examine the fluid that filled the universe with the effects of other parameters involved in its expression.

2.7 Principle of Holography:

This principle states that "we can obtain information about the occupied volume from its enveloping surface that bound this volume". These information may be in the form of entropy of BH, in the form of interference pattern on the photographic film or by a physical theory, which can be converted into another form. The name holographic is due to the fact that the enveloping surface/boundary works as “hologram" (a 3D image produced by holography). It is the characteristic of the hologram that each point on its surface receives reflected light from every part of the illuminated object. Therefore, its each point contains the complete visual record of the object as a whole. It seems in a complete 3D form with highly realistic perspective effects. In this way, the image possessed the entire visual properties like the original have.

2.8 Dark Energy Models:

The dynamical nature of DE can be understood from several models. Here, we discuss some DE models which are frequently used in the literature.

2.8.1 Cosmological Constant:

The cosmological constant, which is given by *Einstein*, is used to describe the unknown nature of DE. This constant agreements with vacuum energy of space and is denoted by Greek letter Λ. This simplest model suffer with some problems associated to the explanation of DE. The two main problems are coincidence and fine-tuning problem. The “*fine-tuning problem*" appears because of the large gaps between observed and expected results of ρ_Λ while the coincidence problem defines that if ρ_{DM} and ρ_{DE} are comparable, then why the current accelerating cosmic expansion is observed?

2.8.2 Quintessence Model:

The quintessence model can be obtained by considering potential term in scalar field Lagrangian, which is a time varying quantity. In this model, the universe accelerates slowly as compared to Λ and has the range $-1 < \omega < -\frac{1}{3}$. A few quintessence models display finite-time singularity, which is also known as sudden singularity in future where P tends to infinity even if ρ has finite value as $t \to t_s$ as $a \to a_s$

2.8.3 Phantom Model:

This type of model is considered through scalar fields and violates weak energy condition having the negative kinetic term, *i.e.*, $\rho + p \geq 0,\ \rho \geq 0$. In phantom model for finite time, energy density increases and tends to infinity. At that time, the quantities P and $a(t)$ approach to infinity to attain future singularity, identified as BR singularity. Due to increase in the *phantom energy*, the gravitational repulsion increases, which will destroy the bound system and galaxies along with the elementary particles.

2.8.4 Quintom Model:

This model is a combination of phantom and quintessence DE models. This model can cross the PD line, *i.e.*, $\omega = -1$, from the both sides. Moreover phantom and quintessence models separately do not cross the PD line, *i.e.*, $\omega = -1$. Feng et al. [29] proposed such a crossing phenomenon by using effect of SN type Ia and cosmic age limits. They determined that through the cosmic age limits, the upper bound on the variation of amplitude of ω_{DE} can be lowered. Current observations favor moderately to quintom scenario such that the singularities such as BR and BB would be avoided. Two types of quintom model are given below.

- *Type A Quintom*: When EoS parameter ω crosses PD line from $\omega > -1$ to $\omega < -1$, it is represented by type A quintom model.

- *Type B Quintom*: In this type of model, the universe transits from $\omega < -1$ to $\omega > -1$ by crossing PD line.

2.8.5 Holographic Dark Energy Model:

The HDE model is one of the best models to explain the unknown nature of DE. The HDE in any compact (bounded and finite) system is defined as

$$\rho_\Lambda = \frac{3c^2}{L^2}, \tag{2.12}$$

where c is some constant and L be the size of the system. This expression is obtained by the HP that "total amount of energy in a physical system of size L cannot exceed the amount of mass of a BH having the same size."

2.8.6 Ricci Dark Energy Model:

Ricci DE is known to be a kind of HDE with the inverse square root of the Ricci scalar as its IR cutoff. As we know, the ρ_{HDE} is inversely proportional to the square of an IR cutoff L as $\rho_{HDE} \sim L^{-2}$. Therefore, in a flat universe, the ρ_{RDE} is given as follows

$$\rho_{RDE} = \frac{\alpha}{2}R = 3\alpha(\dot{H} + 2H^2), \tag{2.13}$$

where, we set $8\pi G = 1$ and α is a dimensionless parameter, which will used to determine the behavior of RDE. In a flat FRW model, the scalar R is calculated as under

$$\text{“}R = 6(\dot{H} + 2H^2)\text{”}, \tag{2.14}$$

which implies that $R_{CC} \propto R$. Therefore, if one selects the causal connection (CC) as an IR cutoff, *i.e.*, $L = R_{CC}^{-1}$. In this way, the RDE model can be obtained.

2.9 Gravitational or Spacetime Singularity:

Mathematically, singularity is a physical point where our derived solution becomes undefined. "Gravitational singularity" or "spacetime singularity" is an area at which the physical quantities (like curvature, mass and density) that are utilized to calculate gravitational field went to infinity and do not depend upon coordinate system. Moreover, “singularity is a location where all physical laws are indistinguishable". There are two types of gravitational singularities: “Coordinate Singularity", arises due to improper choice of coordinates and can be removed by using appropriate coordinates. “Essential Singularity", which is additionally known as curvature singularity appears where we have infinitely large spacetime curvature.

2.10 Types of Singularity:

Some important types of future singularities have been found in literature and studied in detail as follows.

2.10.1 Big-Rip Singularity:

This singularity exists for $\rho \to \infty,\ t \to t_s,\ a \to \infty$ and $\mid p \mid\to \infty$. This case leads to incomplete time-like and null geodesics. Therefore, it represents a genuine space-time singularity.

2.10.2 Sudden Singularity:

For $\rho \to \rho_s$, $t \to t_s$, $a \to a_s$ and $\mid p \mid\to \infty$. Geodesics are complete and observers are not essentially crushed (weak singularity).

2.10.3 Big Freeze Singularity:

For $\rho \to \infty$, $t \to t_s$, $a \to a_s$ and $\mid p \mid\to \infty$. Geodesically complete solutions, which can be either weak or strong.

2.10.4 Generalized Sudden Singularity:

For $\rho \to \rho_s$, $t \to t_s$, $a \to a_s$ and $\mid p \mid \to p_s$. but, second and higher derivatives of H diverge. Geodesics are complete with the weak singularity.

2.10.5 ω Singularity:

For $\rho \to 0$, $t \to t_s$, $a \to \infty$ and $\mid p \mid \to 0$, but $\omega \to \infty$, these singularities are weak as well.

Chapter 3

Dynamical Study of an Interacting Ricci Dark Energy Model with Dark Matter

In this chapter, we analyze the behavior of the universe in the background of non-flat FRW spacetime using Ricci cut-off and taking a positive interaction term between RDE and DM. Using this scenario, we will evaluate the corresponding solution of field equations in terms of RS using CPL type parametrization. We will constraint the involved parameters to examine the accelerating behavior of the universe by plotting trajectories $s-r$ and $q-r$.

The chapter is organized as follows: In section **3.1**, the interacting scenario is being discussed using a CPL type parametrization for $r(z)$. In this scenario, we calculate the analytical solutions for the field equations. Section **3.2** is devoted to evaluate the type of future singularity. In section **3.3**, a detailed analysis of some cosmological parameters is presented along with graphical trajectories.

