Coupled Parallel Fluid Flow in Porous Medium Adjacent to Free Flow Domain

By

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I, Ifrah Tehreem, **CIIT/SP17-RMT-023/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research project has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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It is certified that Ifrah Tehreem (CIIT/SP17-RMT-023/LHR) has carried out all the research work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore campus and the work fulfills the requirement for award of MS degree.

Dated: _____________

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Dr. Mohsan Hassan
Assistant Professor

Head/Incharge Of Deepartment

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Dr. Sarfraz Ahmad
Associate Professor
Department of Mathematics

DEDICATED TO

My loving Parents, Teachers
and Friends who always encouraged
me to work hard and guided me towards
the right way and destination.

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ABSTRACT

Fluid flow through a partially filled porous channel, in which porous layer is considered at center of the channel bounded by free flow regions. Fluid flow over a partially filled porous channel is directed by different types of mathematical model with various kinds of boundary conditions at interface. In **chapter 2**, we consider the fluid flow through fluid-porous channel in which the flow in free region is governed by Stokes equation and Darcy-Forchheimer model. At interface, we assume the velocity for the free fluid is proportional to the shear rate of the permeable region. In this chapter, we examined the effects of the Darcy number, interfacial velocity slip coefficient and Knudsen number on the velocity distribution. In **chapter 3,** we consider the same channel in which porous media, but here mathematical model is govern by Stokes equation for free region and Brinkman model for porous region. In addition, at the interface, we use the continuity condition of velocity and shear stress with jump effect to find the fluid flow behavior through the channel. In this case, the effect of different values of Darcy number on the velocity distribution is analyzed and compare with the results of Chapter 2.

Contents

Chapter 1

Preliminaries

1.1 Introduction

A porous material or medium is known as the one having pores in it and its characteristics are highly dependent on the level of porosity, the shape of these pore or openings and the connectedness level of these pores [1]. We use such mediums and materials every-day in our daily life both in natural and technological applications. For example, wood, bones, soil and rocks and artificial materials are the everyday examples of porous mediums [2]. The flow through porous medium such as crude oil extraction [3], soil pollution [4] and groundwater flow [5] are engineering applications.

Composite domain consists of a porous medium adjacent to an open channel (free flow domain) where flow occurs in and over the porous medium [6]. The Fluid flow through the composite domain is known as coupled flow. Study of fluid flow through such mediums has become a prominent field of interest in the last decade as it finds its applications in various aspects of life. Some prominent examples can be seen as the fuel reservoirs, heat exchangers and heat enhancement techniques [7]. Recently these materials are also finding their applications in ocean cleaning and oil recovery processes [8].

To investigate the fluid flow in domain, several flow conditions are proposed at the interface. At interface, Beavers et al. [9] assumed the slip velocity for the fluid in free region is proportional to the shear rate at the porous wall. Moreover, they consider the "Stroke's flow" for free flow region and apply the "Darcy's Law" in the porous region. Ochoa-Tapia et al. [10, 11] presented a stress jump boundary condition for obtaining continuous velocity profile across the interface and the continuity of the 'modified' normal stress by a volume averaging technique of the momentum equations in the interface region. In their work, they retain the continuity of velocity but conduct a jump in shear stress by comparing Stokes and Brinkman equations. Later on Neale and Nader [12] presented the velocity continuity and stress across continuity over the interface as the Stokes flow and Brinkman equations have same order. After the above studies, different authors used these boundary conditions at interface and get the solution of different mathematical for free flow and porous regions. The first complete analytical solution of the interfacial conditions between the porous medium and free fluid region is presented by Vafai and Thiyagaraja [13]. Vafai and Kim [14] reviewed Beavers and Joseph's porous-fluid coupling. Xu et al. [15, 16] used the velocity and shear stress continuity at the interface and get the analytical and numerical solutions of the flow between the porous medium and free fluid regions.

Development of a coupled model which could account for the processes in real time for both domains is still an intricate task so often simplified models or decoupled concepts are used [17]. The processes that occur between the porous and free fluid regions are not only operate at the interface in real, but these two regions also interrelate in a transition region where there may be strong gradients and fluid properties changes. The fluid flow in a geometry consist of three layers were analytically shown by Nield and Kuznetsoy [18]. Dumen et al. [19] describe the problem of coupling by analyzing the effects of change in shape at the boundary. Goharzadeh et al. [20, 21] and Morad and Khalili [22] experimentally examined fluid flow through the three-layer domain in which porous media is bounded with free fluid regions. Dauben et al. [23] presented some basic flow mechanics in the porous medium and presented certain conditions which can cause some vital mobility effects with the interaction of porous media properties due to transition region.

In this thesis, we study the fluid flow through a partially filled porous channel, in which porous layer is considered at center of the channel bounded by free flow regions. Fluid flow over a free-porous regions is directed by different types of mathematical model with various kinds of boundary conditions at interface as purposed in literature and get the solution in the form of velocity profile for disscusion.

1.2 Fluid

A fluid is a substance having particles that flow continuously under the action of shear stress. It does not matter how shear stress may be smaller or larger.

1.3 Fluid Properties

The following are some of the important fluid's properties

1.3.1 Density

Density is defined as the ratio between mass m and volume V.

$$\rho = \frac{m}{V} \tag{1.1}$$

Where ρ denotes the density and unit of density is $\frac{kg}{m^3}$.

1.3.2 Viscosity

Viscosity is the property of fluid which defines the interaction between the moving particles of fluid. In simple words, friction between the molecules of fluid. Formally, viscosity is the ratio of the shear stress to the velocity gradient.

$$\text{Viscosity} = \frac{\text{Shear stress}}{\text{velocity}} \tag{1.2}$$

And, viscosity Eq. (1.2) becomes,

$$\mu = \frac{F/A}{\frac{\Delta v_x}{\Delta z}} \tag{1.3}$$

1.3.3 Pressure

Pressure is defined as the force per unit area of fluid. In simple words, the ratio between force of fluid to the area of fluid held perpendicular to the direction of force**.** Pressure is by 'p' and its unit is N/m^2 .

