Chirped and Dipole Soliton in Nonlinear Negative Index Materials

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Certificate

It is certified that **Adnan Khalil(CIIT/FA14-BSM-012/LHR)** has carried out all the work related to this report under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus, and the work fulfills the requirement for award of BS degree.

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DEDICATION

To

My Parents

ACKNOWLEDGEMENT

I'm so much thankful to my God, who gave me the ability and strength of completing my project. Secondly I'm thankful to the Head of Department of Mathematics for providing us research facilities. After that I would like to acknowledge my supervisor Dr. Syed Tahir Raza Rizvi . His support, co-operation, helped me in a truly sense. A special thanks to Dr. Kashif Ali. He gave courage to do my work very effectively and flourished my ideas. My parents have provided me with this great opportunity so a very special thanks to them. The continuing support and love of my brother and sisters also contributed a lot in the completion of my work.

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ABSTRACT

Chirped and Dipole Soliton in Nonlinear Negative Index Materials

A wave which propagates without changing its shape and speed is known as soliton. Whenever two of these waves collide they do not change their shape and speed. Solitons have a lot of applications in other fields like, fluid, biology and engineering. This dissertation find out the chirped and dipole soliton for nonlinear negative-index materials with quadratic-cubic nonlinearity. We will use sub-ODE method [26] to find chirped soliton and ansatz method of Choudhuri and Porsezian [16] to get dipole soliton.

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Chapter 1

Introduction

1.1 Differential Equation

A differential equation refers to a special type of mathematical equation that associates some function to with its derivatives.There are two types of differential equation. One is Ordinary Differential Equation(ODE) and second is partial differential equation(PDE). A differential equation is said to be ODE if it contains ordinary derivative of dependent variable w.r.t a single independent variable. If an equation contains partial derivatives of one or more dependents variables of two or more independent variables is said to a PDE. Highest derivative in a diffrintial equation is said to be a order of a differential equation. A first order PDE for an unknown function $x(r,s)$ is a linear equation if it can be written in the form

$$x(r,s)\frac{\partial x(r,s)}{\partial r}+b(r,s)\frac{\partial x(r,s)}{\partial s}+c(r,s)x(r,s)=d(r,s) \tag{1.1.1}$$

If $d(r,s)$=0, then equation said to be a homogeneoues equation.

1.2 Famous Nonlinear Partial Differential Equation

Benjamin-Ono

$$u_t+Hu_{xx}+uu_x=0 \tag{1.2.1}$$

Boussinesq Equation

$$u_{tt}-u_{xx}-u_{xxxx}-3(u^2)_{xx}=0 \tag{1.2.2}$$

Burgers's Equation

$$u_t+uu_x-vu_{xx}=0 \tag{1.2.3}$$

Benjamin-Bono-Mahony' Equation

$$w_t + w_x + ww_x - w_{xxt} = 0 \tag{1.2.4}$$

1.3 Soliton

The word soliton refers to a special type of wave that travel without loss of energy and maintains its shape and speed. When two soliton collide with each other no effect occur on the both soliton. John Scott Russell [41] observed the solitary wave in a canal, he said that, "I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped ? not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation".

Optical solitons is an emerging field of nonlinear optics. Optical Solitons have brought a revolution in the field of physics and mathematics. Optical solitons are light pulses in time or space, that are localized and during their travel in some host medium they do not disperse or diffract. Optical solitons are of great importance in the present as well as the future telecommunication systems like internet industry. There are two main types of solitons in optics. one is the, temporal solitons hold their temporal width in fibers even if they travel distances of several hundred of kilometers [36]. Spatial soliton is formed when optical soliton propagate in a non linear medium without diffraction, throughout the propagation wave weidth doesn't change. Spatial soliton that propagate several diffraction lengths but not broadening [24]. Incoherent solitons are constructed from white light from a regular incandescent light-bulb. Some other forms are spatio-temporal solitons [30], discrete solitons [20] and bragg solitons [21]. Existence of solitons in nonlinear optical fibres was first predicted by Hasegawa and Tappert in 1973 and its experimental verication came in 1980. In the telecommunicaton saytem, optical soliton are vital significant in present as well as the up comming time in the communication syatem.

1.4 Nonlinear Optics

The special branch of optics that describes the behavior of light in nonlinear media is known as nonlinear optics (NLO), media in which the dislectric polarization P responds nonlinearly to electric field E of the light. Nonlinearity is observed at high light intensities like provided by lasers. The superposition principle in NLO no longer holds. NLO describes nonlinear response of properties such as polarization, phase frequency or path of incident light.

1.5 Literature Review

Alshaery et al. [5] found the optical solitons in multiply-core couplers. They have studied the nonlinearities in detail and gave a critical analysis of kerr law, power low, parabolic law and the dual power law. Biswas et al. [9] observed the general form of the NLSE to get bright and drak soliton. Biswas et al. [10] found soliton solution of the NLSE that manages the propagation of solitons through optical fibers. Biswas et al. [11] obtained the 1-soliton solution of birefringent fiber with the help of Kerr law nonlinearity. They also studied perturbation term with the help of inter model dispersion. Biswas et al. [12] used G'/G to find bright and dark 1-soliton solution to the NLSE with the help of parabolic law nonlinearity.