3.1 Interacting DM-DE Scheme:

In this section, we are going to describe briefly the dynamics of interacting scheme for the DM-DE components. We have a tendency to show that beneath the selection of a cut-off, which is given by *Hubble* scale in the form of ρ_{DE}, we are able to construct a particular interaction term between these dark components, which is function of some model parameters as well as the cosmological RS. In the case of non-flat FLRW spacetime, the Friedmann constraint can be defined as follows

$$E^2(z) = \frac{1}{3H_0^2}(\rho_{DE}(z) + \rho_{DM}(z)) + \Omega_k(z), \tag{3.1}$$

where $E(z) = \frac{H(z)}{H_0}$ be a normalized *Hubble* parameter, $1+z = \frac{a_0}{a}$, ρ_{DM}, the energy density of DM and ρ_{DE} be the DE density. Also, Ω_k is the curvature parameter described as $\Omega_k(z) = \Omega_k(0)(1+z)^2$, where $\Omega_k(0) = -\frac{k}{a_0^2 H_0^2}$. The continuity equations in the form of energy densities are defined as

$$\rho'_{DE} - 3\left(\frac{1+\omega_{DE}}{1+z}\right)\rho_{DE} = \frac{Q}{H_0 E(z)(1+z)}, \tag{3.2}$$

$$\rho'_{DM} - \left(\frac{3}{1+z}\right)\rho_{DM} \;=\; -\frac{Q}{H_0E(z)(1+z)}, \qquad (3.3)$$

where ω_{DM} is supposed to be zero in the above equations and prime represents derivative with respect to z. Also, the Q-term conclude the behavior of the interaction between DM and DE.

Differentiating Eq. (3.1) with respect to z and simplifying, we get

$$1 + \frac{\omega_{DE}(z)}{1+r(z)} = \frac{2E^2(z)}{3\rho_{DE}(1+r(z))}\left[\frac{3H_0^2}{2}\frac{(1+z)d\ln E^2(z)}{dz} - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]. \qquad (3.4)$$

We can evaluate an expression used in the above equation using (3.1), given as follows

$$\frac{E^2(z)}{\rho_{DE}(1+r(z))} = \left[3H_0^2 - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]^{-1},$$

putting this expression in the above Eq.(3.4), we get

$$1 + \frac{\omega_{DE}(z)}{1+r(z)} = \frac{2}{3}\left(\frac{1}{2}(1+z)\frac{d\ln E^2(z)}{dz} - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right)\left[1 - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]^{-1}, \qquad (3.5)$$

where $r(z)$ is said to be the coincidence parameter defined as $r = \frac{\rho_{DM}}{\rho_{DE}}$. Also, Eq.(3.5) can be written in the form of deceleration parameter, using following definition of q (converting formula of q in terms of z) as

$$1 + q(z) = \frac{1}{2}(1+z)\left(\frac{d\ln E^2(z)}{dz}\right), \qquad (3.6)$$

we get modified Eq.(3.5) as under

$$1 + \frac{\omega_{DE}(z)}{1+r(z)} = \frac{2}{3}\left(1 + q(z) - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right)\left[1 - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]^{-1}. \qquad (3.7)$$

Performing the calculations for Eq. (3.7) at present time *i.e.,* at $z = 0$, we can have an approximation for the present value of q as follows

$$q_0 = \frac{1}{2}\left(1 + \frac{3\omega_{DE,0}}{1+r_0}\right)(1 - \Omega_k(0)). \qquad (3.8)$$

If we assume the expression given in (3.7) along with values of $\Omega_k(0)$ and $r(z)$ that are given in Ref. [30, 31] at $z = 0$, we get an interval for $\omega_{DE,0} \in [-1.4746, -0.008746]$, which represents the crossing from phantom region ($\omega < -1$) to quintessence region ($\omega > -1$).

3.2 Ricci cut-off for DE:

In this section, we will evaluate some physical quantities like $\rho_{DE},\ \rho_{DM}$ assuming that universe is bounded by a horizon with Ricci cut-off $L = \frac{1}{R}$. The RDE density is given as follows [5]

$$\rho_{DE} = \frac{3c^2}{8\pi}R, \tag{3.9}$$

where *Ricci* scalar for flat FRW universe is given as

$$R = -6(\dot{H} + 2H^2),$$

where, c is a positive constant, which is given in the interval $0 < c^2 < 1$ in order to define an expanding universe. To explain the behavior of RDE, this parameter c has an important role. Moreover, according to recent observations, this choice of ρ_{DE} has a good fit results as shown in Ref. [34]. Now, we substituting value of R in Eq. (3.9), we get ρ_{DE} in terms of z (using expression of $E(z)$) as

$$\rho_{DE}(z) = -\frac{9c^2}{4\pi H_0}\bigg(2H_0E^2(z) + E'(z)\bigg), \tag{3.10}$$

Now, we are able to calculate the quantity ρ_{DM} via Eqs.(3.1) and (3.10) in terms of z as

$$\rho_{DM}(z) = 3H_0^2E^2(z)\bigg[1 - \Omega_k(0)\bigg(\frac{1+z}{E(z)}\bigg)^2 + \frac{3c^2}{2\pi H_0^2}\bigg] + \frac{9c^2}{4\pi H_0}E'(z). \tag{3.11}$$

Dividing Eq.(3.11) to Eq.(3.10) and after some simple calculations the parameter $r(z)$ may be written as

$$r(z) = -1 - \frac{1}{3c^2(2H_0E^2(z) + E'(z))}\bigg[4\pi H_0^3E^2(z)\bigg(1 - \Omega_k(0)\bigg(\frac{1+z}{E(z)}\bigg)^2\bigg)\bigg]. \tag{3.12}$$

Now differentiating Eq. (3.11) with respect to z, we get

$$\begin{aligned}\rho'_{DM}(z) &= \frac{1}{4\pi H_0}\bigg[-24\pi H_0^3(1+z)\Omega_k(0) + 24\pi H_0^3E(z)E'(z) + 36H_0c^2E(z)E'(z) \\ &+ 9c^2E''(z)\bigg],\end{aligned} \tag{3.13}$$

inserting above calculated expression of $\rho'_{DM}(z)$ in Eq.(3.3) and after some calculations, we arrived at

$$\begin{aligned}(1+z)\frac{d\ln E^2(z)}{dz} &= 3+\frac{2(1+z)^2\Omega_k(0)}{E^2(z)}+\frac{9c^2}{2\pi H_0^2}+\frac{9c^2E'(z)}{4\pi H_0^3E^2(z)}-\frac{Q}{3H_0^3E^3(z)}\\ &- \frac{3\Omega_k(0)(1+z)^2}{E^2(z)}-\frac{3c^2E'(z)(1+z)}{\pi H_0^2E(z)}-\frac{3c^2E''(z)(1+z)}{4\pi H_0^3E^2(z)}.\end{aligned} \tag{3.14}$$

