New Solitary Wave Solution for Nonlinear Schrodinger Equation

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I **Urooj Akram** registration number **CIIT/FA16-RMT-005/LHR** hereby declares that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to whatever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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Certificate

It is certified that Urooj Akram **(CIIT/FA16-RMT-005/LHR)** has carried out all the work related to this report under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus, and the work fulfills the requirement for award of MS degree.

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DEDICATION

To

My Family

ACKNOWLEDGEMENT

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ABSTRACT

New Solitary Wave Solutions for Nonlinear Schrodinger Equation

A nonsingular and localized wave which propagates without change of velocity and shape is called a solitary wave. A soliton is a solution of NLPDE which has a permanent form and is localized. A solitary traveling wave does not obey superposition principle; it does not disperse. Solitary waves retain its permanent structure even when it collides with other soliton [25]. Soliton has many applications in other fields like engineering and biology. Soliton solutions can be calculated with the help of different nonlinearities. These nonlinearities are Kerr law, power law, parabolic law, dual power law, exponential law etc. In this thesis we will study, the soliton solutions of Lakshmanan-Porsezian-Danial (LPD) model. In order to obtain exact soliton solutions several powerful methods have been proposed. We will use modified trail equation method with Kerr and parabolic law to obtain some hyperbolic and rational function solutions to LPD model. We also find new and more general exact traveling wave solution by using improved G'/G-expansion method with Kerr and parabolic laws of nonlinearities.

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Chapter 1

Introduction

Partial differential equation (PDE) is such a kind of differential equation which contain unknown multivariable functions and their derivatives. PDE often model multidimensional systems. It relates the partial derivatives of a function of two or more independent variable together. It can be used to describe phenomenons such as, heat, sound, fluid dynamics and quantum mechanics etc. We also define linear PDE's as equation for which the dependent variable and its derivatives appear in term with degree at most one. Some common examples of linear PDE's include heat equation, Laplace equation, wave equation, poisson equation, Klein-Gordon equation etc.

A NLPDE is a equation with nonlinear terms. Thermodynamics, relativity, neurology, elasticity, fluid mechanics, ecology, gas dynamics and many more are often modeled by NLPDE. There have been a lot of research work in this field during last 30 years. Some well known nonlinear evolution equations are nonlinear Schrödinger equation (NLSE), Burgers equation, Korteweg and de vries (Kdv) equation, Sasa-Satsuma equation, Manakov model, Chen-Lee-Liu equation, Complex Ginzburg-Landau equation and several others.

1.1 Solitons

John Scott Russell witnessed a soliton in 1834, while riding on a horse near a Edinburgh-Glasgow canal [32]. He observed some solitary waves in water. Then he came to know about some solitary wave which propagates without change of velocity and shape termed as soliton later on. During their travel through matter these waves do not scattered energy behind them, they have well defined energy and direction. Soliton also appear in the formation of tidal waves. It is nonlinear PDE of traveling wave solution. Solitons is a universal mechanism, it is not limited to water waves only it also appears in many system which are nonlinear and also in support waves [3].

Soliton is a solitary wave packet that unchanged their identity and shape during propagation. It exists in plasma waves, fluids, temporal soliton in mechanical spring and mass system, particle physics and DNA chains [2]. A wave which does not have any temporal evolution in size or shape and propagate with its group velocity is called a solitary wave. The solitary wave has balance between nonlinear and dispersive effects. Solitary waves arises in several cases like light intensity in an optical fiber and on the surface of water there exist solitary waves. Soliton is a special kind of nonlinear solitary wave, because it does not obey superposition principle and it does not disperse. It retains its structure even when collides with other soliton. Thus if two solitary waves are propagating in opposite directions they will pass through each other without any disturbance or breakage.

1.2 Optical Solitons

In nonlinear optics, optical solitons is an emerging field. It has brought revolution in the field of optical telecommunication system. Optical solitons are light pulses in space or time, that are localized and during their travel in some host medium they do not diffract or disperse. Optical solitons are of great importance in present as well as in future telecommunications system like in internet industry (facebook, twitter etc). In optics there are so many types of solitons like discrete solitons, spatio temporal solitons and bragg soliton.