Choudhuri et al. [18] find the Dark-in-the-Bright solitary wave solution of higher-order nonlinear Schrodinger equation. Using first integral method, Eslami et al. [21], Optical solitons for the resonant NLSE with time-dependent coefficients. Fermi et al. [22], studied some nonlinear problems . Younis et al. [30] found optical solitons for ultrashort pulses in nano fibers. Milovic et al. [31] studied the bright and dark solitons of the generalized nonlinear Schrodinger equation. Rizvi et al. [38] observed the propagation of optical solitons defined in the physical modal. They found out the exact solitons through the parabolic law nonlinearity and G'/G expansion scheme. Rizvi et al. [39] explained the optical 1-soliton solution in dual core fiber with the help of inter modal dispersion. They used some nonlinearities like Kerr and Power law with the help of ansatz approach.

Rizvi et al. [40] achieved the dark and singular solitons in dual core fiber by the help of G'/G expansion scheme. They also listed the existence of soliton solution. Younis et al. [41] gave some results on dispersive dark optical soliton in (2+1) dimensions by G'/G expansion with dual power law non-linearity. Savescu et al. [42] found the optical solitons in magneto-optic wave guides with spatial-temporal dispersion. In metamaterials Serge et al. [43] discusse the Optical chirped soliton soution. They [45] also used four wave mixing for kerr law nonlinearity to find the optical solitons in birefringent. Li et al. [46] analyzed the New types of solitary wave solutions (combo solitons) for the higher-order nonlinear Schrodinger equation. Topkara et al. [48] found optical solitons with non-kerr law nonlinearity with inter model dispersion, they used solitary wave ansatz method, was used to find the optical solitons for NLSE with time dependent coefficients. Triki et al. [49] find the dark optical solitons and conservation laws for parabolic and dual-power law nonlinearities in (2+1)-dimensions. For a NLSE on a continuous-wave background Triki et al. [49] find the Chirped solitary pulses solutions. By utilizing spatio-temporal dispersion and parabolic law nonlinearity, Zhou et al. [50] gave the analytical study of thirring optical solitons. Birefringent fibers containing thirring solitons found by Zhou et al. [51], that can be explained by vector coupled nonlinear Schrödinger equation with the help of kerr law nonlinearity.

Chapter 2

Basic Concepts

In this chapter, we present some basic definitions, that will help in understanding our results.

2.1 Nonlinear Schrödinger's Equation

In theoretical physics, the nonlinear variation of Schrödinger equation is known as nonlinear Schrödinger equation (NLSE). NLSE is classical field equation, the propagation of light in nonlinear optical fibers and planar waveguiges are the principal application of NLSE. Integrable model is a 1D NLSE. In optics, Manakov system is a model of wave propagation and NLSE occur in this system. for water waves, The evlution of the envelope of modulated wave groups described by NLSE. There are many applications of Non-Linear Schrödinger's Equation (NLSE) some are below, Base-Einstein, Condensates and Bimolecular dynamics. In NL optics dynamics of picoseconds pulses described by NLSE that travel NL source due to delicate balance between group velocity dispersion (GVD) Kerr linearity.

$$i\partial_x\phi+\Delta\phi+2|\phi|^2\phi=0 \tag{2.1.1}$$

where ϕ is proportional to the slowly varying complex envelope of the electromagnetic field, x is the propagation variable, and Δ denotes the Laplacian with respect to the transverse coordinates.

2.2 Bright Solitons

The bright solitons are in the form of pluse and on a zero background [9]. when the frequency increase in dispresion area then refractive index will be decrease. In optical fiber, nonlinear effects responsible for the formation of soliton. Group velocity dispresion(GVD) is associated with bright and drak soliton. GVD has both negative and positive sign.The GVD is irregular and a wave has constant amplitude continuously for optical fiber. Due to instability, the wave is unstable and break down into a sequence of pulses. These pulses are bright solitons. It does not change its shape and speed, even after colliding with other such waves during propagation in local dispersion. It can be written as,

$$u(x,t) = \lambda Sech^{p}\tau, \qquad \tau = \eta_1 x + \eta_2 y - \nu t \tag{2.2.1}$$

Here λ is the soliton amplitude, ν is the soliton velocity and $\eta_i (i = 1,2)$ are the inverse width of the soliton.

2.3 Dark Solitons

The terms bright and dark soliton are borrowed from optics, where they recognizable as bright area and dark shadows in optical fibres [9]. Dark solitons do occur in case of self-defocussing non-linearities. Dark soliton has no variation and stable and localized pulses. Those pulses occurs only in gaps are dark solitons. Dark solitons has continuous background. Dark soliton not formed simply as bright soliton. Because the dark solitons have two requirement. First requirement is local variation in phase. And second is amplitude change of the back ground wave. Dark solitons mathematically can be written as

$$p(x,t) = ATanh^{p}\tau, \quad where \quad \tau = B(x - \nu t) \quad and \quad p > 0 \tag{2.3.1}$$

Where A is the amplitude of the solitons, B represents the inverse width of soliton and ν is the solitary wave.