1.4 Porous media

A porous media or material is known as the one having pores in it and its characteristics are highly dependent on the level of its porosity, the shape of its pore or openings and the connectedness level of these pores. Several natural objects like the wood, bones, soil and rocks and artificial materials like cement and ceramics are the everyday examples of porous mediums.

1.5 Porosity

Porosity is the quality of being porous, or full of tiny holes. The word “porosity” derives from the Greek word “porous” meaning “passage”.

1.6 Permeability

Permeability is defined as the ability of porous medium to measure that how the fluid passes over it. It depends on porosity of the porous medium, the shape of pores and their level of connectedness.

1.7 Mathematical models for porous media

To find the flow in porous media, different models are introduced by different investigators. For example Darcy law model [24], Brinkman equation model [25] and Darcy-Forchheimer-Brinkman model [26, 27].

1.7.1 Darcy’s law Model

The Flow in porous medium is described by the well-known Darcy’s law which was first found empirically in 1856. In 1972, Flow in porous mediums are represented by Darcy’s law [24]. The relation for the Darcy’s model is presented as

$$u = u_p = -\frac{K}{\mu_f}\nabla p \tag{1.4}$$

Where u_p denotes the velocity in the porous region, u is the fluid velocity, K is the permeability, μ_f is the fluid viscosity and ∇p is the pressure gradient.

1.7.2 Brinkman’s model

Brinkman equations were initially found by Brinkman [25] in 1947 that defines the flows through a porous medium. Many investigators discussed about the non-Darcy model to find the flow through the porous medium. They said that the use of Brinkman's equation is more appropriate to find the fluid flow in porous medium instead of Darcy's law model. Brinkman's equation is used when the porosity is high of the porous medium and the no-slip conditions are imposed on the solid walls. Rajagopal presented the specific continuum model for a combination of two different constituents like sand displayed as the stiff solid and liquid (fluid) [28].

$$-\frac{\mu_f}{K}u+\mu_{eff}\frac{\partial^2 u}{\partial y^2}=f-\frac{dp}{dx}, \tag{1.5}$$

Where μ_{eff} is the effective viscosity of fluid in porous medium and defined as $\mu_{eff}=\mu_f/\varepsilon$, and ε is the porosity of the pore.

1.7.3 Darcy-Forchheimer Brinkman model

Some investigators highlighted the utilization of non-Darcy model for both viscous shear and inertial impacts. Vafai and kim [14] and Vafai and Tien [29], employed a relation which utilized for the viscous shear term (Darcy-Forchheimer-Brinkman model) and an inertial term (The Forchheimer quadratic inertial term) and presented as

$$-\frac{\mu_f}{K}u+\mu_{eff}\frac{\partial^2 u}{\partial y^2}+\frac{dp}{dx}-\frac{\rho_f\varepsilon F}{\sqrt{K}}u^2=0 \tag{1.6}$$

Where F denotes the Forchheimer coefficient.

1.8 Interface Boundary Conditions at Fluid and Porous media

Follow the interface conditions for the fluid flow through the both regions that are exist in literature.

1.8.1 The Beavers-Joseph Interface Condition (Slip flow boundary condition)

Beavers and Joseph (BJ for short) [9] made their first attempt in which they applied a semi-observed interface condition for velocity to solve Stokes and Darcy equations together. Beavers and Joseph [9] provides an huge information of interface's slip flow assumption and suggested the following observed slip-flow condition that approved their experiments:

$$\alpha^* \left(u\big|_{y=0^+} - u_D \right) = \sqrt{K}\, \frac{\partial u}{\partial y}\bigg|_{y=0^+} . \tag{1.7}$$

Where $u\big|_{y=0^+}$ represents the velocity of fluid in free region, u_D denote the Darcy velocity inside the porous media and α^* is Beavers-Joseph constant (slip coefficient) and its range from 0 to 4. The Beavers-Joseph constant α^* only depends on porous medium properties.

1.8.2 Saffman's modifcation of the Beavers-Joseph Interface Condition

In 1971, Saffman proposed a modify Beavers-Joseph condition (BJ’s condition). He claimed that the interface tangential velocity is proportional to the shear stress. Saffman also presented Beavers–Joseph interface condition a ’theoretical’ explanation of the Beavers–Joseph interface condition at a physical consistency level [30]. Saffman showed that the Darcy velocity (fluid velocity in porous medium) is unimportant when it is compared with the free flow velocity in the Beavers-Joseph condition. So, the Beaver-Joseph condition can be written as

$$\alpha^* u\big|_{y=0^+} = \sqrt{K}\, \frac{\partial u}{\partial y}\bigg|_{y=0^+} + O\left(K\right). \tag{1.8}$$

1.8.3 Velocity and Shear-stress Continuity Condition

In 1974, Neale and Nader [12] suggested to consider velocity and shear stress continuity over the fluid-porous interface and defined as

$$\begin{aligned} u\big|_{\Sigma p} &= u\big|_{\Sigma f} \\ n\cdot\left(\mu_{eff}\nabla u - pI\right)\Big|_{\Sigma p} &= n\cdot\left(\mu\nabla u - pI\right)\Big|_{\Sigma f} \end{aligned} . \tag{1.9}$$

Where $\sum p$, $\sum f$ are the same interface seen from the fluid and porous region.

1.8.4 Stress Jump Condition

In 1995, Ochoa-Tapia and Whitaker [10, 11] introduced a jump condition at the fluid-porous interface that is defined as

$$\mu_{eff}\, \frac{du}{dy}\bigg|\left(x,0^-\right) - \mu \frac{du}{dy}\bigg|\left(x,0^+\right) = \beta \frac{\mu}{\sqrt{K}} u . \tag{1.10}$$

where β denotes the adjustable jump coefficient must be of order 1 which is empirically determined. They attained a good convention with the experimental data of Beavers and Joseph by adjusting the coefficient β.