1.2.1 Spatial soliton

If there is no diffraction in the optical beam when it propagates in a nonlinear medium, then spatial solitons are formed. During propagation the diameter of the beam remains unchanged. Spatial solitons represents the exact balance between self-lensing and diffraction induced by or self-focusing effects. When the light throw on convex lens, optical field approaches the lens. The non-uniform phase change the effect of lens, Since the field is optical, refractive index create and change focusing effect. When two effect balance each other, we get perfect field which wave move in fiber. This type of soliton is based on the Kerr law, it create a self phase modulation. According to the intensity self modulation changes the refractive index.

1.2.2 Temporal solitons

Optical temporal solitons are formed when the group velocity dispersion is totally counteracted by nonlinear self-phase modulation effects. These are non-spreading optical pulses. Temporal solitons hold their temporal width in fibers even if they travel distances of several hundred of kilometers. In optical fibers temporal solitons are formed. When there is group velocity dispersion, it creates impulses having non zero bound width and the wire wave move through refractive index that depends on frequency.

1.3 Literature Review

In multiply-core couplers Alshaery et al. [1] found the optical solitons. Biswas et al. [7] obtained 1-soliton solution of the NLSE. The general form of the NLSE was observed by Biswas et al. [8]. Biswas et al. [9] studied the 1-soliton solution of NLSE by using log law nonlinearity. By using Kerr law of nonlinearity, Biswas et al. [10] obtained the 1-soliton solution of birefringent fiber. With the help of inter model dispersion they also studied perturbation term. By using parabolic law of nonlinearity Biswas et al. [11] used ansatz method to NLSE. They found bright and dark soliton solutions. For the study of dynamics of dark optical solitons, Biswas et al. [12] used the G'/G expansion method. With the help of power law, Mirzazadeh [13] used G'/G expansion to obtained the dark optical solitons. In power law medium Biswas et al. [14] obtained dispersive dark optical soliton with Schrödinger-Hirota equation by G'/G-expansion approach. Biswas et al. [15] obtained 1-soliton solution of the (1+2)-dimensions NLSE by using power law of nonlinearity. Biswas et al. [16] obtained the dark solitons for a generalized NLSE by using parabolic and dual power law. In [17] H. Bulut, Y. Pandir used modified trial equation method to obtained single kink solution, rational function solution and periodic solutions to the nonlinear fractional Sharma-Tasso-Olever equation. Eslami et al. [18] found optical solitons for the resonant NLSE by using first integral method. Fermi et al. [19] studied some nonlinear problems. Girgis et al. [20] studied DWDM systems and optical gaussian in birefringent fibers. In [24] Kaplan et al. used improved G'/G-expansion method for finding exact solutions of NLPDE. Liu et al. [28] used same method to get some solitary wave solutions. Rizvi et al. [29] observed the propagation of optical solitons . They obtained the exact solitons by using parabolic law nonlinearity and G'/G expansion scheme. In dual core fiber Rizvi et al. [30] explained the optical 1-soliton solution with the help of inter modal dispersion. By using ansatz approach they used some nonlinearities like Kerr and power law. By using G'/G expansion scheme. Rizvi et al. [31] achieved the dark and singular 1-solitons in dual core fiber. They also listed the existence of soliton solution. With spatial-temporal dispersion Savescu et al. [33] obtained the optical solitons in magneto-optic wave guides. Savescu et al. [34] also discussed the optical solitons with spatio-temporal dispersion in nonlinear directional couplers. Savescu [35] found optical solitons with spatio-temporal dispersion and coupled Hirota equation. For finding the optical solitons in birefringent, they [36] also used four wave mixing for Kerr law nonlinearity. In [37] Y. A. Tandogan, N. Bildik found dark optical soliton by using modified trial equation method to the Time-Fractional Fisher equation. Triki et al. [38] found the dark optical solitons in (2+1)-dimensions. Topkara et al. [39] obtained optical solitons with inter model dispersion, with non-kerr law nonlinearity they fond the optical solitons for NLSE with time dependent coefficients by using solitary wave ansatz method. Younis et al. [40] explained analytical solutions of different types of solitons which had been explained by using nonlinear model named nano-bioelectronics transmission lines with some constraint conditions. Younis et al. [41] used G'/G expansion with dual power law non-linearity and gave some results on dispersive dark optical soliton in (2+1) dimensions. In [42] Younis et al. obtained optical solitons for ultrashort pulses in nano fibers. Zhou et al. [43] found birefringent fibers containing thirring solitons that can be explained by vector coupled NLSE by using Kerr law nonlinearity. Zhou et al. [45] gave the analytical study of thirring optical solitons by utilizing spatio-temporal dispersion and parabolic law nonlinearity.