The dark solitons have fundamental as well as technological importance. Optical dark solitons are stable during the propagation. When temporal solitons are present in the optical fibers, nonlinear effects are weak and most of the case is valid for the cubic NLS equation. Solitons are always stable. Spatial solitons in waveguide or bulk media use the higher power. The nonlinear refractive index shows the non Kerr change in real optical materials with the increase of the light power. Dark solitary waves can become unstable in the non Kerr materials. A black dark soliton is always stable. A dark soliton has zero power at its center.

2.4 Chirped Soliton

Chrip is a special type of [4] signal in which frequency change with time. It refers to the chriping sound made by birds. It is very useful in speed-spectrum communication and in some importants devices like radar and sonar. It is also very useful in solitary wave-based communication links, optical pulse compressors and design of fiber optic amplifiers.

2.5 Dipole Soliton

Dipole soliton is formed by the product of dark and bright solitry waves [18]. Basically, dark in the bright soliton is known as dipole soliton which is used for the higher order NLSE with non-Kerr nonlinearity through some condition and purpose to bring parameters in relation with optical context. Dipole soliton mathmatically can be expressed as

$$F(x,t) = i\beta + \lambda sech[\eta(t-\chi x)]tanh[\xi(t-\chi x)] \tag{2.5.1}$$

Complex envolp function represented by $F(x,t)$, η and ξ represents the pulse width and χ represents the shift of group velocity.

2.6 Combo Soliton

Combined soliton solution of bright and drak is called combo soliton [46]. Combo soliton mathematically written as

$$F(x,t) = i\beta + \lambda tanh[\eta(t-\chi x)] + i\rho sech[\eta(t-\chi x)] \tag{2.6.1}$$

Complex envolp function represented by $F(x,t)$, η represents the pulse width and ξ represents the shift of group velocity.

2.7 The Ansatz Method

In few decade, important enhancement has been done in finding wave solutions for NLPDE. Many efficient ways has been set to deal with NLPDE. By using direct method to get wave solution, the selection of ansatz method is important. For study of new and other general wave solutions we use ansatz. Many singular, spatial, periodic, temporal and solitary wave solutions established. The ansatz has significant importance in giving specific solutions for different phenomena of nonlinearity in engineering sciences. In real life problems higher order non-linear evolution equations occurs which can be solved by ansatz. We present the process of the ansatz. We take the NLPDE as follow

$$A(v, v_t, v_x, v_{tt}, v_{ss}, ...) = 0 \tag{2.7.1}$$

where A is representing a polynomial in $v(t,x)$ and respective derivatives, then our concern is only with highest order derivatives and nonlinear terms. We utilize the conversion of traveling wave

$$v(t,x) = v(\xi), \xi = x + wt \tag{2.7.2}$$

to reduce equation (2.7.1) in following ODE

$$B(v, v', v', v''v'', ...) = 0 \tag{2.7.3}$$

where w are representing the constant. Here, B represent a polynomial in $v(x)$ and its all derivatives, whereas dashes denote the ordinary derivatives with respect to x .

Chapter 3

Chirped and Dipole Soliton in Nonlinear Negative Index Materials

3.1 Mathematical Model

$$iq_t + aq_{xx} + (b_1|q| + b_2|q|^2)q = i\{\alpha q_x + \beta(|q|^2 q)_x + \nu(|q|^2)_x q\}$$
$$+\theta_1(|q|^2 q)_{xx} + \theta_2|q|^2 q_{xx} + \theta_3 q^2 q*_{xx} \tag{3.1.1}$$

In above mathematical model independent variables spatial and temporal are represented by x and t. $q(x,t)$ is the complex valued dependent variable which represents the wave profile. Group velocity dispersion is represented by a. While quadratic - cubic nonlinearities by b_1 and b_2. On other side of mathematical model, inter modal dispersion, self-steepening and nonlinear dispersion are represented by α, β and ν respectively. Here we use the the assumption,

$$q(x,t) = \rho(\xi)e^{i(X(\xi)-\Omega t)} \tag{3.1.2}$$

In above equation $\xi = x - ut$. where $\rho(\xi)$ represents Amplitude function, phase function is denoted by $X(\xi)$, u is the wave velocity and Ω is represents the frequency of wave oscillation

$$iq_t = e^{i(X(\xi)-\Omega t)}[\rho X' u + \rho\Omega - i\rho' u] \tag{3.1.3}$$

$$aq_{xx} = ae^{i(X(\xi)-\Omega t)}[i\rho X'' + 2i\rho' X' + \rho'' - \rho(X')^2] \tag{3.1.4}$$

$$|q(x,t)| = |\rho(\xi)e^{i(X(\xi)-\Omega t)}| = \rho(\xi) \tag{3.1.5}$$

$$b_1|q(x,t)| = b_1\rho \quad b_2|q|^2 = b_2\rho^2 \tag{3.1.6}$$

$$(b_1|q(x,t)| + b_2|q|^2) = (b_1\rho^2 + b_2\rho^3)e^{i(X(\xi)-\Omega t)} \tag{3.1.7}$$

$$\alpha q(x,t)_x = \alpha e^{i(X(\xi)-\Omega t)}[i\rho X' + \rho'] \tag{3.1.8}$$