Putting above result in Eq. (3.4) and after a straightforward calculation, we get interaction term as a function of z as follows

$$\begin{aligned}\frac{Q(z)}{9H_0^3E^3(z)} &= 1+\frac{3c^2}{2\pi H_0^2}\Bigg[1+\frac{E'(z)}{2H_0E^2(z)}-\frac{E''(z)(1+z)}{6H_0E^2(z)}-\frac{2\pi H_0^2\Omega_k(0)(1+z)^2}{3c^2E^2(z)}\\ &- \frac{2E'(z)(1+z)}{3E(z)}\Bigg]-\left(1+\frac{\omega_{DE}(z)}{1+r(z)}\right)\left(1-\Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right).\end{aligned} \tag{3.15}$$

At $z=0$, the term $Q(z)$ is reduced to

$$Q_0=9(1-\Omega_k(0))+\frac{9c^2}{\pi}-9(1-\Omega_k(0))\left(1+\frac{\omega_{DE,0}}{1+r_0}\right). \tag{3.16}$$

If $Q_0>0$, then energy flows from DE to DM and vice versa for $Q_0<0$. As perceived, the Q-term written in Eq. (3.15), which is constructed through physical quantities, *i.e.,* we did not consider a specific parametrization for the Q-term as is often done. In Ref. [33] by using the best fit value of the cosmological parameters, included the previous construction may have the benefit of constraining the interaction term. It is significant to note that the value of Q-term determines the rate at which the universe expanding as $r(z)$ decreases.

Now, from Eq.(3.1), we can write the following expression

$$\rho_{DE}+\rho_{DM}=3H^2(z)-\Omega_k(z). \tag{3.17}$$

Equation (3.10) can be modified as follows

$$\rho_{DE}(z)=-\frac{9c^2(2H^2(z)+H'(z))}{4\pi H_0^2}. \tag{3.18}$$

Differentiating $r(z)$ with respect to z and using Eqs. (3.17) and (3.18), we get

$$\frac{r'}{r} = -\frac{3\omega_{DE}}{1+z} - \frac{Q}{H_0 E(z)(1+z)}\left(\frac{4\pi H_0^2(3H^2(z) - \Omega_k(z))}{-9c^2\rho_{DM}(2H^2(z) + H'(z))}\right), \tag{3.19}$$

using chain rule, $r' = \frac{\dot{r}}{\dot{z}}$, in the above equation, we have

$$\frac{\dot{r}}{r} = 3H(z)\omega_{DE}(z) - Q(z)\left(\frac{4\pi H_0^2(3H^2(z) - \Omega_k(z))}{9c^2\rho_{DM}(z)(2H^2(z) + H'(z))}\right). \tag{3.20}$$

The rate of change for the coincidence parameter can be varied as $\Omega_k(z)$ decreases or increases. In the above equation, assuming $Q = 0$ along with $\omega_{DE} = -1$, the Λ-CDM model can be recovered where $\dot{r} = -3Hr$ [28]. Furthermore, at present time taking $\dot{r} = 0$ in the above equation, we can solve to obtain a specific value r_0 given as under

$$\begin{aligned} r_0 \;=\; & \Bigg[6H_0^2\Bigg(c^2\rho_{DM,0}\omega_{DE,0} - 2H_0c^2 + 2\pi H_0\omega_{DE,0}(1 - \Omega_{k,0})\Bigg) \\ + \; & c^2(6H_0'\rho_{DM,0}\omega_{DE,0} + 4H_0\Omega_{k,0}) + 4\pi H_0\omega_{DE,0}\Omega_{k,0}(\Omega_{k,0} - 1)\Bigg] \\ \times \; & \Bigg[c^2\Bigg(4H_0(3H_0^2 - \Omega_{k,0}) - 3\rho_{DM,0}\omega_{DE,0}(2H_0^2 + H_0')\Bigg)\Bigg]^{-1}. \end{aligned} \tag{3.21}$$

Using Eq.(3.21), we can constraint r_0 at specific values of other model parameters at present time.

From Eq.(3.16) and the positivity condition, $Q_0 > 0$, we get

$$\omega_{DE,0} < (1 + r_o)\left(\frac{c^2}{\pi(1 - \Omega_{k,0})}\right), \tag{3.22}$$

where $\omega_{DE,0} < -0.666485$ or $\omega_{DE,0} < -0.666007$ and $\Omega_k(0) = 0.000^{+0.005(k=-1)}_{-0.005(k=1)}$ [30, 31]. It is important to point out that the parameter $\omega_{DE,0}$ can take positive values, if $k = 1$, which could define a decelerated expansion. Using already mentioned expression given in Eq.(3.6), we can find the value of $q(z)$ from Eq.(3.14) as

$$q(z) \;=\; \frac{1}{2}\Bigg[1 + \frac{2(1+z)^2\Omega_k(0)}{E^2(z)} + \frac{9c^2}{2\pi H_0^2} + \frac{9c^2E'(z)}{4\pi H_0^3E^2(z)} - \frac{3c^2E''(z)(1+z)}{4\pi H_0^3E^2(z)}$$

$$- \quad \frac{3\Omega_k(0)(1+z)^2}{E^2(z)} - \frac{3c^2 E'(z)(1+z)}{\pi H_0^2 E^{(}z)} - \frac{Q}{3H_0^3 E^3(z)}\Bigg]. \tag{3.23}$$

While at present time, it is calculated that $q_0 < \frac{1}{2}$. By using Eq.(3.23), we get interaction term in the following form

$$\begin{aligned} Q(z) \quad < \quad & \frac{27c^2 H_0 E^3(z)}{2\pi} + \frac{27c^2 E(z)E'(z)}{4\pi} - \frac{9c^2 E(z)E''(z)(1+z)}{4\pi} \\ - \quad & \frac{9c^2 H_0 E^2(z)E'(z)(1+z)}{\pi} - 3H_0^3\Omega_k(0)E(z)(1+z)^2. \end{aligned} \tag{3.24}$$

It is significant to point out that the above changes in the sign of Q-interaction term strongly depends on the sign of $\Omega_k(z)$. Moreover, a modification in the sign of Q interaction term used to determine the phase transitions (sign changes in heat capacities) along the cosmic evolution as well as provide data to confirm the validity of SLT [35, 36].

3.3 Future Singularity:

we will know in this model about the presence of future singularity, The most highlighted point is that the mentioned singularity is only assists in a curved universe. we assume a CPL-type parametrization for $r(z)$ to study the singularity. It helps us to recognize the occurrence of singularity in the RS. We are working with a Type III future singularity but not with a genuine BR, moreover, we can observe that cosmic evolution induced by the existence of previous singularity differs from that singularity, which is attained by Λ. We prove that the Q-term will remain positive through the cosmic evolution using the previous results. Using Eq.(3.12), we can form $E(z)$ in terms of $r(z)$ as given below:

$$E(z) = \frac{2\pi H_0^3 \Omega_k(0)}{3c^2 r_0^3}\Bigg[r_0^2(1+z)^2 + 2r_0(r_0+r_c)(1+z) + 2(r_0+r_c)^2 \ln(r(z)-r_c)\Bigg], \tag{3.25}$$

here a constant quantity, $r_c = \frac{1-c^2}{c^2}$. The expression $E(z)$ became singular at given point $r(z) = r_c$. Evaluating Eq.(3.25) at $z = 0$, we can find an expression for r_c, which is given in terms of r_0 and $\Omega_k(0)$ as