Chapter 2

Basic Concepts

In this chapter, we present some basic definitions, that will help in understanding our results.

2.1 Dispersion

It is a process in which the phase velocity of a wave depends on frequency. Rainbow is one of the most familiar example of dispersion in which white light is dispersed into different colors (wavelengths). There are two main sources of dispersion, material dispersion and waveguide dispersion. For the construction of spectrometers and spectoradiometers, the dispersion of light by the glass prism is used. Dispersion causes chromatic aberration in lenses. The phase velocity (v) of wave is defined as,

$$v = \frac{c}{n}$$

where c is speed of light and n is refractive index.

2.2 Group-Velocity Dispersion

Group-Velocity (GV) produces as a result of rate of change of inverse group velocity. In this case IGV depends on angular frequency. In a transparent medium, GVD depends on wavelength.

$$GVD = \frac{\partial}{\partial\psi}\frac{1}{v_g} = \frac{\partial}{\partial\psi}(\frac{\partial k}{\partial\psi}) = \frac{\partial^2 k}{\partial\psi^2}$$

where k is the frequency dependent wave number. The basic units of the group velocity dispersion are $\frac{s^2}{m}$. GVD can have different important effects, depending on situation e.g, it is responsible for ultrashort pulses. The effect of nonlinearities strongly depends on the GVD in optical fibers.

2.3 Self Phase Modulation (SPM)

It is nonlinear optical effect, it leads to spectral broadening of optical pulses. It was first observed by Stolen and Lin in (1978). SPM causes a nonlinear phase delay which has same temporal shape as the optical intensity. It also define as the nonlinear change in refractive index:

$$\Delta n = n_2 I$$

where n_2 is nonlinear index and I is optical intensity. Refractive index depends on optical intensity as Kerr effect. Optical fibers modifies its own phase (SPM). During the propagation of pulse along fiber its spectrum changes because of SPM. SPM is useful for many devices and system applications soliton formation, optical switching, waelength conversion, demultiplexing, all optical regeneration etc.

2.4 Non-Linear Schrödinger Equation

Nonlinear Schrödinger equation (NLSE) is special kind of (NLPDE) that has been derived in many areas of physics and analyzed mathematically over last four decades. In nonlinear optical system NLSE's supporting solitons solutions. NLSE's is the central themes in nonlinear optics for finding the exact solution. The dimensionless form of NLSE is given below:

$$iq_t + \frac{1}{2}q_{xx} + G(|q|^2)q = 0.$$

where p is complex and depend on independent variables x and t, it usually represents the wave profile and x is spatial variable while t is temporal variable. G represents nonlinearity

2.5 Nonlinearities

Optical solitons is the most significant field of study in nonlinear optical fiber. If the process response is proportional to the stimulus given to it, then this process is linear. The linear model are simple mathematical models and are very straight forward they can be solved without the aid of computers. A process which is opposite to the linear process is known as nonlinear process. In case of nonlinear process nonlinearities occur. The nonlinear effects arise in optical fiber during transmission because of an increase in data rates, transmission lengths, number of wavelengths and in optical power levels. Another nonlinearity is fiber nonlinearity which arises because of two mechanisms one is refractive index and second is scattering. There are many distinct forms of nonlinearities of the function G.

2.5.1 Kerr Law :$G(p) = p$

Most of the optical fiber obeys the Kerr law of nonlinearity. It is simplest form of nonlinearity. Kerr law is also called cubic nonlinearity. By Kerr law the NLSE can be integrated by the inverse scattering transformation technique.