Chapter 2

Analysis of Double Slip Model for Coupled Parallel Fluid in Free Flow-Porous Domains

In this section, the straight channel of height H is composed with three layers. The middle layer is filled with a porous medium while the upper and lower layers are used as free fluid regions. The height of lowest fluid layer is H_1 and the distance from the upper wall of the porous layer to the lower wall of the lowest fluid layer is H_2. The boundary conditions including flow slip effect at the upper and lower walls and at interface velocity for the free fluid is proportional to the shear rate at the permeable boundary. This flow is driven by constant pressure gradient throughout the channel. The results are obtained in form of velocity profile for free flow channel and porous medium.

2.1 Mathematical Modeling

Flow Modeling

We considered the laminar and fully developed fluid flow in a channel with the porous medium confined by free fluid regions. The problem under consideration is shown in Fig. 2.1.

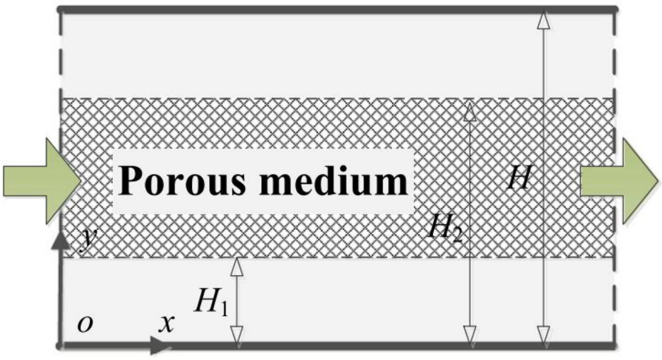

Fig. 2.1: Schematic diagram of the fluid flow.

The modeling is attained through the use of general flow equation that is useable all over the channel. If we consider homogeneous porous region then the equation can be written as [31]

$$\frac{\rho_f}{\varepsilon}\left(\frac{\partial u}{\partial t}+\nabla\left(\frac{u\cdot u}{\varepsilon}\right)\right)=-\nabla p+\mu_{eff}\nabla^2 u-\frac{\mu_f}{K}u-\frac{\rho_f C_E}{\sqrt{k}}|u|u. \qquad (2.1)$$

Where ρ_f denotes the fluid density, ε is the porosity, p is the pressure, $\mu_{eff}=\mu_f/\varepsilon$ denotes the effective viscosity [32], μ_f is the fluid viscosity, u is the fluid velocity vector and K is the permeability. The equations for flow through a channel containing three layers can be written as:

Free region $\left(0 < y < H_1 \text{ and } H_2 < y < 1\right)$

$$\mu_f \frac{\partial^2 u}{\partial y^2} - \frac{dp}{dx} = 0, \tag{2.2}$$

Porous Media $\left(H_1 \le y \le H_2\right)$

$$u = u_p, -\frac{\mu_f}{K} u - \frac{\rho_f C_E}{\sqrt{K}} u^2 - \frac{dp}{dx} = 0. \tag{2.3}$$

In above equation (2.3), K denotes the permeability and C_E denotes the Ergun constant. Since, the relation between these two parameters vary for different kinds of porous mediums. There are different connections of these parameters. The permeability and the Ergun constant are defined as [33]

$$K = 0.00073(1-\varepsilon)^{-0.224} \left(\frac{d_{fiber}}{d_{pore}}\right)^{-1.11} d_{pore}^2, \tag{2.4}$$

$$C_E = 0.00212(1-\varepsilon)^{-0.132} \left(\frac{d_{fiber}}{d_{pore}}\right)^{-1.63}. \tag{2.5}$$

In above equations d_{fiber} and d_{pore} denotes the fiber and pore diameter respectively. The permeable fluid region should not be less than $\sqrt{2K}$ according to Beavers and Joseph [9]. Therefore, the range of H_1 and $H - H_2$ should be

$$H_1 > \sqrt{2K},\ H - H_2 > \sqrt{2K}. \tag{2.6}$$

Now we have to define dimensionless variables that are given below:

$$\left.\begin{aligned} & Y = \frac{y}{H}, Y_1 = \frac{H_1}{H}, Y_2 = \frac{H_2}{H}, U = \frac{u}{-\frac{H^2}{\mu_f}\frac{dp}{dx}}, \\ & \mathrm{Re} = 4\sqrt{\frac{\frac{dp}{dx} \cdot \rho_f \cdot H^3}{f \cdot \rho_f^2}}, \beta_v = \frac{2-\sigma_v}{\sigma_v} Kn, \beta_i = \frac{\sqrt{Da}}{\alpha^*} \end{aligned}\right\}. \tag{2.7}$$

In above, Re denotes the Reynold number and Kn presents the Knudsen number. Later on it is defined as the ratio of average distance travelled by the fluid molecules and to the length of the system and is given as

$$Kn = \frac{\lambda}{H}. \tag{2.8}$$

Where λ represents the average distance and H is the length of the system. The range of Knudsen number for the different flow models is shown in Fig. 2.2.

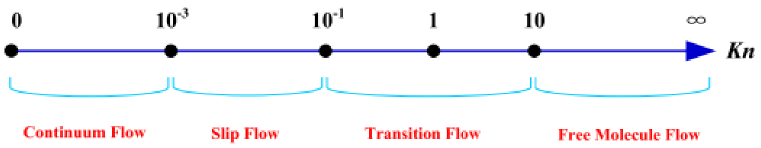

Fig. 2.2: Knudsen number range for different fluid flow models.

