Characterization of Line Total Simplicial Complexes

By

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CIIT/FA16-RMT-033/LHR

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Ayesha Andalib kiran

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I **Ayesha Andalib Kiran CIIT/FA16-RMT-033/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of HEC.

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Certificate

It is certified that **Ayesha Andaib Kiran** (**CIIT/FA16-RMT-033/LHR)** has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus, and the work fulfills the requirement for award of MS degree.

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DEDICATION

My success is really the fruit of sincerest prayers of my father (Prof. M. Ashfaq), and my sister (Prof. Dr. Asma Ashfaq) who always courage me and be my inspiration in study. And support me morally and financially during the entire period of studies. There is a big contribution of my family who ever remained ready to lend me a helping hand during the period of research.

Ayesha Andalib Kiran

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In the name of Allah, the Most Gracious and the Most Merciful
The Prophet (S.A.W) said:
"Seeking knowledge is obligatory for every Muslim"

First, I recognize the creator and finisher of my faith and in memorial of my life, I say thank ALLAH Almighty and Prophet Muhammad (S.A.W) for everything. I am grateful to Almighty ALLAH, without whose endowments it was difficult to finish this proposal.

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Abstract

In 2017, Ahmed and Mahmood introduced the line simplicial complex $\Delta_L(G)$ associated to a simple graph G. The p^{th} Betti number is the rank of the p^{th} homology group of a topological space and it refers to the number of n-dimensional holes on a topological surface. In Theorem 3.0.15, we give formula for Betti numbers of the line simplicial complex $\Delta_L(W_{n+1})$ associated to wheel graph W_{n+1} by writing code in MATLAB.
We define first the line total simplicial complex $\Delta_T(G)$ associated to a simple graph G. In Theorem 4.0.24, we prove that the line total simplicial complex $\Delta_T(G)$ is connected iff G is connected. In Lemma 4.0.19, we find f-vector of the line total simplicial complex $\Delta_T(F_{2n+1})$ associated to friendship graph F_{2n+1}. In Theorem 4.0.20, we give formula for Betti numbers of $\Delta_T(F_{2n+1})$ by coding in MATLAB.

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Chapter 1

Introduction

A graph $G(V, E)$ is an orderd pair set of its vertex set V and the edge set E. The graph G is said to be simple if it has neither loop nor multiple edges. Two vertices (edges) are called adjacent if there is a common edge (vertex) between them. A vertex i and an edge j are said to be incident if the vertex i lies on j. A graph is said to be connected if there exists a path between every pair of its vertices. A collection of finite nonempty sets is called an abstract simplicial complex Δ, if F belongs to Δ then every nonempty subset of F will also belong to Δ. The sets belong to Δ are called simplices while the subsets of a simplex are faces. A simplex is said to be maximal if it is not contained in any other simplex. The maximal simplicies are called facets. The dimension of a simplex is one less than the number of its elements. The dimension of Δ is the largest dimension of one of its simplices. The homomorphism $\partial_p : C_p(\Delta) \to C_{p-1}(\Delta)$ is called boundary operator, where $C_p(\Delta)$ is a free abelian group generated by p-dimensional faces of Δ. The kernel of ∂_p is called the group of p-cycles and image of $\partial_{p+1} : C_{p+1}(\Delta) \to C_p(\Delta)$ is called the group of p-boundries. Let $H_p(\Delta) = \frac{\ker \partial_p}{Im\, \partial_{p+1}}$ is called the p^{th} homology group of the simplicial complex Δ. The Betti number of a simplicial complex Δ is defined as $\beta_p(\Delta) =$ dim(ker ∂_p) - dim($Im\, \partial_{p+1}$).

In Chapter 2, we recall first simplicial complexes then we discuss some important properties of simplicial homology and boundary maps.

In Chapter 3, we define the line simplicial complex $\Delta_L(G)$, see [1].

We give formula for Betti numbers of the line simplicial complex $\Delta_L(W_{n+1})$ associated to wheel graph W_{n+1} by writing code in MATLAB.

In Chapter 4, we define first the line total simplicial complex $\Delta_T(G)$ associated to a simple graph G. In Theorem 4.0.24, we prove that the line total simplicial complex $\Delta_T(G)$ is connected iff G is connected. In Lemma 4.0.19, we find f-vector of the line total simplicial complex $\Delta_T(F_{2n+1})$ associated to the friendship graph F_{2n+1}. In Theorem 4.0.20, we give formula for Betti numbers of the line total simplicial complex $\Delta_T(F_{2n+1})$ by coding in MATLAB.

Chapter 2

Simplicial Homology

The polytope theory is a motivation for simplicial complexes in which simplicial complex appears as a boundary of simplicial polytopes. In combinatorics abstract simplicial complexes are recognized as independence systems, inherited set systems, descending-closed set systems and were first introduced by Pavel Alexandrov as the models of topological spaces. Simplicial complexes are structures that are constructed by points, lines and triangles.

Definition 2.0.1. [1] "A **simplicial complex** Δ with vertex set $X = \{1, ..., n\}$ is a finite collection of subsets of X such that (1) If for $F \in \Delta$ and $K \subseteq F$ then $K \in \Delta$;

(2) If $F, K \in \Delta$ then $F \cap K$ is either empty or is a face of both F and K".

The sets belongs to Δ are called **simplices** while the subsets of a simplex are faces. A simplex is said to be **maximal** if it is not contained in any other simplex. Maximal simplicies are known as **facets**. The largest dimension of one of the simplices of Δ is dimension of Δ. An abstract simplicial complex Δ is called pure if every facet has the same dimension. A graph G is connected if between every pair of its vertices there exists a path.

Definition 2.0.2. "A simplicial complex Δ is called connected if for any two facets F and F' of Δ , there exists a sequence of facets $F = F_0, ..., F_m = F'$ such that $F_i \cap F_{i+1} \neq \emptyset$ for any $i = 0, ..., m-1$."