2.5.2 Power Law :$G(p) = p^s$

This nonlinearity occur in nonlinear plasmas. It appears in semi-conductors and in optics. In power law, to avoid the wave collapse it is necessary to take s in the interval $0 \leq s \leq 2$. Similarly it is compulsory that $s \neq 2$.

2.5.3 Parabolic Law :$G(p) = p + \nu p^2$

Whenever there is interaction between electrons and Langmuir waves then parabolic law holds.

2.5.4 Dual Power Law :$G(p) = p^s + \xi p^{2s}$

Using this form of nonlinearity many organic materials can be modeled. This model describes the saturation of the nonlinear refractive index. Many optical nonlinearities are formed by using this form of nonlinearity. Solitons becomes unstable in this interval $1 \leq s \leq 2$. If we take $s \geq 2$, then these solitons would collapse with each other.

2.5.5 Saturation Law :$G(p) = \frac{\lambda p}{1+\lambda p}$

For $\lambda > 0$, this nonlinearity law is used to describe the variation of di-electrics constant present in the gas vapors, the laser beam is propagated [4].
In case of semi-conductors doped fibers the saturation nonlinearity is used for modeling the soliton propagation inspite of Kerr law of nonlinearity.

2.5.6 Exponential Law :$G(p) = \frac{1}{2\lambda}(1 - e^{-2\lambda p})$

Exponential law of nonlinearity is observed for un magnetized, and laser induced plasmas. Saturable law of nonlinearity is obtained if the coupling equation is combined with plasmas motion of low frequency.

2.5.7 Log Law :$G(p) = a \ln(b^2 p)$

This form of nonlinearity occurs in many different fields of physics. The advantage of this model is that the absence of radiation from periodic soliton because of discrete spectrum only.

2.5.8 Higher order polynomial Law :$G(p) = p + \nu p^2 + \gamma p^3$

Higher order polynomial law is extension of parabolic law. In various physical systems this law is observed [5].

2.5.9 Triple power Law :$G(p) = p^s + \nu p^{2s} + \gamma p^{3s}$

Triple power law is an extension of dual power law and higher order polynomial law. Hamiltonian diagrams are studied in this case.

2.6 Bright solitons

The bright solitons are on a zero background in the form of pulses. When frequency increase the refractive index decreases in dispersion area, the temporal bright solitons occur in this region. GVD has two different signs, it is associated with the bright and dark solitons. In optical fiber, wave has constant amplitude and group velocity dispersion is irregular. The wave is unstable due to instability, it break down into a sequence of pulses. These pulses are bright solitons. It maintains its shape and speed, during propagation even after colliding with other wave in local dispersion. These waves are also called solitary waves and non-topological waves. Its is given as,

$$u(x,t) = \lambda_1 \sec h^{\rho} \tau \quad where \quad \tau = M(x - vt) \tag{2.6.1}$$

In above equations, λ and M are known as unconstrained parameters and where λ represents the amplitude ,M is said to be the inverse width of the soliton and v is basically the velocity of the solitary wave.

2.7 Dark solitons

Dark solitons are intensity dips that exist on the background of the continuous-wave. The spatial bright solitons exist in case of self-focussing while the dark solitons do occur in case of self-defocussing nonlinearities. Dark soliton travels with constant amplitude and that carries energy and propagate through the mediums such as solid, liquid, gas and plasma. It travels at a much higher speed as compared to an ordinary waves, the generation of dark soliton is less straight forward than the bright soliton, because the dark solitons have both two requirement like first is local change and second requirement is amplitude change. Dark soliton rotates its phase twice than the bright soliton phase, so that’s why there was no method to invent the dark soliton. It solitary wave form is represented as,

$$u(x,t) = \lambda \tanh^{\rho} \tau \quad where \quad \tau = \beta(x - vt) \tag{2.7.1}$$

In above equations, λ and β are called unconstrained parameters and λ represents the amplitude, where β is said to be the inverse width of the soliton.