$$(|q(x,t)|^2 q)_x = e^{i(X(\xi)-\Omega t)}[3\rho^2\rho' + i\rho^3 X'] \tag{3.1.9}$$

$$|q(x,t)|_x^2.q(x,t) = 2\rho^2\rho' e^{i(X(\xi)-\Omega t)} \tag{3.1.10}$$

$$\begin{aligned}(|q(x,t)|^2 q)_{xx} = e^{i(X(\xi)-\Omega t)}[&3\rho''\rho^2 + 6\rho(\rho')^2 + 3i\rho^2\rho' X' + i\rho^3 X'' \\ &+3i\rho^2\rho' X' - \rho^3(X')^2]\end{aligned} \tag{3.1.11}$$

$$|q|^2 q(x,t)_{xx} = e^{i(X(\xi)-\Omega t)}[i\rho^3 X'' + 2i\rho^2\rho' X' + \rho^2\rho'' - \rho^3(X')^2] \tag{3.1.12}$$

$$q*_{xx} = e^{i(X(\xi)-\Omega t)}[\rho'' - i\rho' X' - i\rho X'' - i\rho' X' - \rho(X')^2] \tag{3.1.13}$$

$$q(x,t)^2 = \rho^2(\xi).e^{2i(X(\xi)-\Omega t)} \tag{3.1.14}$$

$$q(x,t)^2.q*(x,t)_{xx} = e^{i(X(\xi)-\Omega t)}[\rho^2\rho'' - 2i\rho^2\rho' X' - i\rho^3 X'' - \rho^3(X')^2] \tag{3.1.15}$$

Now substituting $q(x,t)$ and its derivatives into Eq. (3.1.1), we get the following real and imaginary part.

$$\begin{aligned}\rho X'u+\rho\Omega+a\rho''-a\rho(X')^2+(b_1\rho^2+b_2\rho^3)+\alpha\rho X'+\rho^3X'\beta\\ +\theta_1(-3\rho''\rho^2-6\rho(\rho')^2+\rho^3(X')^2)+\theta_2(\rho^3(x')^2-\rho^2\rho'')\\ +\theta_3(\rho^3(X')^2-\rho^2\rho'')=0\end{aligned} \tag{3.1.16}$$

and

$$\begin{aligned}-\rho'u+a\rho X''+\alpha\rho'X''+2\alpha\rho'X'-\alpha\rho'-3\beta\rho^2\rho'-2\nu\rho^2\rho'\\ +\theta_1(-6\rho^2\rho'X'-\rho^3X'')+\theta_2(\rho^3X''-2\rho^2\rho'X')\\ +\theta_3(2\rho^2\rho'X'+\rho^3X'')=0\end{aligned} \tag{3.1.17}$$

Multiply Eq. (3.1.18) by ρ and integrate it, we have the following expression

$$\begin{aligned}-\frac{(u+\alpha)}{2}\rho^2-\frac{3\beta+2\nu}{4}\rho^4+a\rho^2X'+(\frac{3}{2}\theta_1-\frac{1}{2}\theta_2+\frac{1}{2}\theta_3)\rho^4X'\\ +(\frac{1}{2}\theta_1+\frac{3}{4}\theta_2+\frac{3}{4}\theta_3)\int\rho^4X''=0\end{aligned} \tag{3.1.18}$$

If we consider $\theta_1=0$ for a arbitary solution. We have

$$-\frac{(u+\alpha)}{2}\rho^2-\frac{3\beta+2\nu}{4}\rho^4+a\rho^2X'=0 \tag{3.1.19}$$

Solving for X'.

$$a\rho^2X'=\frac{(u+\alpha)}{2}\rho^2+\frac{3\beta+2\nu}{4}\rho^4 \tag{3.1.20}$$

$$X'=\frac{3\beta+2\nu}{4a}\rho^2+\frac{(u+\alpha)}{2a} \tag{3.1.21}$$

Now we consider the following chrip

$$X' = \delta\rho^2 + \eta \tag{3.1.22}$$

Where δ and η are the nonlinear and constant chrip parameters.

$$\delta = \frac{3\beta + 2v}{4a} \tag{3.1.23}$$

$$\eta = \frac{u + \alpha}{2a} \tag{3.1.24}$$

After putting the value of X' in real part of equation, We get

$$\begin{aligned}
\rho(\delta\rho^2+\eta)u + \rho\Omega + a\rho'' - a\rho(\delta\rho^2+\eta)^2 + (b_1\rho^2 + b_2\rho^3) + \alpha\rho(\delta\rho^2+\eta) + \rho^3\delta\rho^2 + \eta\beta \\
+\theta_1(-3\rho''\rho^2 - 6\rho(\rho')^2 + \rho^3(\delta\rho^2+\eta)^2) + \theta_2(\rho^3(\delta\rho^2+\eta)^2 - \rho^2\rho'') \\
+\theta_3(\rho^3(\delta\rho^2+\eta)^2 - \rho^2\rho'') = 0
\end{aligned} \tag{3.1.25}$$