Example 2.0.3. Let $X = \{1, 2, 3, 4\}$ then

$$\Delta = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}\}$$

is a simplicial complex with $\dim(\Delta) = 2$ because $\dim(F) = |1, 2, 3| - 1 = 2$

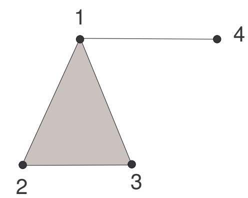


Figure 2.1: Simplicial Complex

Example 2.0.4. Let $X = \{1, 2, 3, 4\}$ then

$$\Delta = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$$

is not a simplicial complex.

Example 2.0.5. Let we have a finite set M then the power set of M is pure abstract simplicial complex. An undirected (simple) graph is a pure 2-dimensional abstract simplicial complex with no isolated vertices.

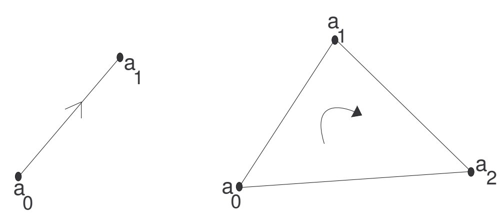
Definition 2.0.6. "Let Δ be a simplicial complex and L be a sub-collection of Δ that contains all faces of its elements, then L is a simplicial complex on its on right, L is called a sub-complex of Δ."

The development of combinatorial methods is one of the major directions in topology after Poincar*é*. Particularly the theory of simplicial complexes, simplicial homology, etc, through which continuous problems about spaces are converted into purely combinatorial problems that can be solved by computer [10].

Definition 2.0.7. [12]

Two orderings of vertex set of a simplex S (either geometric or abstract) are said to equivalent if they are different from each other by an even permutation. If $\dim(S) > 0$, then the ordering of its vertices can be classified in two equivalence classes. Each class is called an **orientation** of S. There is only one class for a 0-simplex S, so there is only one orientation of S. A simplex S together with an orientation is called **oriented simplex**. Let $a_0, ..., a_p$ are independent points then the simplex they span is denoted the symbol $a_0, ..., a_p$, we denote the equivalence class of the particular ordering $(a_0, ..., a_p)$. We denote oriented simplex containing the simplex $a_0, ..., a_p$ by the symbol $[a_0, ..., a_p]$. We use S to denote either a simplex or an oriented simplex.

Example 2.0.8. The 1-simplex can be oriented by drawing an arrow on it. The oriented 1-simplex $[a_0, a_1]$ is shown in following Figure. An arrow is drawn which is pointing from a_0 to a_1. A 2-simplex is figured by drawing circular arrow. The oriented 2-simplex $[a_0, a_1, a_2]$ is shown in following figure by drawing an arrow directed from a_0 to a_1 to a_2. One can check that $[a_1, a_2, a_0]$ and $[a_2, a_0, a_1]$ are specified by the same clockwise arrow. The oppositely oriented simplex is indicated by drawing an arrow in the counterclockwise direction



Definition 2.0.9. "Let us define a Homomorphism

$$\partial_p : C_p(\Delta) \to C_{p-1}(\Delta)$$

called **boundary operator**, where $C_p(\Delta)$ is a free abelian group generated by p-dimensional faces of the simplicial complex Δ. If $S = [a_0, ..., a_p]$ is an oriented simplex with $p > 0$, we define

$$\partial_p(S) = \partial_p[a_0, ..., a_p] = \sum_{t=0}^{p} (-1)^t [a_0, ..., \hat{a}_t, ...a_p] \tag{2.0.1}$$

where the symbol $\hat{a}_t$ means that the vertex a_t is to be deleted from array. Since $C_p(\Delta)$ is the trivial group for $p < 0$, the operator ∂_p is trivial homomorphism for $p \leq 0$.

∂_p is well defined and $\partial_p(-S) =-\partial_p(S)$. The right-hand side of above equation changes signs if we exchange two adjacent vertices in the array $[a_0, ..., a_p]$".

So let us compute expression for,

1 $\partial_p[a_0, ..., a_u, a_{u+1}, ..., a_p]$ and

2 $\partial_p[a_0, ..., a_{u+1}, a_u, ..., a_p]$.

for $t \neq u$, $u+1$, the t^{th} term in these two expression differ precisely by a sign; the terms are same except that a_u, a_{u+1} have been swapped.

what about it u^{th} term for $t = u$ and $t = u + 1$ in first expression, one has $(-1)^u[..., a_{u-1}, a_u, a_{u+1}, a_{u+2}, ...] + (-1)^{u+1}[..., a_{u-1}, a_u, a_{u+1}, a_{u+2}, ...]$

in second expression,one has

$(-1)^u[..., a_{u-1}, a_{u+1}, a_u, a_{u+2}, ...] + (-1)^{u+1}[..., a_{u-1}, a_{u+1}, a_u, a_{u+2}, ...]$

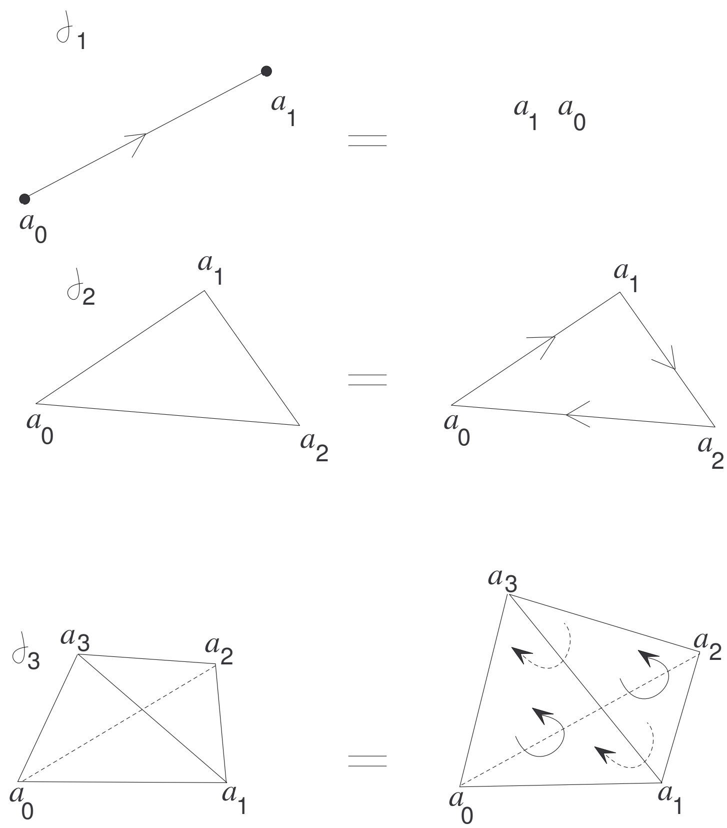
comparing, one sees these two expressions differ by sign [12].