After defining the dimensionless variables, Eq. (2.2) and (2.3) becomes

$$\frac{\partial^2 u}{\partial Y^2} + 1 = 0, \qquad 0 < Y < Y_1 \text{ or } Y_2 < Y < 1 \tag{2.9}$$

$$\frac{C_E \cdot f \cdot \mathrm{Re}^2}{16\sqrt{Da}} U_p{}^2 + \frac{1}{Da} U_p - 1 = 0, \; Y_1 \le Y \le Y_2 \tag{2.10}$$

Boundary Conditions

The boundary conditions including flow slip effect at the upper and lower walls $(Y = 0$ and $Y = H)$ are correspondingly as given below:

$$u\big|_{y=0} = \lambda \frac{2 - \sigma_v}{\sigma_v} \frac{\partial u}{\partial y}\bigg|_{y=0}, \tag{2.11}$$

$$u\big|_{y=H} = \lambda \frac{2 - \sigma_v}{\sigma_v} \frac{\partial u}{\partial y}\bigg|_{y=H}, \tag{2.12}$$

Where σ_v is the tangential momentum coefficient. The range of the value of σ_v is $0 < \sigma_v < 2$, and at solid boundaries the value of the tangential momentum coefficient is 2. Thus, Beavers–Joseph condition is used at fluid-porous interfaces to couple the flow [9]. At the lower and upper fluid-porous interfaces $(y = H_1 \text{ and } y = H_2)$

$$-\alpha^*(u-u_p) = \sqrt{K}\,\frac{\partial u}{\partial y}\bigg|_{y=H_1^-}\,, \tag{2.13}$$

$$\alpha^*(u-u_p) = \sqrt{K}\,\frac{\partial u}{\partial y}\bigg|_{y=H_2^+}\,. \tag{2.14}$$

The value of α^* based on the arrangement and characteristics of the porous medium [34]. The range of α^* is generally between 0.1 and 4 [9].

By considering the dimensionless variable, the boundary conditions in Eqs. (2.11) to (2.14) takes the form

$$U\big|_{Y=0^+} = \beta_v\,\frac{\partial U}{\partial Y}\bigg|_{Y=0^+}\,, \tag{2.15}$$

$$U\big|_{Y=Y_1^-} = U_p - \beta_i\,\frac{\partial U}{\partial Y}\bigg|_{Y=Y_1^-}\,, \tag{2.16}$$

$$U\big|_{Y=Y_2^+} = U_p + \beta_i\,\frac{\partial U}{\partial Y}\bigg|_{Y=Y_2^+}\,, \tag{2.17}$$

$$U\big|_{Y=1^-} = -\beta_V\,\frac{\partial U}{\partial Y}\bigg|_{Y=1^-} \tag{2.18}$$

2.3 Analytical solution

There are two conditions that are employed to obtain the velocity in the porous region. First condition is $f\,\mathrm{Re}^2 = const.$ that is utilized in analytical study. From Eq. (2.9) the fluid velocity in the porous layer is given as:

$$U_p = \frac{-\dfrac{1}{Da} + \sqrt{\dfrac{1}{Da^2} + \dfrac{C_E f\,\mathrm{Re}^2}{4\sqrt{Da}}}}{\dfrac{C_E f\,\mathrm{Re}^2}{8\sqrt{Da}}}\,. \tag{2.19}$$

The velocity distribution in the porous layer for the condition $C_E = 0$ is given from Eq.(2.19) :

$$U_p = \lim_{C_E \to 0}\left(\frac{-\frac{1}{Da}+\sqrt{\frac{1}{Da^2}+\frac{C_E f\,\mathrm{Re}^2}{4\sqrt{Da}}}}{\frac{C_E f\,\mathrm{Re}^2}{8\sqrt{Da}}}\right) = Da\,. \qquad (2.20)$$

The results in Eq. (2.20) also obtained by reducing the quadratic term in Eq. (2.10), that matches with the flow in porous layer. For whole domain, the velocity distributions are given as:

$$U(Y) = -\frac{1}{2}Y^2 + C_1 Y + C_2\,, \quad 0 < Y < Y_1\,, \qquad (2.21)$$

$$U = U_p = \frac{-\frac{1}{Da}+\sqrt{\frac{1}{Da^2}+\frac{C_E f\,\mathrm{Re}^2}{4\sqrt{Da}}}}{\frac{C_E f\,\mathrm{Re}^2}{8\sqrt{Da}}}\,, \quad Y_1 \le Y \le Y_2\,, \qquad (2.22)$$

$$U(Y) = -\frac{1}{2}Y^2 + C_3 Y + C_4 \quad Y_2 < Y < 1\,. \qquad (2.23)$$

Where C_1, C_2, C_3 and C_4 are constants and the values for these constants are

$$C_1 = \frac{\frac{1}{2}Y_1^2 + \beta_i Y_1 + U_p}{Y_1 + \beta_v + \beta_i}\,,\ C_2 = C_1 \beta_v\,,$$

$$C_3 = \frac{\frac{1-Y_2^2}{2} + \beta_v + \beta_i Y_2 - U_p}{1 - Y_2 + \beta_v + \beta_i}\,,\ C_4 = \beta_v + \frac{1}{2} - (\beta_v + 1)C_3\,. \qquad (2.24)$$

Mean Velocity

The calculated velocities(mean) for the free fluid and porous regions are as follows

$$U_{m1} = \frac{1}{Y_1}\int_0^{Y_1} U dY = \frac{-Y_1^2}{6} + \frac{C_1 Y_1}{2} + C_2\,, \qquad (2.25)$$

$$U_{m,p} = U_p \qquad (2.26)$$

$$U_{m2} = \frac{1}{1-Y_2}\int_{Y_2}^{1} U dY = \frac{1 - Y_2 + Y_2^2}{6} + \frac{C_3(1+Y_2)}{2} + C_4\,, \qquad (2.27)$$

$$U_m = \frac{1}{A}\int U dA = \left(Y_2^3 - Y_1^3 - 1\right) + \frac{C_1 Y_1^2 + C_3\left(1 - Y_2^2\right)}{2} + C_2 Y_1 + C_4\left(1 - Y_2\right) + U_p\left(Y_2 - Y_1\right). \tag{2.28}$$