In above equation put the value of δ and η. We have the folowing expression

$$\begin{aligned}
\delta^2(\theta_2+\theta_1+\theta_3)\rho^7 + (2\delta\eta(\theta_1+\theta_2+\theta_3) - a\delta^2 + \beta\delta)\rho^5 + (b_2 + \delta(u+\alpha) \\
+\eta(-2a\delta+\beta) + \eta^2(\theta_1+\theta_2+\theta_3))\rho^3 + (b_1 + \rho''(-(3\theta_1+\theta_2+\theta_3)))\rho^2 \\
+(\Omega + \eta(\alpha - a\eta + u) - 6\theta_1(\rho')^2)\rho + a\rho'' = 0
\end{aligned} \tag{3.1.26}$$

$$A\rho^7 + B\rho^5 + C\rho^3 + (b_1 + \rho''D)\rho^2 + (E - 6\theta_1(\rho')^2)\rho + a\rho'' = 0 \tag{3.1.27}$$

$$A = \delta^2(\theta_2 + \theta_1 + \theta_3) \tag{3.1.28}$$

$$B = 2\delta\eta(\theta_1 + \theta_2 + \theta_3) - a\delta^2 + \beta\delta \tag{3.1.29}$$

$$C = b_2 + \delta(u + \alpha) + \eta(-2a\delta + \beta) + \eta^2(\theta_1 + \theta_2 + \theta_3) \tag{3.1.30}$$

$$D = -(3\theta_1 + \theta_2 + \theta_3) \tag{3.1.31}$$

$$E = \Omega + \eta(\alpha - a\eta + u) \tag{3.1.32}$$

Multiplying with ρ', and integrating, we get

$$A\frac{\rho^8}{8} + B\frac{\rho^6}{6} + C\frac{\rho^4}{4} + E\frac{\rho^2}{2} + a\frac{\rho'^2}{2} = 0 \tag{3.1.33}$$

$$(\rho')^2 = -\frac{A}{4a}\rho^8 - \frac{B}{3a}\rho^6 - \frac{C}{2a}\rho^4 - \frac{E}{a}\rho^2 \tag{3.1.34}$$

$$(\rho')^2 = a_1\rho^8 + a_2\rho^6 + a_3\rho^4 + a_4\rho^2 \tag{3.1.35}$$

$$a_1 = \frac{-A}{4a}, \;\; a_2 = \frac{-B}{3a}, \;\; a_3 = \frac{-C}{2a}, \;\; a_4 = \frac{-E}{a}$$

$$a_1 = \frac{-\delta^2(\theta_2 + \theta_1 + \theta_3)}{4a}$$

$$a_2 = \frac{-2\delta\eta(\theta_1 + \theta_2 + \theta_3) - a\delta^2 + \beta\delta}{3a}$$

$$a_3 = \frac{-b_2 + \delta(u+\alpha) + \eta(-2a\delta+\beta) + \eta^2(\theta_1+\theta_2+\theta_3)}{2a}$$

$$a_4 = \frac{-\Omega + \eta(\alpha - a\eta + u)}{a}$$

By considering the following cases, we get the soliton solution.

Case 1: $a_1 > 0$, $a_2 < 2a_1$, $a_3 = \frac{a_2^2}{4a_1} - a_1$

Eq. (3.1.35) gives the positive solution .

$$\rho(\xi) = \frac{1}{cosh(2\sqrt{a_1})\xi - \frac{a_2}{2a_1}} \tag{3.1.36}$$

In Eq. (3.1.36) if we put $a_2 = 0$, we have the bright soliton solution.

$$\rho(\xi) = sech(2\sqrt{a}_1\xi)^{\frac{1}{2}} \tag{3.1.37}$$

we know that

$$a_2 = \frac{-2\delta\eta(\theta_1+\theta_2+\theta_3) - a\delta^2 + \beta\delta}{3a}$$

$$0 = -2\delta\eta(\theta_1+\theta_2+\theta_3) - a\delta^2 + \beta\delta$$

$$\delta = 0$$

Put the value of $\delta = \frac{3\beta+2v}{4a}$ And we have the β

$$\beta = -\frac{2v}{3} \tag{3.1.38}$$

From Eq. (3.1.37) we have

$$\rho(\xi)^2 = sech(2\sqrt{a}_1\xi) \tag{3.1.39}$$

$$q(x,t) = \rho(\xi)e^{i(X(\xi)-\Omega t)} \tag{3.1.40}$$

put the value of $\rho(\xi)$ in Eq. (3.1.40)

$$q(x,t) = sech(2\sqrt{a}_1\xi)^{\frac{1}{2}}e^{i(X(\xi)-\Omega t)} \tag{3.1.41}$$