Example 2.0.10. For a 1-simplex we compute $\partial_1[a_0, a_1] = a_1 - a_0$. In case of a 2-simplex, we have

$$\partial_2[a_0, a_1, a_2] = [a_1, a_2] - [a_0, a_2] + [a_0, a_1].$$

similarly we have formula for 3-simplex

$$\partial_3[a_0, a_1, a_2, a_3] = [a_1, a_2, a_3] - [a_0, a_2, a_3] + [a_0, a_1, a_3] - [a_0, a_1, a_2]$$



Lemma 2.0.11. [12] " $\partial_{p-1} \circ \partial_p = 0$.

Proof.

The proof is straightforward. We compute

$$\partial_{p-1}\partial_p[a_0, ..., a_p] = \sum_{t=0}^{p}(-1)^t \partial_{p-1}[a_0, ..., \hat{a}_t, ..., a_p]$$

$$= \sum_{u<t}(-1)^t(-1)^u[..., \hat{a}_u, ..., \hat{a}_t, ...]$$

$$+ \sum_{u>t}(-1)^t(-1)^{u-1}[..., \hat{a}_t, ..., \hat{a}_u, ...].$$

the terms of these two summations cancel in pairs."

Definition 2.0.12. [12] "The kernel of boundary map $\partial_p : C_p(\Delta) \to C_{p-1}(\Delta)$ is known as the group of p-cycles and is denoted $Z_p(\Delta)$ similarly the image of $\partial_{p+1} : C_{p+1}(\Delta) \to C_p(\Delta)$ is known as the group of p-boundaries and is denoted by $B_p(\Delta)$ by proceeding lemma, each boundary of a $p+1$ chain is automatically a p-cycles. That is

$$B_p(\Delta) \subset Z_p(\Delta).$$

We define

$$H_p(\Delta) = Z_p(\Delta)/B_p(\Delta),$$

and call it the p^{th} **homology group** of Δ."

Definition 2.0.13. [12] "A chain complex Δ is a family $\{C_p, \partial_p\}$ of abelian groups C_p and homomorphisms

$$\partial_p : C_p \to C_{p-1},$$

indexed with the integers, such that $\partial_p \circ \partial_{p+1} = 0$ for all p.

If C_p is free abelian group for each p, then Δ is called a **free chain complex**".

Definition 2.0.14. [12] The **simplicial homology group** $H_p(\Delta)$ is defined as $H_p(\Delta) = \frac{\ker \partial_p}{Im\, \partial_{p+1}}$

is known as the p^{th} homology group of the chain complex Δ.

The simplicial homology can be computed reasonably easily by given a chain complex structure on Δ .

Definition 2.0.15. [10] The **Betti numbers** are topological objects. *Poincaré* proved Betti numbers as invariants and he used them to extend the polyhedral formula to higher dimensional spaces. The rank of the p^{th} homology group of a topological space is known as p^{th} Betti number . Informally, the number of p-dimensional holes on a topological surface is referred by p^{th} Betti number [10]. The p^{th} Betti number of a simplicial complex Δ is defined as

$$\beta_p(\Delta) = \dim(\ker \partial_p) - \dim(Im\, \partial_{p+1}).$$

Chapter 3

Line Simplicial Complexes

The line Simplicial Complex is a well studied graph and was first introduced by Harry and Norman [11] in 1960. Many authors discussed various Properties of line simplicial complexes, and a most recent book by Prisoner [13] explain many interesting generalization of line graph. Much research has been done on the study and application of line graph a comprehensive survey of results is found in [14].

Definition 3.0.16. [1] "Let G be a finite simple graph. The graph $L(G)$ is said to be line graph of G if each vertex of $L(G)$ represents an edge of G and two vertices of $L(G)$ are adjacent if and only if their corresponding edges are adjacent in G."

Example 3.0.17. For the graph G its line graph shown in figure below

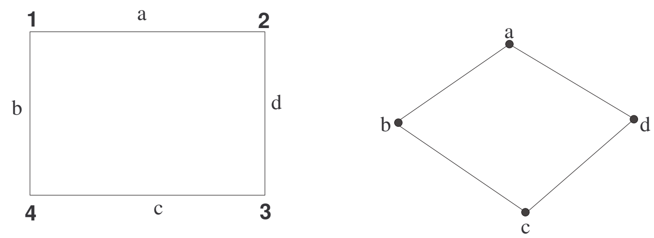


Figure 3.1: simple graph and its line graph $L(G)$

Definition 3.0.18. [1] "Let G be a finite simple graph on the vertex set $V(G)$. Let $E(G) = \{e_{i,j} = \{i,j\} | i,j \in V(G)\}$ be the edge set of G. We define the set of line indices $\Upsilon(G)$ associated to the graph G as the collection of subsets of such that if $e_{i,j}$ and $e_{j,k}$ are adjacent in $L(G)$, then $F_{i,j,k} = \{i,j,k\} \in \Upsilon(G)$ or if $e_{i,j}$ is an isolated vertex in $L(G)$ then $F_{i,j} = \{i,j\} \in \Upsilon(G)$."

Definition 3.0.19. [1] "A line simplicial complex $\Delta_L(G)$ of G is a simplicial complex on the vertex set $V(G)$ such that $\Delta_L(G) =< F | F \in \Upsilon(G) >$, where $\Upsilon(G)$ is the set of line indices of G."

Theorem 3.0.20. Let $\Delta_L(W_{n+1})$ be a line simplicial complex associated to wheel graph W_{n+1} having $n+1$ vertices and $2n$ edges. Then, the Betti numbers of $\Delta_L(W_{n+1})$ are given by

$$(\beta_0, \beta_1, \beta_2) = (1, 0, n)$$

for $n \geq 3$.