2.5 Results and Discussion

In this section, we will talk over the findings for velocity distribution in three layered channel. The velocity distribution for different values Darcy number, slip coefficient and Knudsen number in the channel are shown in Fig. 2.3-2.5. Fig. 2.3 graphically shown the effect of Darcy number on the velocity distribution. The velocity of fluid in the free and porous regions is increased with an increasing of the Darcy number at the fixed value of $f\,\mathrm{Re}^2$. Behind the increasing of Darcy number, permeability increases which leads to enhancement in fluid flow. As the Darcy number decreases the porous region become dense and due to less permeable the fluid is forced to escape the clear region that increases the jump at fluid-porous interface. In Fig. 2.4 we can see the graphical representation of the effect of slip coefficient on the velocity distribution. We can also gotten that by increasing the value of slip coefficient, both the value of jump velocity at the porous-fluid interface and fluid velocity in the free region decreases accordingly. As jump discontinuity seen in Fig. 2.4, it is clear that the given function makes the function discontinuous by jumping from on point to another. It is also noted that after the pick-up, the nature of function will not remain same as before the starting point. First half represents the velocity jump at fluid-porous interface, while on the other point the straight line denotes the velocity in the porous layer. In Fig.2.5, we can see the effect of Knudsen number on the velocity distribution in the channel graphically. By increasing the Knudsen number, fluid velocity increases in the free region and also at the solid walls.

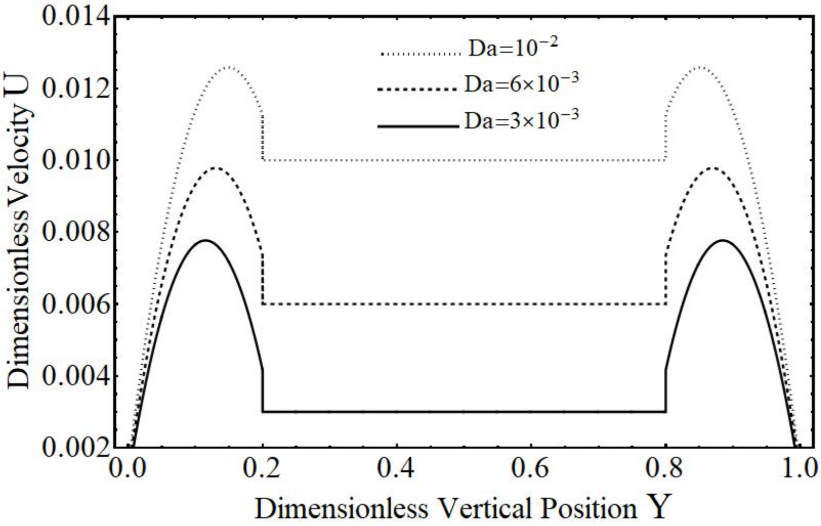

Fig. 2.3: Effect of the Darcy number on the velocity distribution.

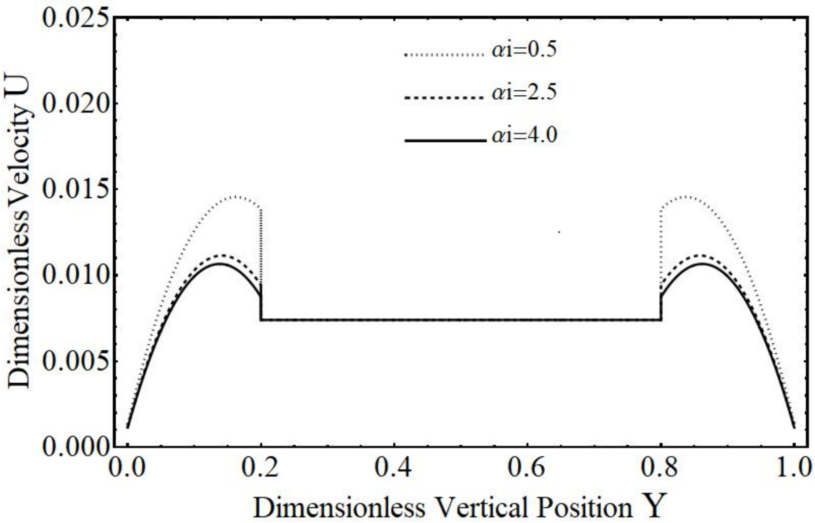

Fig. 2.4: Effect of the slip coefficient on the velocity distribution.

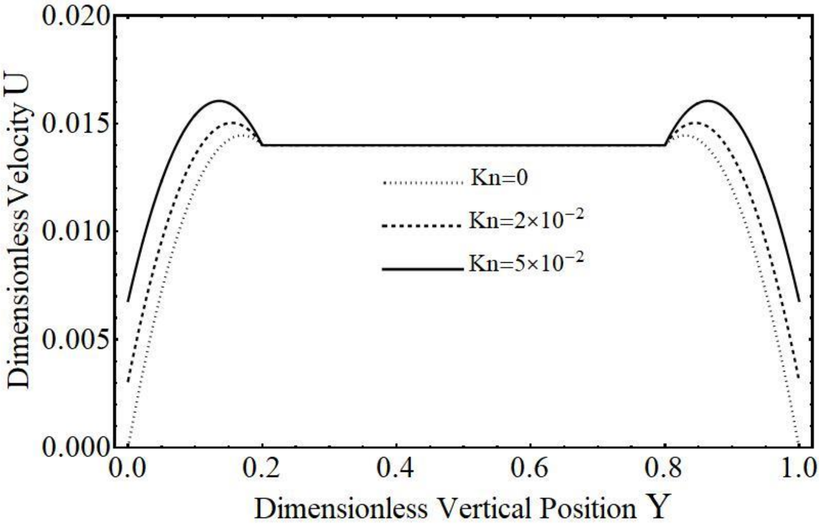

Fig. 2.5: Effect of the Knudsen number on the velocity distribution.