<u>**Case 2**</u>: $a_1 > 0$, $a_3 < 0$, $a_2 = -2\sqrt{(a_1a_3)}$

Eq. (3.1.35) also gives the positive solution. And put the $a_2 = -2\sqrt{(a_1a_3)}$ and solve for β

$$\frac{a\delta^2 - \beta\delta}{3a} = -2\sqrt{a_1a_3} \tag{3.1.42}$$

we have the β

$$\beta = \frac{6a\sqrt{a_1a_3} + a\delta^2}{\delta} \tag{3.1.43}$$

$$\rho(\xi) = \left(\sqrt{\frac{a_1}{a_3}}(\frac{1}{2} + \frac{1}{2}tanh(\sqrt{a_1a_2}))\right)^{\frac{1}{2}} \tag{3.1.44}$$

$$\rho(\xi)^2 = \sqrt{\frac{a_1}{a_3}}(\frac{1}{2} + \frac{1}{2}tanh(\sqrt{a_1a_2})) \tag{3.1.45}$$

$$q(x,t) = \rho(\xi)e^{i(X(\xi)-\Omega t)} \tag{3.1.46}$$

put the value of $\rho(\xi)$

$$q(x,t)=\left(\sqrt{\frac{a_1}{a_3}}(\frac{1}{2}+\frac{1}{2}tanh(\sqrt{a_1a_2}))\right)^{\frac{1}{2}}e^{i(X(\xi)-\Omega t)} \tag{3.1.47}$$

Case 3: $a_1=0$, $a_2=1$, $a_3<0$

Eq. (3.1.35) also gives the positive solution. By putting the $a_2=1$ we have

$$a\delta^2-\beta\delta=3a \tag{3.1.48}$$

$$\beta=\frac{a\delta^2-1}{\delta} \tag{3.1.49}$$

$$\rho(\xi)=\sqrt{\frac{1}{\xi^2-c}} \tag{3.1.50}$$

$$q(x,t)=\rho(\xi)e^{i(X(\xi)-\Omega t)} \tag{3.1.51}$$

$$q(x,t)=\sqrt{\frac{1}{\xi^2-c}}e^{i(X(\xi)-\Omega t)} \tag{3.1.52}$$

Case 4: $a_1=-1$, $a_2=2\sigma$, $a_3=1-\sigma^2$

Eq. (3.1.35) also gives the positive solution. Taking $a_2=2\sigma$

$$a\delta^2-\beta\delta=6a\sigma \tag{3.1.53}$$

we get β

$$\beta=\frac{a\delta^2-6a\sigma}{\delta} \tag{3.1.54}$$

$$\rho(\xi) = \sqrt{\frac{1}{\sigma + sin2\xi}} \tag{3.1.55}$$

$$\delta\omega(\xi) = (\delta\rho^2 + \eta) \tag{3.1.56}$$

$$q(x,t) = \sqrt{\frac{1}{\sigma + sin2\xi}} e^{i(X(\xi) - \Omega t)} \tag{3.1.57}$$

In the following section, we will study dipole soliton for Eq. (3.1.36)

3.2 Dipole Soliton

Here we assume the following Hypothesis

$$q(x,t) = P(x,t)e^{i\psi(x,t)} \tag{3.2.1}$$

where $P(x,t)$ is represents the amplitude portion of soliton

$$\psi(x,t) = -\kappa x + \omega t + \theta \tag{3.2.2}$$

In the above equation, frequency of soliton, wave number and phase constant are represented by κ, ω, and θ respectively. By using Eq. (3.2.1) and its derivatives into Eq. (3.1.1) we get real and imaginary parts

$$q_t = iP(x,t)e^{i\phi(x,t)} \frac{\partial\psi}{\partial x} + e^{i\psi(x,t)} \frac{\partial P}{\partial x} \tag{3.2.3}$$

$$q_{xx} = iP(x,t)e^{i\psi(x,t)}\frac{\partial^2\psi}{\partial x^2} + 2i\frac{\partial p}{\partial x}\frac{\partial\psi}{\partial x}e^{i\psi(x,t)} - Pe^{i\psi(x,t)}(\frac{\partial\psi}{\partial x})^2 + \frac{\partial^2\psi}{\partial x^2}e^{i\psi(x,t)} \quad (3.2.4)$$

$$|q|^2 = P^2 \quad (3.2.5)$$

$$(b_1|q| + b_2|q|^2)q = (b_1P^2 + b_2P^3)e^{i\psi(x,t)} \quad (3.2.6)$$

$$(|q|^2q)_x = 3P^2\frac{\partial P}{\partial x}e^{i\psi(x,t)} + iP^3\frac{\partial\psi}{\partial x}e^{i\psi(x,t)} \quad (3.2.7)$$

$$|q|_x^2 q = 2P^2\frac{\partial P}{\partial x}e^{i\psi(x,t)} \quad (3.2.8)$$

$$\begin{aligned}(|q|^2q)_{xx} = {} & 3P^2\frac{\partial^2 P}{\partial x^2}e^{i\psi(x,t)} + 6P(\frac{\partial P}{\partial x})^2e^{i\psi(x,t)} + 6iP^2\frac{\partial P}{\partial x}\frac{\partial\psi}{\partial x}e^{i\psi(x,t)} \\ & + iP^3\frac{\partial^2 P}{\partial x^2}e^{i\psi(x,t)} - P^3(\frac{\partial\psi}{\partial x})^2e^{i\psi(x,t)} \quad (3.2.9)\end{aligned}$$

$$\begin{aligned}|q|^2q_{xx} = {} & iP^3(x,t)e^{i\psi(x,t)}\frac{\partial^2\psi}{\partial x^2} + 2P^2i\frac{\partial\psi}{\partial x}\frac{\partial P}{\partial x}e^{i\psi(x,t)} \\ & - P^3e^{i\psi(x,t)}(\frac{\partial\psi}{\partial x})^2 + P^2\frac{\partial^2\psi}{\partial x^2}e2^{i\psi(x,t)} \quad (3.2.10)\end{aligned}$$

$$\begin{aligned}q*_{xx} = {} & \frac{\partial^2 P}{\partial x^2}e^{-i\psi(x,t)} - i\frac{\partial P}{\partial x}\frac{\partial\phi}{\partial x}e^{-i\psi(x,t)} - iP\frac{\partial^2\psi}{\partial x^2}e^{-i\psi(x,t)} \\ & - P(\frac{\partial P}{\partial x})^2e^{-i\psi(x,t)} \quad (3.2.11)\end{aligned}$$