Proof. Let $\Delta_L(W_{n+1})$ be a line simplicial complex of the wheel graph W_{n+1} having $n+1$ vertices and $2n$ edges with $n \geq 3$. As show in figure below

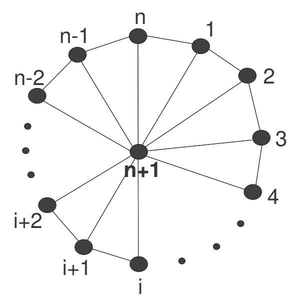


Figure 3.2: W_{n+1}

We write code in MATLAB to compute the Betti numbers of $\Delta_L(W_{n+1})$.

```
clear all
clc
r= 'Enter value of n ≥ 3 ';
n = input(r)
M = [ ];
N = [ ];
storex6=[ ];
for i=1:n+1
for j=i+1:n+1
res6 = sym(['v',num2str(j),'-','v',num2str(i)])
storex6=[storex6;res6]
end
end
p=[storex6]
[A] = equationsToMatrix([p])
textnew_a = A;
new_a( any(A,2),:)=[ ];
M=[M;new_a]
```

```
storex1=[ ];
for i=1
for j=2
for k=n
res1 = sym(['e',num2str(j),num2str(k),'-','e',num2str(i),num2str(k)
,'+','e',num2str(i),num2str(j)])
storex1=[storex1;res1]
end
end
end
storex2= [ ];
for i = 1:n
for k =n+1
for j= i+1:n
res2 = sym(['e',num2str(j),num2str(k),'-','e',num2str(i),num2str(k)
,'+','e', num2str(i),num2str(j)])
storex2=[storex2;res2]
end
end
end
storex3= [ ];
for i=1:n-2
res3 = sym(['e',num2str(i+1),num2str(i+2),'-','e',num2str(i),num2str(i+2)
,'+','e', num2str(i),num2str(i+1)])
storex3=[storex3;res3]
end
```

```
storex4= [ ];
for i = 1
for j = n-1
for k =n
res4 = sym(['e',num2str(j),num2str(k),'-','e',num2str(i),num2str(k)
,'+','e', num2str(i),num2str(j)])
storex4= [storex4;res4]
end
end
end
r=[(storex1);(storex2);(storex3);(storex4)]
[B] = equationsToMatrix([r]) new_b = B;
new_b( any(B,2),:)=[ ];
N=[N;new_b]
D1 = transpose(M)
rankD1 = rank(D1)
Z1 = null(D1)
nullityOfD1 = size(Z1, 2)
D2 =transpose(N)
rankD2 = rank(D2)
Z2 = null(D2)
nullityOfD2 = size(Z2, 2)
f0=n+1
f1=size(p,1)
f2=size(r,1)
b0 = (n+1)-rankD1
```

b1 = nullityOfD1-rankD2

b2= nullityOfD2-0 □

Chapter 4

Line Total Simplicial Complexes

The total graph was first introduced by Behzad and Chartrand [2]. Most of the properties of the total graphs are discovered in the literature (see [3], [4], [5], [6]). Behzad obtained a characterization of total graphs.

Definition 4.0.21. [9] “The total graph $T(G)$ of a graph G is defined to be the graph having

$$V(T(G)) = V(G) \cup E(G)$$

where $V(G)$ be the vertex set and $E(G)$ be the edge set of G then $\{i, j\} \in E(T(G))$ if

(1) i, j are adjacent vertices in G or;

(2) i is incident to j or j is incident to i or;

(3) i and j are adjacent edges in G.”

Definition 4.0.22. Let $G(V, E)$ be a finite simple graph line total indices $\Upsilon_T(G)$ of the graph G defined as the collection of subsets of $(V(G) \cup E(G))$ such that

(1) If i, j and k are three adjacent vertices in G then $F_{i,j,k} = \{i, j, k\} \in \Upsilon_T(G)$;

(2) If i, j and k are three adjacent edges in G then $F_{i,j,k} = \{i, j, k\} \in \Upsilon_T(G)$;

(3) If i and j are adjacent vertices and k is an edge which is incident to i or j then $F_{i,j,k} = \{i, j, k\} \in \Upsilon_T(G)$;

(4) If i and j are adjacent edges and k is a vertex such that i or j is incident to k, then $F_{i,j,k} = \{i, j, k\} \in \Upsilon_T(G)$;

(5) If i be an isolated vertex in G then $i \in \Upsilon_T(G)$.

Definition 4.0.23. The line total simplicial complex $\Delta_T(G)$ of a graph G defined over $V(G) \cup E(G)$ such that $\Delta_T(G) =< F | F \in \Upsilon_T(G) >$, where $\Upsilon_T(G)$ is the set of line total-indices of G.

Theorem 4.0.24. Let G be a finite simple graph with vertex set $V(G)$ and edge set $E(G)$. Then G is connected iff the line total simplicial complex $\Delta_T(G)$ is connected.

Proof. Let G be a finite simple graph with vertex set $V(G)$ and edge set $E(G)$ such that $V(G) \cup E(G) = [n]$. For $n \leq 3$, the result is trivial. Therefore, we take $n \geq 4$. Let G be a connected graph and let us assume on contrary that the line total simplicial complex $\Delta_T(G)$ is not connected. Since G is connected, therefore the line total simplicial complex is pure of dimension 2. Then there exist atleast two facets $F_{p,q,r}$ and $F_{s,t,u}$ such that $F_{p,q,r} \cap F_{s,t,u} = \emptyset$. Consequently, any facet that belongs to the line total simplicial complex $\Delta_T(G)$ either has vertices from a subset V_1 or V_2 of $V(G)$. Therefore, there exist atleast two non-empty subsets V_1 and V_2 of $V(G)$ such that $V(G) = V_1 \cup V_2$ with $V_1 \cap V_2 = \emptyset$, a contradiction. Now we prove the converse implication. Let us consider contrary that G is not connected then there exist two vertices $p, q \in G$ such that there is no path between p and q it implies that there is no face $\Delta_T(G)$ containing both vertices p and q i.e $\Delta_T(G)$ is not connected. Hence proved. $\square$

Lemma 4.0.25. Let F_{2n+1} be a friendship graph having $2n+1$ vertices and $3n$ edges. Then the f-vector of line total simplicial complex $\Delta_T(F_{2n+1})$ associated to F_{2n+1} is
$(f_0, f_1, f_2) = (5n+1, 10n^2+5n, \frac{4n^3+42n^2+14n}{3})$.