$$r_c = \frac{1-c^2}{c^2} = \frac{9(1+r_0)}{4\pi(\Omega_k(0)-1)}. \tag{3.26}$$

Now for $r(z)$, we will assume a CPL-type parametrization given as follows [37]

$$r(z) = r_0 + \epsilon_0 \frac{z}{1+z}, \tag{3.27}$$

where $\epsilon_0 = r_0'$ can be obtained from Eq. (3.27) after taking its derivative with respect to z. One can see that at $z=-1$, the preceding parametrization will become singular for high values of z for which $r(z)$ has a bounded nature and declares a linear behavior for low values of z, also shows the sensitive behavior to observational data [38]. Comparing the Eqs. (3.26) and (3.27), we can find the following value of z_s at which $E(z)$ will become singular

$$z_s = -\left[\frac{4\pi r_0^2(1-\Omega_k(0)) + 9(1+r_0)(2r_0-r_c)}{4\pi r_0(r_0+\epsilon_0)(1-\Omega_k(0)) + 9\epsilon_0(1+r_0)(1+(2r_0-r_c)/\epsilon_0)}\right]. \tag{3.28}$$

In the future, we must have $-1 < z_s < 0$ for a singular behavior. From the previous condition and by using Eqs.(3.26) and (3.27), we get

$$r(z) - r_c = \epsilon_0\left[\frac{z-z_s}{(1+z_s)(1+z)}\right] \geq 0 \Longrightarrow z \geq z_s. \tag{3.29}$$

It is noted that $r \to r_c$ as the limit $z \to z_s$ in the above expression. Evaluating previous equation at present time, we get an inequality which must be satisfied

$$-\epsilon_0\frac{z_s}{(1+z_s)} \geqslant 0, \tag{3.30}$$

and is consistent with the interval $-1 < z_s < 0$. The Eq.(3.25) can be re-written using previous results

$$\begin{aligned} E(z) &= \frac{4\pi H_0^3\Omega_k(0)(r_0+r_c)}{3c^2r_0^2}(1+z) + \frac{4\pi H_0^3\Omega_k(0)(r_0+r_c)^2}{3c^2r_0^3}\ln\left(\frac{z-z_s}{\eta c^2(1+z)}\right) \\ &+ \frac{2\pi H_0^3\Omega_k(0)}{3c^2r_0}(1+z)^2, \end{aligned} \tag{3.31}$$

where $\eta = \frac{(1+zs)}{c^2\epsilon_0} > 0$ if $\epsilon_0 > 0$. The behavior of equation (3.31) can be examined

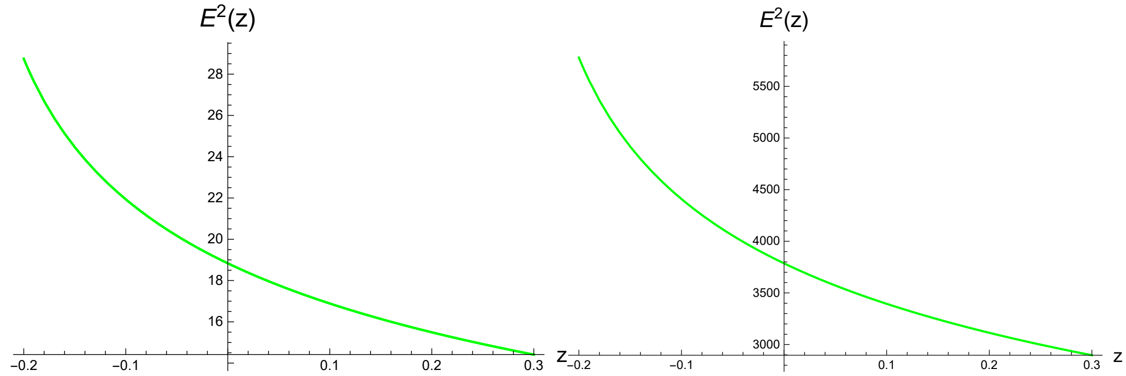

Figure 3.1: Left plot of $E^2(z)$ versus z for $\Omega_k(0) = -0.005,\ c = 0.681476,\ \epsilon_0 = 0.2,\ z_s = -0.26,\ H_0 = 1,\ r_0 = 0.11,\ r_c = 0.4674$; Right plot of $E^2(z)$ versus z for $\Omega_k(0) = -1.005$.

from Fig.(**3.1**). Left plot of Fig.(**3.1**) shows behavior of $E^2(z)$ as z progresses for specific values of the model parameters taken from literature [30, 31]. It is clear from the trajectory plotted in the left graph, $E^2(z) - z$ that during closed universe ($\Omega_k(0) = -0.005$ for $k = 1$), $E^2(z)$ has decreasing behavior as z transits from past to future era. Right plot of Fig.(**3.1**) is plotted for $\Omega_k(0) = -1.005$, which also admits the same decreasing behavior as left plot but range of $E^2(z)$ is too much large. So, generally it is noted that the behavior of $E^2(z)$ remains the same throughout the closed universe. We have also checked that for positive values of $\Omega_k(z)$ (representing an open universe $(k = 1)$), the normalized function $E^2(z)$ becomes negative, which is not a physical result according to recent observations. Note that a change in the sign of the $\Omega_k(0)$ can cause two types of cosmic evolution but in both cases the value z is the same.

Inserting $E(z) = \frac{H}{H_0},\ H = \frac{\dot{a}}{a}$ along with $1 + z = \frac{a_o}{a}$ in Eq.(3.31) and perform some simplifications, we get an analytical solution for $a(t)$ as follows

$$a(t) = \exp\left[\left(\frac{4\pi H_0^4 \Omega_k(0)(r_0 + r_c)^2}{3c^2 r_0^3 - 4\pi H_0^4 \Omega_k(0)(r_0 + r_c)(r_0 a_0)}\right) \ln\left(\frac{a_0}{1 + z_s}\right) t\right]. \tag{3.32}$$

It is clear from Eq.(3.32), as $\Omega_k(0) \to 0$ then $a(t)$ converges to a constant value, which shows a static universe. This type of universe model is in conflict with the SLT, which is one of the famous universal laws of nature [39], so a singular universe along with a negative value of $\Omega_k(z)$ is more preferred explanation of cosmic expansion. Furthermore, since both of the energy densities ρ_{DM} and ρ_{DE} are directly associated to the quantity $E(z)$ and its first derivative over the Friedmann constraint as $z \to z_s$, the $\rho_{DM},\ \rho_{DE} \to \infty$, therefore the quantity P_{DE} also diverges. From this behavior, we have concluded that our model with RDE admits a future singularity of Type III similar to HDE [40, 41].