Chapter 3

Coupled Parallel Fluid Flow in Porous Medium Adjacent to Free Flow Domain

In this chapter, we are going to discuss the fluid flow through partially filled porous channel as proposed in Fig. 2.1. Fluid flow through porous medium is governed by Brinkman's equation. The boundary conditions including flow no-slip effect at the upper and lower walls and at interface of free flow and permeable regions the velocity and shear stress are considered to be equal. The results in form of velocity profile for both regions are displaced for discussion.

3.1 Mathematical Modeling

Equations governing for fluid flow in each region are given by the momentum transfer equation which is valid all over the channel, if we consider homogeneous porous region, the equation takes the form [29]

$$\frac{\rho_f}{\varepsilon}\left(\frac{\partial u}{\partial t}+\nabla\left(\frac{u\cdot u}{\varepsilon}\right)\right)=F-\nabla p+\mu_{eff}\nabla^2 u-\frac{\mu_f}{K}u\,. \tag{3.1}$$

Where ρ_f denotes the fluid density, ε is the porosity of the porous medium, u represents the fluid velocity vector, F represents the body forces, p represents the pressure, effective viscosity μ_{eff} and fluid viscosity μ_f. Permeability is infinitely large in fluid flow region and Eq. (3.1) reprobate into Stokes equation. So the simplified equations in the three layered channel which contain porous layer in its center bounder by free flow regions can be written as:

Free region ($0<y<H_1$ and $H_2<y<1$**)**

$$\mu_f\frac{\partial^2 u}{\partial y^2}-\frac{dp}{dx}=0, \tag{3.2}$$

Porous region ($H_1\leq y\leq H_2$)

Brinkman model with constant permeability is used to find the fluid flow in the porous region with effective viscosity μ_{eff}.

$$u=u_p,\ -\frac{\mu_f}{K}u+\mu_{eff}\frac{\partial^2 u}{\partial y^2}-\frac{dp}{dx}=0. \tag{3.3}$$

Defining the dimensionless variables that are given below

$$Y = \frac{y}{H},\ U = \frac{u}{\frac{dp}{dx}\frac{H^2}{\mu_f}},\ Da = \frac{K}{H^2},\ M_2 = \frac{\mu_{eff}}{\mu}. \tag{3.4}$$

After using these dimensionless variables, Eq. (3.2) and (3.3) becomes

$$\frac{\partial^2 U}{\partial Y^2} + 1 = 0 \qquad 0 < Y < Y_1 \ \text{or}\ Y_1 < Y < 1, \tag{3.5}$$

$$\frac{\partial^2 U_p}{\partial Y^2} - \lambda_2^2 U_p + \frac{1}{M_2} = 0 \qquad Y_1 \le Y \le Y_2 \tag{3.6}$$

Where $\lambda_2 = \frac{1}{[M_2 Da]^{1/2}}$.

3.2. Boundary Conditions

In this chapter, the continuity conditions of velocity and shear stress utilize at the porous-fluid interface $(y = H_1 \text{ and } y = H_2)$ and no-slip conditions at solid walls $(y = 0 \text{ and } y = H)$ are considered which given as

$$u = 0 \quad \text{at} \quad y = 0, \tag{3.7}$$

$$u = u_p \quad \text{and} \quad \mu_f \frac{du}{dy} = \mu_{eff} \frac{du_p}{dy} \quad \text{at} \quad y = H_1, \tag{3.8}$$

$$u_p = u \quad \text{and} \quad \mu_{eff} \frac{du_p}{dy} = \mu_f \frac{du}{dy} \quad \text{at} \quad y = H_2. \tag{3.9}$$

$$u = 0 \quad \text{at} \quad y = H \tag{3.10}$$

By the dimensionless variables defined above, we can write boundary conditions in Eqs. (3.7) to (3.10) in the following form

$$U\big|_{Y=0} = 0 \tag{3.11}$$

$$U\big|_{Y=Y_1} = U_p\big|_{Y=Y_1}$$
$$\mu_f \frac{dU}{dY}\bigg|_{Y=Y_1} = \mu_{eff} \frac{dU_p}{dY}\bigg|_{Y=Y_1} \tag{3.12}$$

$$U\big|_{Y=Y_2} = U\big|_{Y=Y_2}$$
$$\mu_{eff} \frac{dU_p}{dY}\bigg|_{Y=Y_2} = \mu_f \frac{dU}{dY}\bigg|_{Y=Y_2} \tag{3.13}$$

$$U\big|_{Y=1} = 0 \tag{3.14}$$

3.3 Analytical solution

The general solution of the Eq. (3.5) and (3.6) are given below

$$U(Y) = -\frac{1}{2}Y^2 + C_1 Y + C_2 \qquad 0 < Y < Y_1 \tag{3.15}$$

$$U = U_p = \frac{1}{M_2\lambda_2^2} + e^{Y\lambda_2}\mathrm{C}_3 + e^{-Y\lambda_2}\mathrm{C}_4 \qquad Y_1 \le Y \le Y_2 \tag{3.16}$$

$$U(Y) = -\frac{1}{2}Y^2 + C_5 Y + C_6 \qquad Y_2 < Y < 1 \tag{3.17}$$

By using conditions in Eqs. (3.10) to (3.13) in Eqs. (3.14) to (3.16), we get the values of constants as

$$C_1 = \frac{-\left(\begin{array}{l} 2\left(e^{2Y_1\lambda_2} - e^{2Y_2\lambda_2}\right)M_2 Y_1\lambda_2 - \delta^2\left(e^{2Y_1\lambda_2} - e^{2Y_2\lambda_2}\right) \\ \left(-1+Y_2\right)\lambda_2\left(2+M_2 Y_1^2\lambda_2^{\,2}\right) \\ -\delta\left(e^{2Y_1\lambda_2} + e^{2Y_2\lambda_2}\right)\left(2+M_2 Y_1\left(2+Y_1-2Y\right)_2\lambda_2^{\,2}\right) \\ -2\delta \mathrm{e}^{(Y_1+Y_2)\lambda_2}\left(-2+M_2\left(-1+Y_2\right)^2\lambda_2^{\,2}\right) \end{array}\right)}{\left(2M_2Y_2\left(\begin{array}{l} -e^{2Y_1\lambda_2}\left(-1+\delta Y_1\lambda_2\right)\left(-1+\delta\left(-1+Y\right)_2\lambda_2\right)+ \\ e^{2Y_1\lambda_2}\left(1+\delta Y_1\lambda_2\right)\left(1+\delta\left(-1+Y_2\right)\lambda_2\right) \end{array}\right)\right)} \tag{3.18}$$