Proof. We compute first the total number of two dimensional face of line total simplicial complex $\Delta_T(F_{2n+1})$ associated to F_{2n+1}.

1. $|F_{i,i+1,i+2}| = n$ with $i = 1, 4, ..., 3n-2$;

2. $|F_{i,j,k}| = 2n$ with $i = 1, 4, ..., 3n-2$, $j = 3n+1, ..., 5n$, $k = 5n+1$;

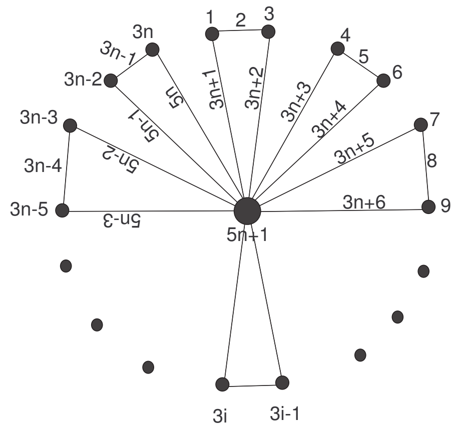


Figure 4.1: F_{2n+1}

3. $|F_{i,j,k}| = 2n$ with $i = 3, 6, ..., 3n,\ j = 3n+1, ..., 5n,\ k = 5n+1$;

4. $|F_{3n+i,3n+j,3n+k}| = \frac{2(n)(n-1)(2n-1)}{3}$ with $i = 1, ..., 2n,\ j = i+1, ..., 2n,\ k = j+1, ..., 2n$;

5. $|F_{3n+i,3n+j,k}| = n(2n-1)$ with $i = 1, ..., 2n$, $j = i+1, ..., 2n$, $k = 5n+1$;

6. $|F_{i,i+1,j}| = 2(2n)$ for $i = 1, 4, ..., 3n-2,\ j = 3n+1, 3n+k+1$ and $k = 1, 3, ..., 2n$ also for $i = 2, 5, ..., 3n-1$ and same value of j and k;

7. $|F_{i,i+1,k}| = 2n$ with $k = 5n+1$, $i = 1, 4, ..., 3n-2$ and $i = 2, 5, ..., 3n-1$;

8. $|F_{i,j,k}| = n(2n-1)$ with $k = 5n+1$, $i = 1, 4, ..., 3n-2$, $j = i+3, ..., 3n$ and also for $j = j-1,\ i = 1, 4, ..., 3n-2$ and for $i = 3, 6, ..., 3n$, $j = i+1$,for $i = 3, 6, ..., 3n,\ j = j+2$ and for $i = 1, 4, ..., 3n-2$, $j = 3n$;

9. $|F_{i,3n+j,5n+1}| = 2n$ with $i = 2, 5, ..., 3n-1$, $j = 1, 3, ..., 2n$ and $j = j+1$;

10. $|F_{3n-i,5n-k,5n-j}| = 2n(n-1)$ with $i = 1, 4, ..., 2n-1$, $j = 0, 2, 4, ..., 2n-3$, $k = j+2, ..., 2n-1$; also for $i = 1, 4, ..., 2n-1$, $j = 1, 3, ..., 2n-2$ and $k = j+1, ..., 2n-1$;

11. $|F_{i,i+2,3n+j}| = 2n$ with $i = 1, 4, ..., 3n-2$, $j = 1, 3, ..., 2n$ and $j = j+1$;

12. $|F_{i,3n+j,3n+k}| = n(2n-1)$ with $i = 1, 4, ..., 3n-2$, $j = 1, 3, ..., 2n$ and $k = j+1, ..., 2n$ also $i = 3, 6, ..., 3n$, $j = 2, 4, ..., 2n$, $k = j+1, ..., 2n$;

13. $|F_{3n-i,5n-k,5n-j}| = n(2n-1)$ with $i = 2, 5, ..., 3n-3$, $j = 1, 3, ..., 2n-2$ and $k = j+1, ..., 2n-1$; also $i = 0, 3, 6, ..., 3n-2$, $j = 0, 2, 4, ..., 2n-2$ and $k = j+1, ..., 2n-1$;

    Hence $f_2 = n + 4n^2 + n(2n-1) + 4n + 2n + n(2n-1) + 2n$
    $+2n+n(2n-1)+2n(n-1)+n(2n-1)+n(2n-1)+2/3(n)(n-1)(2n-1)$
    $= (4n^3 + 42n^2 + 14n)/3$.

Now, we compute the number of one dimensional faces.

1. $|\{i,j\}| = n(2n-1)$ with $i = 1, 3, 4, ..., 3n$, $j = i+1, ..., 3n$;

2. $|\{i,j\}| = 2n$ with $i = 1, 3, ..., 3n$, $j = 5n+1$;

3. $|\{i,j\}| = n$ with $i = 1, 4, 7, ..., 3n-2$, $j = 2, 5, ..., 3n-1$;

4. $|\{i,j\}| = 4n^2$ with $i = 1, 3, 4, 6, ..., 3n$, $j = 3n+1, ..., 3n$;

5. $|\{i,j\}| = n(2n+2)$ with $i = 2, 5, 8, ..., 3n-1$, $j = 3, 6, 9, ..., 3n$; also $j = 3n+1, .., 3n$ and $j = 5n+1$

6. $|\{i,j\}| = n(2n+1)$ with $i = 3n+1, ..., 5n$, $j = 3n+2, ..., 5n$; also $j = 5n+1$

Therefore, $f_1 = n(2n-1)+2n+n+4n^2+n(2n+2)+n(2n+1) = 10n^2+5n$. $\square$

Theorem 4.0.26. Let $(\Delta_T(F)_{2n+1})$ be the line total simplicial complex associated to friendship graph F_{2n+1} with $2n+1$ vertices. Then the Betti numbers of $(\Delta_T(F)_{2n+1})$ is

$$(\beta_0, \beta_1, \beta_2) = (1, 0, \frac{4n^3 + 42n^2 + 14n}{3})$$

for $n \geq 3$.