Now, we are able to evaluate Eq.(3.4) in terms of z only using obtained solution of $E(z)$ given in Eq.(3.31) as

$$\begin{aligned}
1+\frac{\omega_{DE}(z)}{1+r(z)} &= \bigg[12(r_0+r_c)r_0^3\theta(z)(1+z)^2+4(r_0+r_c)^2r_0^2\theta(z)(1+z)\{(1+\theta(z))\\
&+ 2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}+8(r_0+r_c)^3r_0\{(1+z)(\theta(z)-1)^2\\
&+ \theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}+4(r_0+r_c)^4(\theta(z)-1)^2+4r_0^4\theta(z)(1+z)^3\\
&+ \frac{9c^4r_0^6\theta(z)(1+z)^2}{2\pi^2H_0^6\Omega_k(0)}\bigg]\bigg[24(r_0+r_c)^3r_0\theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&+ 12(r_0+r_c)r_0^3\theta(z)(1+z)^2+12(r_0+r_c)^2r_0^2\theta(z)(1+z)\{1\\
&+ \ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}+12(r_0+r_c)^4\frac{\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&- \frac{27c^4r_0^6\theta(z)(1+z)}{4\pi^2H_0^6\Omega_k(0)}+3r_0^4\theta(z)(1+z)^3\bigg]^{-1},
\end{aligned}\tag{3.33}$$

where $\theta(z)=\frac{(1+z)}{(z-z_s)}$. Consequently, by using aforementioned equation, we get a divergent behavior for ω_{DE}, which can be written as $\omega_{DE}(z\to z_s)\to-\infty$, negative sign is due to range of z_s. For early universe, the RS parameter $z\to\infty$. By using this assumption in CPL parametrization, we obtained $r(z\to\infty)\to r_0+\epsilon_0$ and the right hand side of Eq.(3.33) becomes $\frac{4}{3}$ while the bounded value of $\omega_{DE}(z\to\infty)\to\frac{1+r_0+\epsilon_0}{3}$. We observe that in early universe, ω_{DE} takes greater values than those attained at present time. To evaluate the interaction term Q, put Eq.(3.33) in Eq.(3.15) and after solving it, we get

$$\frac{Q(z)}{3H_0^3} = 3\bigg[\frac{4\pi^2H_0^6\Omega_k^2(0)}{9c^4r_0^2}\bigg(\frac{4(r_0+r_c)^2(1+z)^2}{r_0^2}+\frac{4(r_0+r_c)^4}{r_0^4}(\ln(\eta^{-1}c^{-2}\theta^{-1}(z)))^2$$

$$
\begin{aligned}
&+ \quad (1+z)^4 + \frac{4(r_0+r_c)}{r_0}(1+z)^3 + \frac{8(r_0+r_c)^3(1+z)}{r_0^3}\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&+ \quad \frac{4(r_0+r_c)^2(1+z)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\Bigg)\Bigg]^{3/2}\Bigg[\Bigg(12(r_0+r_c)r_0^3\theta(z)(1+z)^2 \\
&\times \quad (2\pi H_0^2+3c^2-1) + 4(r_0+r_c)^2r_0^2\theta(z)(1+z)\{6\pi H_0^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&+ \quad 6\pi H_0^2 + 9c^2 + 9c^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) - 1 - \theta(z) + 2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} \\
&+ \quad 8(r_0+r_c)^3r_0\theta(z)\{6\pi H_0^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) + 9c^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&- \quad (1+z)(\theta(z)-1)^2 - \ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} - 4(r_0+r_c)^4(\theta(z)-1)^2 \\
&+ \quad 12(r_0+r_c)^4\frac{\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))(2\pi H_0^2+3c^2) + r_0^4\theta(z)(1+z)^3(6\pi H_0^2 \\
&+ \quad 9c^2-4) - \frac{9c^4r_0^6\theta(z)(1+z)}{2\pi H_0^4\Omega_k(0)}\Bigg(3+\frac{9c^2}{2\pi H_0^2}+\frac{1+z}{\pi H_0^2}\Bigg)\Bigg)\Bigg(\{2\pi H_0^2\}\{\triangle\}\Bigg)^{-1}\Bigg] \\
&+ \quad \frac{2\pi H_0^3\Omega_k^2(0)(1+z)^2}{c^2r_0}\Bigg[\frac{2(r_0+r_c)(1+z)}{r_0} + \frac{2(r_0+r_c)^2}{r_0}\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&+ \quad (1+z)^2\Bigg]\Bigg[\Bigg(4(r_0+r_c)^2r_0^2\theta(z)(1+z)\{-2+\theta(z)-\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} \\
&+ \quad 8(r_0+r_c)^3r_0\{(1+z)(\theta(z)-1)^2 - 2\theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} \\
&+ \quad 4(r_0+r_c)^4\{(\theta(z)-1)^2 - \frac{3\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} \\
&+ \quad r_0^4\theta(z)(1+z)^3 + \frac{9c^4r_0^6\theta(z)(1+z)}{2\pi^2H_0^6\Omega_k(0)}(z+\frac{5}{2})\Bigg)\Bigg(\triangle\Bigg)^{-1}\Bigg] \\
&+ \quad \frac{8\pi H_0^4\Omega_k^2(0)}{3c^2r_0^2}\Bigg[\Bigg(\Bigg\{\frac{2(r_0+r_c)(1+z)}{r_0} + \frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&+ \quad (1+z)^2\Bigg\}\Bigg\{(1+z) + \frac{(r_0+r_c)}{r_0} + \frac{(r_0+r_c)^2(\theta(z)-1)}{r_0^2(1+z)}\Bigg\}\Bigg)\Bigg(\frac{3}{4H_0} \\
&- \quad \frac{2\pi H_0^3\Omega_k(0)(1+z)}{3c^2r_0}\Bigg\{\frac{2(r_0+r_c)(1+z)}{r_0} + \frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&+ \quad (1+z)^2\Bigg\}\Bigg)\Bigg],
\end{aligned} \tag{3.34}
$$

where

$$
\begin{aligned}
\triangle \quad = \quad & 12(r_0+r_c)r_0^3\theta(z)(1+z)^2 + 12(r_0+r_c)^2r_0^2\theta(z)(1+z)\{1+\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} \\
+ \quad & 24(r_0+r_c)^3r_0\theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) + 12(r_0+r_c)^4\frac{\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
- \quad & \frac{27c^4r_0^6\theta(z)(1+z)}{4\pi^2H_0^6\Omega_k(0)} + 3r_0^4\theta(z)(1+z)^3.
\end{aligned}
$$

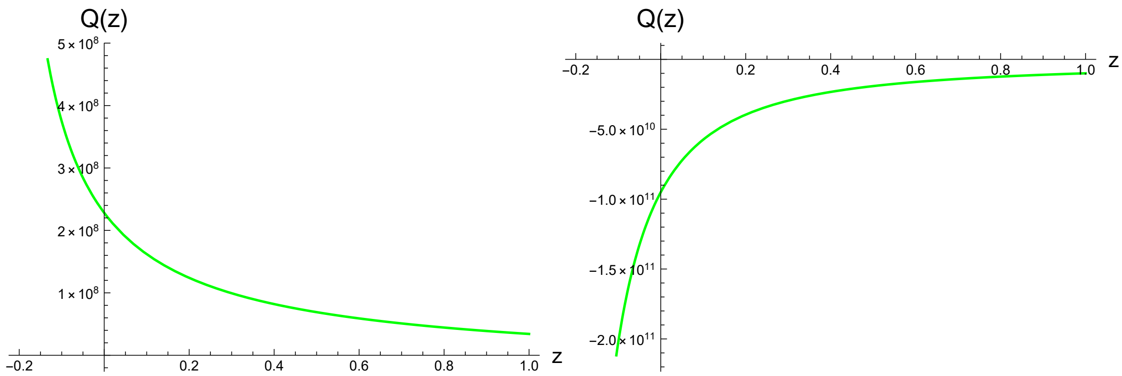

Figure 3.2: Left plot $Q(z)$ versus z for $\Omega_k(0) = -0.005,\ c = 0.681476,\ \epsilon_0 = 0.2,\ z_s = -0.26,\ H_0 = 1,\ r_0 = 0.11,\ r_c = 0.4674$; Right plot for $\Omega_k(0) = 0.005$.