UUsing Eq. (3.2.1)- Eq. (3.2.11) into Eq. (3.1.1) we get real and imaginary parts,

$$-P\frac{\partial\psi}{\partial t}-aP\left(\frac{\partial\psi}{\partial x}\right)^2+a\frac{\partial^2P}{\partial x^2}+\left(b_1P^2+b_2P^3\right)+\alpha P\frac{\partial\psi}{\partial x}+\beta P^3\frac{\partial\psi}{\partial x}$$
$$-\theta_1P^3\left(\frac{\partial\psi}{\partial x}\right)^2+3P^2\frac{\partial^2P}{\partial x^2}+6P\left(\frac{\partial P}{\partial x}\right)^2-\theta_2P^2\frac{\partial^2P}{\partial x^2}$$
$$-\theta_3P^2\frac{\partial^2P}{\partial x^2}+P^3\left(\frac{\partial\psi}{\partial x}\right)^2=0 \tag{3.2.12}$$

$$\frac{\partial P}{\partial t}+2a\frac{\partial P}{\partial x}\frac{\partial\psi}{\partial x}-\alpha\frac{\partial P}{\partial x}-\beta\left(3P^2\frac{\partial P}{\partial x}+2vP^2\frac{\partial P}{\partial x}\right)$$
$$-\theta_1\left(3P^2\frac{\partial P}{\partial x}+3P^2\frac{\partial P}{\partial x}\frac{\partial\psi}{\partial x}\right)-2\theta_2P^2\frac{\partial\psi}{\partial x}\frac{\partial P}{\partial x}$$
$$-\theta_3P^2\frac{\partial P}{\partial x}\frac{\partial\psi}{\partial x}=0 \tag{3.2.13}$$

To get dark-in-the-bright soliton, we consider the following hypothesis

$$P(x,t)=i\beta+\lambda sech[\eta(t-\chi x)]tanh[\rho(t-\chi x)] \tag{3.2.14}$$

In Eq. (3.2.14) ρ, η are represents the plus widths and shift of inverse group velocity dispersion is represented by χ. If $\beta=\eta=0$ than the amplitude function $P(x,t)$ transform into dark soliton. From above equation we can fine the magnitude.

$$|P(x,t)|=\sqrt{\beta^2+(\lambda sech[\eta(t-\chi x)]tanh[\rho(t-\chi x)])^2} \tag{3.2.15}$$

and non-linear phase shift function is

$$\psi(x,t)=arctan(\frac{\beta}{\lambda sech\eta(t-\chi x)tanh(t-\chi x)}) \tag{3.2.16}$$

$$P^2(x,t) = -\beta^2 + 2i\beta\lambda sech[\eta(t-\chi x)]tanh[\rho(t-\chi x)]$$
$$\lambda^2 sech^2[\eta(t-\chi x)]tanh^2[\rho(t-\chi x)] \qquad (3.2.17)$$

$$P^3(x,t) = -i\beta^3 + 3i\beta\lambda^2 sech[\eta(t-\chi x)]tanh^2[\rho(t-\chi x)]$$
$$+\lambda^3 sech^3[\eta(t-\chi x)]tanh^3[\rho(t-\chi x)]$$
$$-3\beta^2\lambda sech[\eta(t-\chi x)]tanh[\rho(t-\chi x)] \qquad (3.2.18)$$

$$\frac{\partial p}{\partial x} = \lambda\Big(\eta\chi sech[\eta(t-\chi x)]tanh[\eta(t-\chi x)]tanh[\rho(t-\chi x)]$$
$$-\rho\chi sech[\eta(t-\chi x)]sech^2 tanh[\rho(t-\chi x)]\Big) \qquad (3.2.19)$$

$$\frac{\partial^2 p}{\partial x^2} = \lambda\chi^2\Big(-\rho\eta sech[\eta(t-\chi x)]tanh[\eta(t-\chi x)]sech^2[\rho(t-\chi x)] - \eta^2 sech^3[\eta(t-\chi x)]tanh[\rho(t-\chi x)]$$
$$+\eta^2 tanh^2[\eta(t-\chi x)]tanh[\rho(t-\chi x)]sech[\eta(t-\chi x)]$$
$$-\rho\eta sec[\eta(t-\chi x)]tanh[\eta(t-\chi x)]sech^2[\rho(t-\chi x)]$$
$$-2\rho^2 sech[\eta(t-\chi x)]sech^2[\rho(t-\chi x)]tanh[\rho(t-\chi x)]\Big) \qquad (3.2.20)$$

$$(\frac{\partial p}{\partial x})^2 = \lambda^2\big(\eta^2\chi^2 sech^2[\eta(t-\chi x)]tanh^2[\eta(t-\chi x)]tanh^2[\rho(t-\chi x)]$$
$$+\rho^2\chi^2 sech^2[\eta(t-\chi x)]sech^4[\rho(t-\chi x)]$$
$$-2\rho\eta\chi^2 sech^2[\eta(t-\chi x)]tanh[\eta(t-\chi x)]tanh[\rho(t-\chi x)]sech^2[\rho(t-\chi x)]\big) \qquad (3.2.21)$$