Proof. We write code in MATLAB to compute the Betti numbers of $(\Delta_T(F)_{2n+1})$.

```
clear all
clc
r= ' Enter value of n ≥ 1 ';
n = input(r)
N=[ ];
M=[ ];
storex1= [ ];
for i=1:3:3*n-2
resx1=[sym(['v',num2str(i+1),'-','v',num2str(i)]);
sym(['v',num2str(i+2),'-','v',num2str(i)]);
sym(['v',num2str(i+2),'-','v', num2str(i+1)])]
storex1=[storex1;resx1]
end
V1=[storex1]
storex2=[ ];
for i =1:3:3*n
for j = 3*n+1:5*n
for k =5*n+1
resx2=[sym(['v',num2str(j),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v', num2str(j)])]
```

```
storex2=sym([storex2;resx2])
end
end
end
storex3=[ ];
for i =3:3:3*n
for j = 3*n+1:5*n
for k =(5*n)+1
resx3=[sym(['v',num2str(j),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v', num2str(j)])]
storex3=sym([storex3;resx3])
end
end
end
V2=[storex2;storex3]
storex4=[ ];
for i = 1:2*n
for j = i+1:2*n
for k = j+1:2*n
resx4=[sym(['v',num2str(3*n+j),'-','v',num2str(3*n+i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(3*n+i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(3*n+j)])]
storex4=sym([storex4;resx4])
end
end
```

```
end
V3=[storex4]
storex5=[ ];
for i = 1:2*n
for j = i+1:2*n
for k = 5*n+1
resx5=[sym(['v',num2str((3*n)+j),'-','v',num2str((3*n)+i)]);
sym(['v',num2str(k),'-','v',num2str((3*n)+i)]);
sym(['v',num2str(k),'-','v',num2str((3*n)+j)])]
storex5=sym([storex5;resx5])
end
end
end
V4=[storex5]
storex15=[ ];
storex7=[ ];
for i=1:3*n-1
for k=1:2:2*n
resx15=[sym(['v',num2str(i+1),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i+1)])]
storex15=[storex15;resx15]
resx7=[sym(['v',num2str(i+1),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k+1),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k+1),'-','v',num2str(i+1)])]
storex7=[storex7;resx7]
```

```
i=i+3
end
break
end
storex19=[ ];
storex9=[ ];
for i=2:3*n-1
for k=1:2:2*n
resx19=[sym(['v',num2str(i+1),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i+1)])]
storex19=[storex19;resx19]
resx9=[sym(['v',num2str(i+1),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k+1),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k+1),'-','v',num2str(i+1)])]
storex9=[storex9;resx9]
i=i+3
end
break
end
V5=([storex15;storex7;storex19;storex9])
storex10=[ ];
for i=1:3:3*n-1
for k=5*n+1
resx10=[sym(['v',num2str(i+1),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
```

```
sym(['v',num2str(k),'-','v',num2str(i+1)])]
storex10=[storex10;resx10]
i=i+3
end
end
storex11=[ ];
for i=2:3:3*n-1
for k=5*n+1
resx11=[sym(['v',num2str(i+1),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i+1)])]
storex11=[storex11;resx11]
end
end
V6=([storex10;storex11])
storex12=[ ];
for i = 1:3:3*n
for j = i+3:3:3*n
for k = 5*n+1
resx12=[sym(['v',num2str(j-1),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(j-1)])]
resx12a = [sym(['v',num2str(j),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(j)])]
storex12=[storex12;resx12;resx12a]
```

```
end
end
end
storex13=[ ];
for i = 3:3:3*n
for j = i+1:3:3*n
for k = 5*n+1
if i =j
resx13=[sym(['v',num2str(j),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(j)])]
resx13a = [sym(['v',num2str(j+2),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(i)]);
sym(['v',num2str(k),'-','v',num2str(j)])]
storex13=[storex13;resx13;resx13a]
end
end
end
end
storex14=[ ];
for i = 1:3:3*n-2
resx14=[sym(['v',num2str(3*n),'-','v',num2str(i)]);
sym(['v',num2str(5*n+1),'-','v',num2str(i)]);
sym(['v',num2str(5*n+1),'-','v',num2str(3*n)])]
storex14=[storex14;resx14]
end
```

```
V7=[storex12;storex13;storex14]
storex15=[ ];
for i = 2:3*n-1
for j = 1:2:2*n
resx15=[sym(['v',num2str(3*n+j),'-','v',num2str(i)]);
sym(['v',num2str(5*n+1),'-','v',num2str(i)]);
sym(['v',num2str(5*n+1),'-','v',num2str(3*n+j)])]
resx15a=[sym(['v',num2str(3*n+j+1),'-','v',num2str(i)]);
sym(['v',num2str(5*n+1),'-','v',num2str(i)]);
sym(['v',num2str(5*n+1),'-','v',num2str(3*n+j+1)])]
storex15=[storex15;resx15;resx15a]
i=i+3
end
break
end
V8=[storex15]
storex16=[ ];
for i = 2:3*n
for j = 1:2:2*n
for k=j+1:2*n
resx16=[sym(['v',num2str(3*n+j),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(3*n+j)])]
storex16=[storex16;resx16]
end
i=i+3
```

```
end
break
end
storex17=[ ];
for i = 2:3*n
for j = 2:2:2*n
for k=j+1:2*n
resx17=[sym(['v',num2str(3*n+j),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(3*n+j)])]
storex17=[storex17;resx17]