It is obvious that Q can be positive and negative defining two different regions of cosmos. The calculated interaction term Q is plotted versus z in left and right plot of Fig.(**3.2**). It is important to note that the value of $\Omega_k(z)$ shows a crucial role in order to have real Q-function. The left plot of Fig.(**3.2**) verifies the natural phenomena that interaction term remains positive throughout the domain of z for $\Omega_k(0) = -0.005$ during closed universe. As $Q > 0$ represents the conversion of DE dominated era to DM dominated era. Although the observations suggests that Q must be positive, but right plot of Fig.(**3.2**) for $\Omega_k(0) = 0.005$ is not in good agreement with recent observations. So, in closed universe, DM is converted in to DE. we have to assume the possible implications at thermodynamic level for late-time universe, this is to study the fulfillment of SLT and the occurring possibility of phase transitions.

Substituting Eq.(3.25) in (3.6), we obtain $q(z)$ as given below

$$q(z) = \frac{1}{2}\left[\frac{r_0^2(1+z)^2 + 2(r_0+r_c)^2\left(\frac{1+z}{r(z)-r_c}r'(z) - \ln(r(z)-r_c)\right)}{\frac{r_0^2}{2}(1+z)^2 + r_0(r_0+r_c)(1+z) + 2(r_0+r_c)^2\ln(r(z)-r_c)}\right]. \tag{3.35}$$

To check the singular behavior of $q(z)$, we have to convert the above expression

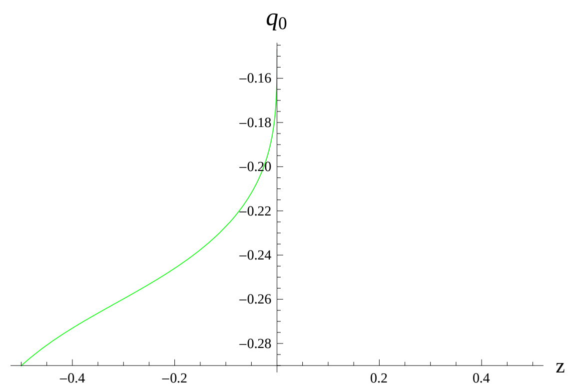

Figure 3.3: Plot of q_0 verses $z = z_s$ for $c = 0.681476,\ \Omega_k(0) = -0.005,\ r_c = 0.4674.$

in terms of z and z_s by putting Eq.(3.31) into Eq.(3.6) as under

$$q(z) = \frac{(1+z)^2 + \frac{2(r_0+r_c)^2}{r_0^2}\left[\left(\frac{1+z_s}{z-z_s}\right) - \ln\left(\frac{\epsilon_0(z-z_s)}{(1+z_s)(1+z)}\right)\right]}{(1+z)^2 + \frac{2(r_0+r_c)}{r_0}(1+z) + \frac{2(r_0+r_c)^2}{r_0^2}\ln\left(\frac{\epsilon_0(z-z_s)}{(1+z_s)(1+z)}\right)}. \tag{3.36}$$

Note that $q(z \to z_s) \to -\infty$ and $q(z \to \infty) \to 0$, *i.e.*, the universe expanding more rapidly. If we evaluate $\theta(z)$ at $z = 0$ then we get $\theta_0 = -\frac{1}{z_s}$, which is always positive. At present time, the expression of $q(z)$ can be modified as follows

$$q_0 = -\left[\frac{-1 + 2\left(\frac{(r_0+r_c)^2}{r_0}\right)\left(-\theta_0(1+z_s) + \ln\left(\frac{-\epsilon_0 z_s}{1+z_s}\right)\right)}{1 + \frac{2(r_0+r_c)}{r_0} + \frac{2(r_0+r_c)^2}{r_0^2}\ln\left(\frac{-\epsilon_0 z_s}{1+z_s}\right)}\right] < 0. \tag{3.37}$$

The above inequality is proved in Fig.(**3.3**) for specified values of the model parameters. We have checked that this expression is independent of $\Omega_k(0)$. The term q_0 remains negative in the range $-1 < z < 1$ for any positive or negative value of $\Omega_k(0)$.

Equations (3.33) and (3.34) at present time has the following expression

$$\begin{aligned}
\omega_{DE,0} &= \Bigg[-1 + \Bigg(12(r_0+r_c)r_0^3\theta_0 + 4(r_0+r_c)^2 r_0^2\theta_0\{(1+\theta_0) + 2\ln(\eta^{-1}c^{-2}\theta_0^{-1})\} \\
&+ 8(r_0+r_c)^3 r_0\{(\theta_0-1)^2 + \theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})\} + 4(r_0+r_c)^4(\theta_0-1)^2 + 4r_0^4\theta_0 \\
&- \frac{9c^4 r_0^6\theta_0}{2\pi^2 H_0^6\Omega_k(0)}\Bigg)\Bigg(12(r_0+r_c)r_0^3\theta_0 + 12(r_0+r_c)^2 r_0^2\theta_0\{1 + \ln(\eta^{-1}c^{-2}\theta_0^{-1})\}
\end{aligned}$$

$$
\begin{aligned}
&+ \quad 24(r_0+r_c)^3 r_0\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1}) + 12(r_0+r_c)^4\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})\\
&- \quad \frac{27c^4r_0^6\theta_0}{4\pi^2H_0^6\Omega_k(0)} + 3r_0^4\theta_0\Big)^{-1}\Big](1+r_0), \qquad (3.38)
\end{aligned}
$$