$$C_2 = 0 \tag{3.19}$$

$$C_3 = \frac{\left(\begin{array}{l} e^{Y_1\lambda_2}(1+\delta\left(-1+Y_2\right)\lambda_2)\left(2+3M_2Y_1^2\lambda_2^{\,2}\right) \\ +e^{Y_2\lambda_2}\left(-1+\delta Y_1\lambda_2\right)\left(-2+M_2\left(-1+Y_2\right)^2\lambda_2^{\,2}\right) \end{array}\right)}{\left(2M_2\lambda_2^{\,2} - e^{2Y_2\lambda_2}\left(\begin{array}{l} -1+\delta Y_1\lambda_2\left(-1+\delta\left(-1+Y_2\right)\lambda_2\right) \\ +e^{2Y_1\lambda_2}\left(1+\delta Y_1\lambda_2\right)\left(1+\delta\left(-1+Y_1\right)\lambda_1\right) \end{array}\right)\right)} \tag{3.20}$$

$$C_4 = \frac{\left(e^{(Y_1+Y_2)\lambda_2}\left(\begin{array}{l} -e^{Y_2\lambda_2}\left(-1+\delta\left(-1+Y_2\right)\lambda_2\right) \\ \left(2+3M_2Y_1^2\lambda_2^{\,2}\right)+\mathrm{e}^{Y_1\lambda_2}\left(1+\delta^{Y_1\lambda_2}\right) \end{array}\right) - e^{2Y_2\lambda_2}\right)\left(-2+M_2\left(\left(-1+Y_2\right)^2\lambda_2^{\,2}\right)\right)}{\left(2M_2Y_2\left(\begin{array}{l} -e^{2Y_1\lambda_2}\left(-1+\delta Y_1\lambda_2\right)\left(-1+\delta\left(-1+Y\right)_2\lambda_2\right) \\ +e^{2Y_1\lambda_2}\left(1+\delta Y_1\lambda_2\right)\left(1+\delta\left(-1+Y_2\right)\lambda_2\right) \end{array}\right)\right)} \tag{3.21}$$

$$C_5 = \frac{\left(\begin{array}{l} \left(e^{2Y_1\lambda_2} - e^{2Y_2\lambda_2}\right)M_2Y_2\lambda_2 - 2\delta e^{(Y_1+Y_2)\lambda_2}\left(2+3M_2 2Y_1^2\lambda_2^{\,2}\right) \\ +\delta^2\left(e^{2Y_1\lambda_2} - e^{2Y_2\lambda_2}\right)Y_1\lambda_2\left(2+M_2\left(-1+Y_2^{\,2}\right)\lambda_2^{\,2}\right) \\ +\delta\left(e^{2Y_1\lambda_2} + e^{2Y_2\lambda_2}\right)\left(2+M_2\left(-1+2Y_1Y_2+Y_2^{\,2}\right)\lambda_2^{\,2}\right)\right) \end{array}\right)}{\left(2M_2\lambda_2\left(\begin{array}{l} -e^{2Y_2\lambda_2}\left(-1+AY_1\lambda_2\right)\left(-1+\delta\left(-1+Y_2\right)\lambda_2\right)\right) \\ +e^{2Y_1\lambda_2}\left(1+\delta Y_1\lambda_2\right)\left(1+\delta\left(-1+Y_2\right)\lambda_2\right) \end{array}\right)\right)} \tag{3.22}$$

$$C_6=\frac{-\begin{pmatrix}\left(e^{2Y_1\lambda_2}-e^{2Y_2\lambda_2}\right)M_2\left(-1+2Y_2\right)\lambda_2\\-6\delta e^{(Y_1+Y_2)\lambda_2}M_2Y_1^2\lambda_2^2+\delta^2\left(e^{2Y_1\lambda_2}-e^{2Y_2\lambda_2}\right)\\Y_1\lambda_2\left(2+M_2\left(-1+Y_2\right)Y_2\lambda_2^2\right)\\+\delta\left(e^{2Y_1\lambda_2}+e^{2Y_2\lambda_2}\right)\\\left(2+M_2\left(\left(-1+Y_2\right)Y_2+Y_1\left(-1+2Y_2\right)\right)\lambda_2^2\right)\end{pmatrix}}{\left(2M_2\lambda_2\begin{pmatrix}-e^{2Y_2\lambda_2}\left(-1+\delta Y_2\lambda_2\right)\left(-1+\delta\left(-1+Y_2\right)\lambda_2\right)\\+e^{2Y_1\lambda_2}\left(1+\delta Y_1\lambda_2\right)\left(1+\delta\left(-1+Y_2\right)\lambda_2\right)\end{pmatrix}\right)} \tag{3.23}$$

Mean velocity

$$U_m=U_{m,1}+U_{m,p}+U_{m,2}=\int_0^{Y_1}U_{m,1}+\int_{Y_1}^{Y_2}U_{m,p}+\int_{Y_2}^{1}U_{m,2} \tag{3.24}$$