end
i=i+3
end
break
end
storex18=[ ];
for i = 1:2*n-1
for j = 0:2:2*n-3
for k=j+2:2*n-1
resx18=[sym(['v',num2str(5*n-j),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-k),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-j),'-','v',num2str(5*n-k)])]
storex18=[storex18;resx18]
end
i=i+3
```

```
end
break
end
storex19=[ ];
for i = 1:2*n-1
for j = 1:2:2*n-2
for k=j+1:2*n-1
resx19=[sym(['v',num2str(5*n-j),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-k),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-j),'-','v',num2str(5*n-k)])]
storex19=[storex19;resx19]
end
i=i+3
end
break
end
V9=[storex16;storex17;storex18;storex19]
storex19b=[ ];
for i = 1:3*n-1
for j = 1:2:2*n
resx19=[sym(['v',num2str(i+2),'-','v',num2str(i)]);
sym(['v',num2str(3*n+j),'-','v',num2str(i)]);
sym(['v',num2str(3*n+j),'-','v',num2str(i+2)])]
resx19a=[sym(['v',num2str(i+2),'-','v',num2str(i)]);
sym(['v',num2str(3*n+j+1),'-','v',num2str(i)]);
sym(['v',num2str(3*n+j+1),'-','v',num2str(i+2)])]
```

```
storex19b=[storex19b;resx19;resx19a]
i=i+3
end
break
end
V10=[storex19b]
storex20=[ ];
for i = 1:3*n-2
for j = 1:2:2*n
for k=j+1:2*n
resx20=[sym(['v',num2str(3*n+j),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(3*n+j)])]
storex20=[storex20;resx20]
end
i=i+3
end
break
end
storex21=[ ];
for i = 3:3*n
for j = 2:2:2*n
for k=j+1:2*n
resx21=[sym(['v',num2str(3*n+j),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(i)]);
sym(['v',num2str(3*n+k),'-','v',num2str(3*n+j)])]
```

```
storex21=[storex21;resx21]
end
i=i+3
end
break
end
storex22=[ ];
for i = 2:3*n-3
for j = 1:2:2*n-2
for k=j+1:2*n-1
resx23=[sym(['v',num2str(5*n-j),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-k),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-j),'-','v',num2str(5*n-k)])]
storex22=[storex22;resx23]
end
i=i+3
end
break
end
storex23=[ ];
for i = 0:3*n-2
for j = 0:2:2*n-2
for k=j+1:2*n-1
resx23=[sym(['v',num2str(5*n-j),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-k),'-','v',num2str(3*n-i)]);
sym(['v',num2str(5*n-j),'-','v',num2str(5*n-k)])]
```

```
storex23=[storex23;resx23]
end
i=i+3
end
break
end
V11=[storex20;storex21;storex22;storex23]
V=([V1;V2;V3;V4;V5;V6;V7;V8;V9;V10;V11])
p=unique(V)
[A] = equationsToMatrix([p])
textnew_a = A;
new_a( any(A,2),:)=[ ];
M=[M;new_a]
storey1 = [ ];
for i=1:3:3*n-2
resy1 = sym(['e',num2str(i),num2str(i+1),'-',
'e',num2str(i),num2str(i+2),'+','e', num2str(i+1),num2str(i+2)])
storey1=[storey1;resy1]
end
F1=[storey1]
storey2=[ ];
for i =1:3:3*n-2
for j = (3*n)+1:(5*n)
for k =(5*n)+1
resy2 = [sym(['e',num2str(i),num2str(j),'-',
'e',num2str(i),num2str(k),'+','e', num2str(j),num2str(k)])]
```

```
storey2=sym([storey2;resy2])
end
end
end
storey3=[ ];
for i =3:3:(3*n)
for j = (3*n)+1:(5*n)
for k =(5*n)+1
resy3 = [sym(['e',num2str(i),num2str(j),'-',
'e',num2str(i),num2str(k),'+','e', num2str(j),num2str(k)])]
storey3=sym([storey3;resy3])
end
end
end
F2=[storey2;storey3]
storey4=[ ];
for i = 1:2*n
for j = i+1:2*n
for k = j+1:2*n
resy4 = [sym(['e',num2str(3*n+i),num2str(3*n+j),'-',
'e',num2str(3*n+i),num2str(3*n+k),'+','e', num2str(3*n+j),num2str(3*n+k)])]
storey4=sym([storey4;resy4])
end
end
end
F3=[storey4]
```

```
storey5=[ ];
for i = 1:2*n
for j = i+1:2*n
for k = 5*n+1
resy5 = [sym(['e',num2str((3*n)+i),num2str((3*n)+j),'-',
'e',num2str((3*n)+i),num2str(k),'+','e', num2str((3*n)+j),num2str(k)])]
storey5=sym([storey5;resy5])
end
end
end
F4=[storey5]
storey16=[ ];
storey7=[ ];
for i=1:3*n-2
for k=1:2:2*n
resy16 = sym(['e',num2str(i),num2str(i+1),'-',
'e',num2str(i),num2str(3*n+k),'+','e',num2str(i+1),num2str(3*n+k)])
storey16=[storey16;resy16]
resy7 = sym(['e',num2str(i),num2str(i+1),'-',
'e',num2str(i),num2str(3*n+k+1),'+','e',num2str(i+1),num2str(3*n+k+1)])
storey7=[storey7;resy7]
i=i+3
end
break
end
storey21=[ ];
```

```
storey9=[ ];
for i=2:3*n-1
for k=1:2:2*n
resy21 = sym(['e',num2str(i),num2str(i+1),'-',
'e',num2str(i),num2str(3*n+k),'+','e',num2str(i+1),num2str(3*n+k)])
storey21=[storey21;resy21]
resy9 = sym(['e',num2str(i),num2str(i+1),'-',
'e',num2str(i),num2str(3*n+k+1),'+','e',num2str(i+1),num2str(3*n+k+1)])
storey9=[storey9;resy9]
i=i+3
end
break
end
F5=([storey16;storey7;storey21;storey9])
storey10=[ ];
for i=1:3:3*n-2
for k=5*n+1
resy10 = sym(['e',num2str(i),num2str(i+1),'-',