$$
\begin{aligned}
\frac{Q_0}{H_0^3} &= 9\Big[\frac{4\pi^2H_0^6\Omega_k^2(0)}{9c^4r_0^2}\Big(1+\frac{4(r_0+r_c)^2}{r_0^2}+\frac{4(r_0+r_c)^4}{r_0^4}(\ln(\eta^{-1}c^{-2}\theta_0^{-1}))^2\\
&+ \frac{4(r_0+r_c)}{r_0}+\frac{8(r_0+r_c)^3}{r_0^3}\ln(\eta^{-1}c^{-2}\theta_0^{-1})+\frac{4(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta_0^{-1})\Big)\Big]^{3/2}\\
&\times \Big[\Big(12(r_0+r_c)r_0^3\theta_0(2\pi H_0^2+3c^2-1)+4(r_0+r_c)^2r_0^2\theta_0\{6\pi H_0^2+9c^2\\
&+ \ln(\eta^{-1}c^{-2}\theta_0^{-1})(9c^2+6\pi H_0^2+2)-1-\theta_0\}-4(r_0+r_c)^4(\theta_0-1)^2\\
&+ 8(r_0+r_c)^3r_0\theta_0\{\ln(\eta^{-1}c^{-2}\theta_0^{-1})(6\pi H_0^2+9c^2-1)-(\theta_0-1)^2\}\\
&+ 12(r_0+r_c)^4\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})(2\pi H_0^2+3c^2)+r_0^4\theta_0(6\pi H_0^2+9c^2-4)\\
&- \frac{9c^4r_0^6\theta_0}{2\pi H_0^4\Omega_k(0)}\Big(3+\frac{9c^2}{2\pi H_0^2}+\frac{1}{\pi H_0^2}\Big)\Big)\Big(\{2\pi H_0^2\}\{\nabla\}\Big)^{-1}\Big]\\
&+ \frac{6\pi H_0^3\Omega_k^2(0)}{c^2r_0}\Big[\frac{2(r_0+r_c)}{r_0}+\frac{2(r_0+r_c)^2}{r_0}\ln(\eta^{-1}c^{-2}\theta_0^{-1})+1\Big]\Big[\Big(r_0^4\theta_0\\
&+ 4(r_0+r_c)^2r_0^2\theta_0\{-2+\theta_0-\ln(\eta^{-1}c^{-2}\theta_0^{-1})\}+8(r_0+r_c)^3r_0\{(\theta_0-1)^2\\
&- 2\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})\}+4(r_0+r_c)^4\{(\theta_0-1)^2-3\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})\}\\
&+ \frac{45c^4r_0^6\theta_0}{4\pi^2H_0^6\Omega_k(0)}\Big)\Big(\nabla\Big)^{-1}\Big]+\frac{8\pi H_0^4\Omega_k^2(0)}{c^2r_0^2}\Big[\Big(\Big\{\frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta_0^{-1})\\
&+ \frac{2(r_0+r_c)}{r_0}+1\Big\}\Big\{1+\frac{(r_0+r_c)}{r_0}+\frac{(r_0+r_c)^2(\theta_0-1)}{r_0^2}\Big\}\Big)\Big(\frac{3}{4H_0}\\
&- \frac{2\pi H_0^3\Omega_k(0)}{3c^2r_0}\Big\{\frac{2(r_0+r_c)}{r_0}+\frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta_0^{-1})+1\Big\}\Big)\Big], \qquad (3.39)
\end{aligned}
$$

where

$$
\begin{aligned}
\nabla &= 12(r_0+r_c)r_0^3\theta_0+12(r_0+r_c)^2r_0^2\theta_0\{1+\ln(\eta^{-1}c^{-2}\theta_0^{-1})\}+24(r_0+r_c)^3r_0\theta_0\\
&\times \ln(\eta^{-1}c^{-2}\theta_0^{-1})+12(r_0+r_c)^4\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})-\frac{27c^4r_0^6\theta_0}{4\pi^2H_0^6\Omega_k(0)}+3r_0^4\theta_0.
\end{aligned}
$$

From Eqs.(3.38), (3.39) and (3.37), we can observe that the both conditions: cosmic evolution determined by the quintessence fluid can be always sure by fulfilling condition $\eta\theta_0^2 > 0$ and the positive interaction term Q at present time.

Finally, using Eq.(3.27) for $r(z)$ and its derivative with respect to z at present

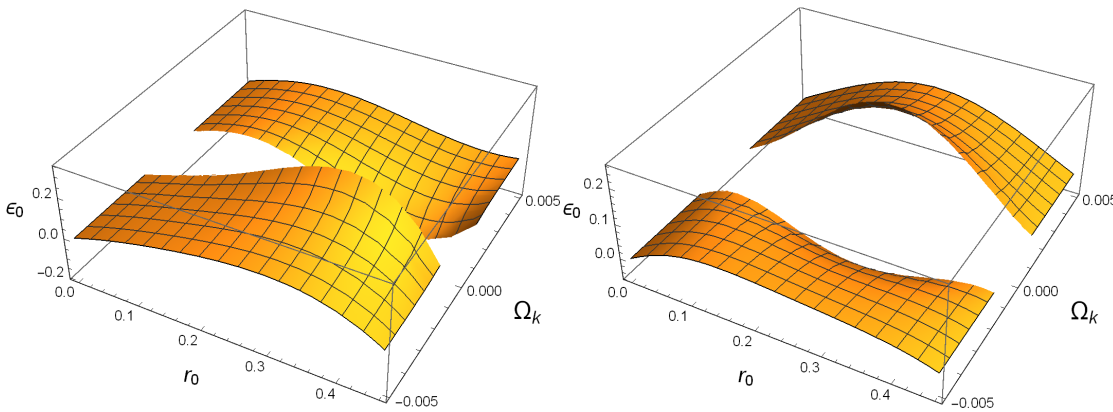

Figure 3.4: Left plot ϵ_0 in terms of r_0 and $\Omega_k(0)$ for $c = 0.681476,\ \omega_{DE,0} = -0.95,\ r_c = 0.4674$; Right plot for $c = 0.681476;\ \omega_{DE,0} = -1.5,\ r_c = 0.4674$.

time along with differentiating both sides of Eq.(3.25) with respect to z, we have

$$r'(z) = \frac{r(z) - r_c}{(r_0 + r_c)^2}\left[\frac{3c^2 r_0^3}{4\pi H_0^3 \Omega_k(0)} E'(z) - r_0^2(1+z) - r_0(r_0 + r_c)\right]. \tag{3.40}$$

Now Eq.(3.40) can be modified using the condition $r_0' = \epsilon_0$ as

$$\epsilon_0 = \frac{1}{(r_0 + r_c)^2}\left[\frac{3c^2 r_0^3 (r_0 - r_c)}{4\pi \Omega_k(0)} E_0' - 2r_0^3 + r_0 r_c (r_0 + r_c)\right]. \tag{3.41}$$

Using Eqs.(3.10) and (3.11) at $z = 0$, then we have a an important result for $E'(z)$ as given below

$$E_0' = \frac{3}{2}\left(1 + \frac{\omega_{DE,0}}{1 + r_0}\right)(1 - \Omega_k(0)) + \Omega_k(0). \tag{3.42}$$

Using this value in the above equation, we determined the value of ϵ_0 as follows

$$\begin{aligned} \epsilon_0 \ &= \ \frac{1}{(r_0 + r_c)^2}\Bigg[\frac{3c^2 r_0^3 (r_0 - r_c)}{4\pi} + \frac{9c^2 r_0^3 (r_0 - r_c)}{8\pi \Omega_k(0)}\left(1 + \frac{\omega_{DE,0}}{1 + r_0}\right)(1 - \Omega_k(0)) \\ &- \ 2r_0^3 + r_0 r_c (r_0 + r_c)\Bigg]. \end{aligned} \tag{3.43}$$

Note that, we can obtained in a similar way the whole analysis developed for the considered model, to find a suitable value for parameter ϵ_0 in which $\Omega_k(z)$ has a crucial role. It is important to point out that for flat and closed universe,

$\Omega_k(0) \leq 0$ implying $\epsilon_0 > 0$, which is the most exciting case in this account. Also for open universe, we have $\Omega_k(0) > 0$ for which $\epsilon_0 > 0$.