Using Eq. (3.1.14) and its derivatives into real and imaginary parts

$$\lambda + 2\alpha\kappa\chi + \alpha\lambda\chi - 3\lambda\chi\beta^3 - 2\lambda\beta^3\chi\nu - 3\theta_3\beta^2\chi\lambda + \alpha\lambda\chi + 3\theta_1\kappa\beta^2\chi\lambda$$
$$+2\theta_2\kappa\beta^2\chi\lambda + \theta_3\kappa\beta^2\chi\lambda = 0 \qquad (3.2.22)$$

$$\theta_3\kappa\beta^2\chi+1+2\alpha\kappa\chi+\alpha\chi-3\beta^3\chi-2\beta^3\nu\chi$$
$$-3\theta_1\beta^2\chi+3\theta_1\kappa\chi\beta^2+2\theta_2\kappa\beta^2\chi=0 \tag{3.2.23}$$

$$3\theta_1+5\beta-3\theta_1\kappa-2\theta_2-\theta_3\kappa=0 \tag{3.2.24}$$

$$6\beta+4\beta\nu\lambda-2\theta_3\kappa\lambda=0 \tag{3.2.25}$$

$$\beta^2(3-\theta_2-\theta_3)-a=0 \tag{3.2.26}$$

$$3\theta_1-3\theta_1\kappa-2\theta_2\kappa-\theta_3\kappa=0 \tag{3.2.27}$$

$$6\beta^2+4\nu\beta^2\chi\eta+6\theta_1\beta\chi\eta+6\theta_1\kappa\beta\eta\chi-2\theta_3\kappa\eta\chi=0 \tag{3.2.28}$$

$$\lambda^2\chi^2\rho^2\eta=0 \tag{3.2.29}$$

$$2\beta\nu\lambda^3\rho\chi=0 \tag{3.2.30}$$

$$\lambda^2(b_2-\kappa\beta-\theta_1\kappa^2+\kappa^2)=0 \tag{3.2.31}$$

$$3\beta\lambda^3\rho\chi=0 \tag{3.2.32}$$

$$4\theta_2\beta\lambda^2\eta\chi=0 \tag{3.2.33}$$

$$-4\theta_2\beta\lambda^2\rho\chi = 0 \tag{3.2.34}$$

$$6\lambda^3\chi^2\eta^2 = 0 \tag{3.2.35}$$

$$3 - \theta_2 - \theta_3 = 0 \tag{3.2.36}$$

$$3\beta^2(b_2 - \kappa\beta - \theta_1\kappa^2 + \kappa^2) + \omega + a\kappa^2 + \alpha\kappa = 0 \tag{3.2.37}$$

$$\beta(\omega + a\kappa(\kappa - 1) - k) + b_2 - \theta_1\kappa^2 + \kappa = 0 \tag{3.2.38}$$

Here we will consider the different casses to get the soliton solution. We need to impose the some condition on parameters. $\beta = 0$ and $\lambda \neq 0$ or $\beta \neq 0$ and $\lambda = 0$. If we take $\lambda = 0$ its means we get no soliton solution. so we must take $\lambda \neq 0$ and $\beta = 0$ to get dipole solution. Eq. (3.1.14) becomes

$$P(x,t) = \lambda sech[\eta(t - \chi x)]tanh[\rho(t - \chi x)] \tag{3.2.39}$$

$$\beta = 0 \tag{3.2.40}$$

$$\kappa = \frac{3\theta_3}{3\theta_1 + 2\theta_2 + \theta_3} \tag{3.2.41}$$

$$\chi = \frac{3\theta_1 + 2\theta_2 + \theta_3}{\alpha(9\theta_1 + 2\theta_2 + \theta_3)} \tag{3.2.42}$$

$$\lambda = \frac{-6\theta_3}{15\theta_1 + 6\theta_2 + 3\theta_3} \tag{3.2.43}$$

$$\omega = \frac{-b_2(a+\alpha)}{\theta_1 - 1} \tag{3.2.44}$$

Thus we obtain the solitary wave solution for Eq. (3.2.1)

$$q(x,t) = \lambda sech[\eta(t-\chi x)]tanh[\rho(t-\chi x)]e^{i\psi(x,t)} \tag{3.2.45}$$

Chapter 4

Conclusions

4.1 Conclusions

In this article, we obtained solitary wave solutions for nonlinear negative-index materials under quadratic-cubic nonlinearity. We obtained two types of soliton solutions; one is chirped soliton and other is dipole soliton solution. We used sub-ODE method to get chirped soliton and studied dipole soliton with the help of ansatz method of Choudhuri. The obtained results will be used in field of telecommunication.

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