'e',num2str(i),num2str(k),'+','e',num2str(i+1),num2str(k)])
storey10=[storey10;resy10]
i=i+3
end
end
storey11=[ ];
for i=2:3:3*n-1
for k=5*n+1
```

```
resy11 = sym(['e',num2str(i),num2str(i+1),'-',
'e',num2str(i),num2str(k),'+','e',num2str(i+1),num2str(k)])
storey11=[storey11;resy11]
end
end
F6=([storey10;storey11])
storey12=[ ];
for i = 1:3:3*n-2
for j = i+3:3:3*n
for k = 5*n+1
resy12 = sym(['e',num2str(j-1),num2str(k),'-',
'e',num2str(i),num2str(k),'+','e',num2str(i),num2str(j-1)])
resy13a = sym(['e',num2str(j),num2str(k),'-',
'e',num2str(i),num2str(k),'+','e',num2str(i),num2str(j)])
storey12=[storey12;resy12;resy13a]
end
end
end
storey14=[ ];
for i = 3:3:3*n
for j = i+1:3:3*n
for k = 5*n+1
resy14 = sym(['e',num2str(j),num2str(k),'-',
'e',num2str(i),num2str(k),'+','e',num2str(i),num2str(j)])
resy15 = sym(['e',num2str(j+2),num2str(k),'-',
'e',num2str(i),num2str(k),'+','e',num2str(i),num2str(j+2)])
```

```
storey14=[storey14;resy14;resy15]
end
end
end
storey15=[ ];
for i = 1:3:3*n-2
resy15 = sym(['e',num2str(i),num2str(3*n),'-',
'e',num2str(i),num2str(5*n+1),'+','e',num2str(3*n),num2str(5*n+1)])
storey15=[storey15;resy15]
end
F7=[storey12;storey14;storey15]
storey16=[ ];
for i = 2:3*n-1
for j = 1:2:2*n
resy16 = sym(['e',num2str(i),num2str(3*n+j),'-',
'e',num2str(i),num2str(5*n+1),'+','e',num2str(3*n+j),num2str(5*n+1)])
resy16a = sym(['e',num2str(i),num2str(3*n+j+1),'-',
'e',num2str(i),num2str(5*n+1),'+','e',num2str(3*n+j+1),num2str(5*n+1)])
storey16=[storey16;resy16;resy16a]
i=i+3
end
break
end
F8=[storey16]
storey17=[ ];
for i = 2:3*n
```

```
for j = 1:2:2*n
for k=j+1:2*n
resy17= sym(['e',num2str(i),num2str(3*n+j),'-',
'e',num2str(i),num2str(3*n+k),'+','e',num2str(3*n+j),num2str(3*n+k)])
storey17=[storey17;resy17]
end
i=i+3
end
break
end
storey18=[ ];
for i = 2:3*n
for j = 2:2:2*n
for k=j+1:2*n
resy18= sym(['e',num2str(i),num2str(3*n+j),'-',
'e',num2str(i),num2str(3*n+k),'+','e',num2str(3*n+j),num2str(3*n+k)])
storey18=[storey18;resy18]
end
i=i+3
end
break
end
storey19=[ ];
for i = 1:2*n-1
for j = 0:2:2*n-3
for k=j+2:2*n-1
```

```
resy19=sym(['e',num2str(3*n-i),num2str(5*n-k),'-',
'e',num2str(3*n-i),num2str(5*n-j),'+','e',num2str(5*n-k),num2str(5*n-j)])
storey19=[storey19;resy19]
end
i=i+3
end
break
end
storey20=[ ];
for i = 1:2*n-1
for j = 1:2:2*n-2
for k=j+1:2*n-1
resy20=sym(['e',num2str(3*n-i),num2str(5*n-k),'-',
'e',num2str(3*n-i),num2str(5*n-j),'+','e',num2str(5*n-k),num2str(5*n-j)])
storey20=[storey20;resy20]
end
i=i+3
end
break
end
F9=[storey17;storey18;storey19;storey20]
storey2 1=[ ];
for i = 1:3*n-1
for j = 1:2:2*n
resy21 = sym(['e',num2str(i),num2str(i+2),'-',
'e',num2str(i),num2str(3*n+j),'+','e',num2str(i+2),num2str(3*n+j)])
```

```
resy21a=sym(['e',num2str(i),num2str(i+2),'-',
'e',num2str(i),num2str(3*n+j+1),'+','e',num2str(i+2),num2str(3*n+j+1)])
storey21=[storey21;resy21;resy21a]
i=i+3
end
break
end
F10=[storey21]
storey22=[ ];
for i = 1:3*n-2
for j = 1:2:2*n
for k=j+1:2*n
resy22= sym(['e',num2str(i),num2str(3*n+j),'-',
'e',num2str(i),num2str(3*n+k),'+','e',num2str(3*n+j),num2str(3*n+k)])
storey22=[storey22;resy22]
end
i=i+3
end
break
end
storey23=[ ];
for i = 3:3*n
for j = 2:2:2*n
for k=j+1:2*n
resy24= sym(['e',num2str(i),num2str(3*n+j),'-',
'e',num2str(i),num2str(3*n+k),'+','e',num2str(3*n+j),num2str(3*n+k)])
```

```
storey23=[storey23;resy24]
end
i=i+3
end
break
end
storey25=[ ];
for i = 2:3*n-3
for j = 1:2:2*n-2
for k=j+1:2*n-1
resy25=sym(['e',num2str(3*n-i),num2str(5*n-k),'-',
'e',num2str(3*n-i),num2str(5*n-j),'+','e',num2str(5*n-k),num2str(5*n-j)])
storey25=[storey25;resy25]
end
i=i+3
end
break
end
storey26=[ ];
for i = 0:3*n-2
for j = 0:2:2*n-2
for k=j+1:2*n-1
resy26=sym(['e',num2str(3*n-i),num2str(5*n-k),'-',
'e',num2str(3*n-i),num2str(5*n-j),'+','e',num2str(5*n-k),num2str(5*n-j)])
storey26=[storey26;resy26]
end
```

```
i=i+3
end
break
end
F11=[storey22;storey23;storey25;storey26]
F=([F1;F2;F3;F4;F5;F6;F7;F8;F9;F10;F11])
r=([F])
[B] = equationsToMatrix([r])
textnew _ b = B;
new _ b( any(B,2),:)=[ ];
N=[N;new _ b]
D1 = transpose(M)
rankD1 = rank(D1)
Z1 = null(D1)
nullityOfD1 = size(Z1, 2)
D2 = transpose(N)
rankD2 = rank(D2)
Z2 = null(D2)
nullityOfD2 = size(Z2, 2)
f0=5*n+1
f1=size(p,1)
f2=size(F,1)
b0 = (5*n+1)-rankD1
b1 = nullityOfD1-rankD2
b2= nullityOfD2-0
```
□

Chapter 5

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