Where U_m represents the mean velocity and $U_{m,1},U_{m,p}$ and $U_{m,2}$ are the mean velocities in lower fluid flow region, porous medium and upper fluid flow region respectively and are given as

$$\begin{aligned}U_{m,1}&=\int_0^{Y_1}U_{m,1}=\int_0^{Y_1}\left(-\frac{1}{2}Y^2+C_1Y+C_2\right)dY\\&=-\frac{Y_1^3}{6}+\frac{C_1Y_1^2}{2}+C_2Y_1\end{aligned} \tag{3.25}$$

$$\begin{aligned}U_{m,p}&=\int_{Y_1}^{Y_2}U_{m,p}=\int_{Y_1}^{Y_2}\left(\frac{1}{M_2\lambda_2^2}+e^{Y\lambda_2}C_3+e^{-Y\lambda_2}C_4\right)dY\\&=\frac{C_3}{\lambda_2}\left(e^{Y_2\lambda_2}-e^{Y_1\lambda_2}\right)+\frac{C_4}{\lambda_2}\left(e^{-Y_2\lambda_2}-e^{-Y_1\lambda_2}\right)+\frac{\left(Y_2-Y_1\right)}{M_2\lambda_2^2}\end{aligned} \tag{3.26}$$

$$\begin{aligned}U_{m,2}&=\int_{Y_2}^{1}U_{m,2}=\int_{Y_2}^{1}\left(-\frac{1}{2}Y^2+C_5Y+C_6\right)dY\\&=-\frac{1}{6}+\frac{Y_2^3}{6}-\frac{C_5Y_2^2}{2}-C_6Y_2+\frac{C_5}{2}+C_6\end{aligned} \tag{3.27}$$

By considering the above Eq. (3.24) to (3.26), Eq. (3.23) takes the form

$$\begin{aligned}U_m&=\frac{1}{6}\left(Y_2^3-Y_1^3-1\right)+\frac{C_1Y_1^2-C_5\left(Y_2^2-1\right)}{2}+C_2Y_1\frac{C_3}{\lambda_2}\begin{pmatrix}e^{Y_2\lambda_2}\\-e^{Y_1\lambda_2}\end{pmatrix}\\&+\frac{C_4}{\lambda_2}\left(e^{-Y_2\lambda_2}-e^{-Y_1\lambda_2}\right)-C_6Y_2+C_6+\frac{\left(Y_2-Y_1\right)}{M_2\lambda_2^2}\end{aligned} \tag{3.28}$$

3.4 Results and Discussion

In this section, the results for the velocity distribution under the effect of Darcy number with taking different length of porous region are shown in Fig. 3.1 and 3.2. The velocity profile throughout the channel for the case $Y_1 = 0.2,\ Y_2 = 0.8$ under the effect of Darcy number is shown in Fig. 3.1. In this case, the velocity in the fluid region as well as in the porous region increases with the increase in the Darcy number.

Fig. 3.2, is the representation of the velocity distribution under the influence of Darcy number by taking short length of porous region for example $Y_1 = 0.3,\ Y_2 = 0.7$ as compare to pervious Fig. 3.1. We get the same results as in Fig. 3.1 for the Darcy number, but velocities in free flow regions are boosted by decreasing the length of porous region.

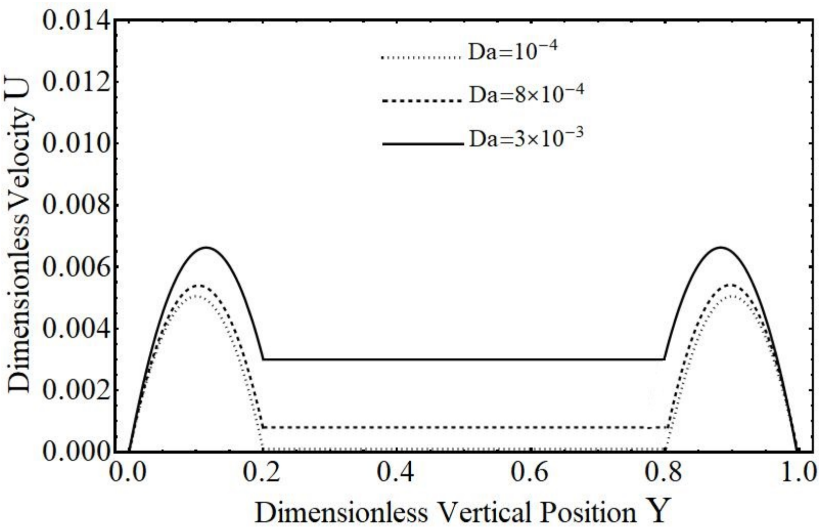

Fig. 3.1: Velocity profile in partially filled porous channel for $Y_1 = 0.2$, $Y_2 = 0.8$, $M_2 = 3.2\times10^{-3}$ and $\delta = 0.01$.

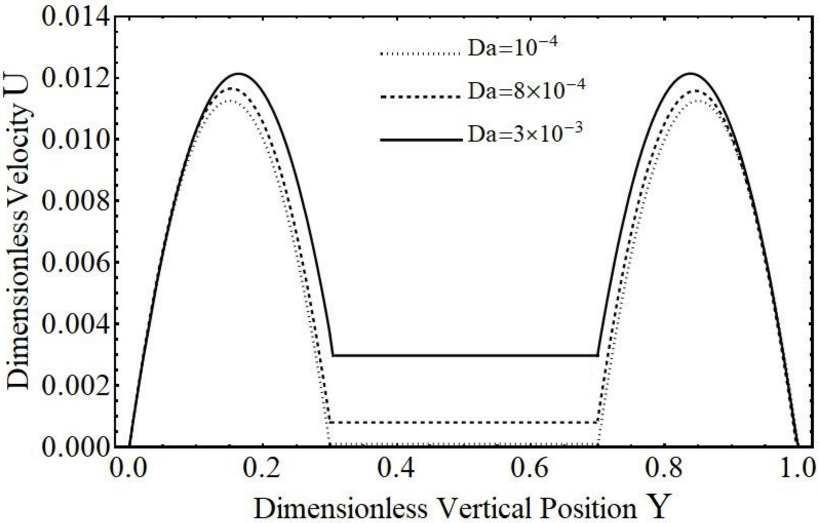

Fig. 3.2: Velocity profile in partially filled porous channel for $Y_1 = 0.3, Y_2 = 0.7$, $M_2 = 2.2\times10^{-3}$ and $\delta = 0.01$.

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