A relationship between ϵ_0 (calculated in the above equation) and $\Omega_k(0)$ can be seen from both three dimensional graphs of Fig.(**3.4**). The model parameters are constrained in the caption of the figure whereas the value of $\omega_{DE,0}$ has been taken from [32]. The left graph is plotted for quintessence region taking value of $\omega_{DE,0} > -1$ while right plot admits the phantom region ($\omega_{DE,0} < -1$). We can see from the left plot that ϵ_0 starts from zero and goes on decreasing with an increment in $\Omega_k(0)$ in the interval $-0.005 < \Omega_k(0) < 0$ (representing the closed universe). In the open universe where $0 < \Omega_k(0) < 0.005$, the graph has monotonic behavior as $\epsilon_0 > 0$ increases firstly with the increase of $\Omega_k(0)$ for $0 < r_0 < 0.2$ after that ϵ_0 has decreasing behavior for some region then again increasing and process continues. In phantom region (right plot of Fig.(**3.4**)), the curves show the same behavior as of quintessence. In region $-0.005 < \Omega_k(0) < 0$, an inversely proportional relation holds between ϵ_0 and $\Omega_k(0)$ while for $0 < \Omega_k(0) < 0.005$, the curve shows monotonic behavior firstly increases to a specific value then decreases while space of validity r_0 increases as compared to previous description for quintessence in the left plot.

3.4 Statefinder Diagnosis:

In this section, we will discuss about the Satefinder diagnosis for our model that once if we have obtained future singularity, this diagnosis shows that current universe evolution is far away from behavior that it would have, if the responsible factor for the cosmic evolution of the universe is Λ. For ΛCDM model, the above parameters corresponds to a fixed point $(r, s) = (1, 0)$ in the $s - r$ plane.

Taking the derivatives of $a(t)$ (calculated in Eq.(3.32)) up to third order, we are able to evaluate the expressions of r and s mentioned in chapter **2** of this thesis. These expression directly reduced to ΛCDM. The declaration parameter q is equal to -1, which represents an accelerating expansion of the universe.

The values found from ΛCDM model $(q = -1)$ are always greater than current values. It is observed that we will get a static universe in our RDE model by assuming a null curvature parameter. Based on previous results, we can conclude that to induce a phantom behavior in the model, the value of curvature parameter must be negative. In spite of the previous value for curvature parameter describes a closed universe, this is due to the existence of the future singularity for which universe will not collapse as we have attained in the standard cosmology.

Chapter 4

Concluding Remarks

This thesis is devoted to explore the nature of RDE model using a special type of parametrization, *i.e.*, CPL parametrization of the coincidence parameter. The work has been done in curved FLRW background bounded by a Ricci cutoff taking an unknown interaction term, Q as a function of redshift parameter z between two dark ingredients of cosmic fluid, which are uniformly distributed in the universe. The considered two interacting dark components are RDE and DM. Via CPL type of parametrization, we are able to evaluate the solutions of field equations. The evaluated expressions of energy densities of both components and pressure of RDE lead us to prove that our model admits a future singularity of type III at a specific point $z = z_s$.

In this scenario, we get an analytical solution of normalized Hubble parameter defined by $E^2(z)$ and it is graphically represented in Fig.(**3.1**) for closed universe. It is clear from the trajectory plotted in the left graph, $E^2(z) - z$ that during closed universe ($\Omega_k(0) = -0.005$ for $k = 1$), $E^2(z)$ has decreasing behavior as z transits from past to future era. Right plot of Fig.(**3.1**) is plotted for $\Omega_k(0) = -1.005$, which also admits the same decreasing behavior as left plot but range of $E^2(z)$ is too much large. So, generally it is noted that the behavior of $E^2(z)$ remains the same throughout the closed universe. We have also checked that for positive values of $\Omega_k(z)$ (representing an open universe $(k = 1)$), the normalized function $E^2(z)$ becomes negative, which is not a physical result according to recent observations. Note that a change in the sign of the $\Omega_k(0)$ can cause two types of cosmic evolution but in both cases the value z is the same.

We have calculated the expression of $Q(z)$ and explored its nature graphically. It is obvious that Q can be positive and negative defining two different regions of cosmos. The calculated interaction term Q is plotted versus z in left and right plot of Fig.(**3.2**). It is important to note that the value of $\Omega_k(z)$ shows a crucial role in order to have real Q-function. The left plot of Fig.(**3.2**) verifies the natural phenomena that interaction term remains positive throughout the domain of z

for $\Omega_k(0) = -0.005$ during closed universe. As $Q > 0$ represents the conversion of DE dominated era to DM dominated era. Although the observations suggests that Q must be positive to satisfy the natural SLT, but right plot of Fig.(**3.2**) for $\Omega_k(0) = 0.005$ is not in good agreement with recent observations. So, in closed universe, DM is converted in to DE.

A relationship between ϵ_0 and $\Omega_k(0)$ can be seen from both three dimensional graphs of Fig.(**3.4**). The model parameters are constrained in the caption of the figure whereas the value of $\omega_{DE,0}$ has been taken from [32]. The left graph is plotted for quintessence region taking value of $\omega_{DE,0} > -1$ while right plot admits the phantom region ($\omega_{DE,0} < -1$). We can see from the left plot that ϵ_0 starts from zero and goes on decreasing with an increment in $\Omega_k(0)$ in the interval $-0.005 < \Omega_k(0) < 0$ (representing the closed universe). In the open universe where $0 < \Omega_k(0) < 0.005$, the graph has monotonic behavior as $\epsilon_0 > 0$ increases firstly with the increase of $\Omega_k(0)$ for $0 < r_0 < 0.2$ after that ϵ_0 has decreasing behavior for some region then again increasing and process continues. In phantom region (right plot of Fig.(**3.4**)), the curves show the same behavior as of quintessence. In region $-0.005 < \Omega_k(0) < 0$, an inversely proportional relation holds between ϵ_0 and $\Omega_k(0)$ while for $0 < \Omega_k(0) < 0.005$, the curve shows monotonic behavior firstly increases to a specific value then decreases while space of validity r_0 increases as compared to previous description for quintessence in the left plot.

Further, it is observed from statefinder diagnostic that our model exactly generate ΛCDM as the expressions of r and s parameters are independent of any model parameters. Similarly the limit $q = -1$ is attained directly without choosing any value of the model parameters. So, the considered interacting model of RDE and DM within non-flat FRW background using Ricci cutoff represent an accelerating expanding universe. Moreover, it is observed that the closed model is more physical as compared to open one. As a general comment, with a Ricci cutoff, this work is more complicated as compared to HDE done in [28]. For HDE model, the ΛCDM model can not be achieved as the authors claimed that there model represent an over acceleration.

It is worthy to mention here that this work can be extended for some other DE models to explore the nature of the universe. Also, it can be extended in some modified theories of gravity.

Chapter 5

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