2.8 Singular Soliton

The dependence of singular wave on function of distance x and time t only and is given by,

$$u(x,t) = \delta \csc h^{\rho} \xi \qquad where \qquad \xi = \kappa(x - vt) \tag{2.8.1}$$

2.9 Description of Methods

2.9.1 The Modified Trial equation method

This method was first introduced by Liu [26, 27]. We consider these steps for applying this method:

Step 1 :Suppose that we have a fractional PDE.

$$Q(p, D_t^{\alpha} p, p_x, p_{xx}, p_{xxx}, ...) \tag{2.9.1}$$

and take the wave transformation,

$$p(x,t) = p(\xi), \qquad where \qquad \xi = kx - \frac{\lambda t^{\alpha}}{\Gamma(1+\alpha)} \tag{2.9.2}$$

where $\lambda \neq 0$. Substituting Eq. (2.9.2) into Eq. (2.9.1) yields a nonlinear ODE, and take the transformation,

$$M(p, p', p'', p''', ...) = 0 \tag{2.9.3}$$

Step 2: Take Trail equation as follow:

$$p' = \frac{R(p)}{T(p)} = \frac{\sum_{i=0}^{n} a_i p^i}{\sum_{j=0}^{l} b_j p^j} \tag{2.9.4}$$

$$p'' = \frac{R(p)(R'(p)T(p) - R(p)T'(p))}{T^3(p)} \tag{2.9.5}$$

where $R(P)$ and $T(P)$ are polynomials. By putting above relation into Eq. (2.9.3) yields an equation of polynomials $\Omega(p)$ of p:

$$\Omega(p) = p_s u^s + ... + p_1 u + p_o = 0 \tag{2.9.6}$$

By using balance principle, we can get a relation of n and l.

Step 3: Let the coefficients of $\Omega(p)$ all be zero will yield an algebraic equation system:

$u_i = 0, \qquad\qquad i = 0, ...s$

Solving this system, we will specify the values of $a_o, ...a_n$ and $b_o, ...b_1$

Step 4: Now by reducing Eq. (2.9.4) to elementary integral form we have,

$$\pm(\mu - \mu_o) = \int \frac{R(p)}{T(p)} dp \tag{2.9.7}$$

Using a complete discrimination system for polynomial to classify the roots of $R(p)$, we solve Eq. (2.9.7) with the aid of Maple and classify the exact solutions to the Eq.

(2.9.3). In addition, we can write the exact traveling wave solutions to the Eq. (2.9.1), respectively.

2.9.2 Improved G'/G expansion method

Consider a general PDE.

$$H(\tau, \tau_t, \tau_s, \tau_{tt}, \tau_{ss}, ...) = 0, \tag{2.9.8}$$

where x and t represents independent variables,and H is representing a polynomial in $\tau(s,t)$ and respective derivatives, then our concern is only with highest order derivatives and nonlinear terms. We utilize the conversion of traveling wave

$$\tau(s,t) = \tau(\xi), \quad \xi = s - vt \tag{2.9.9}$$

where v denotes the wave velocity. Reduce equation (2.9.8) in following ordinary differential equation(ODE)

$$Q(\tau, \tau', \tau', \tau''\tau'', ...) = 0 \tag{2.9.10}$$

where v is representing the constant. Here, Q represent a polynomial in $\tau(\xi)$ and its all derivatives, whereas dashes denote the derivatives with respect to ξ
Secondly, We suppose that equation (2.9.10) has the solution:

$$\tau\xi) = \sum_{i=0}^{n} a_i \frac{G'(\xi)}{G(\xi)}^i \tag{2.9.11}$$

where $a_i = (i = 0, ..., l)$ are representing the constants will resolute afterward where n is a balancing number, determined by homogeneous balance principle, also $G(\zeta)$

satisfies the given ODE in the form:

$$GG'' = KG^2 + LGG' + M(G')^2, \tag{2.9.12}$$

where K, L and M are the real parameters. Now Eq. (2.9.12) reduces into:

$$\frac{d}{d\xi}\left(\frac{G'}{G}\right) = K + L\left(\frac{G'}{G}\right) + (M-1)\left(\frac{G'}{G}\right)^2 \tag{2.9.13}$$

Now by using Eq. (2.9.13), we have the following different cases:

Case 1: If $L \neq 0$ and $\Delta = L^2 + 4K - 4KM < 0$

$$\frac{G'(\xi)}{G(\xi)} = \frac{L}{2(1-M)} + \frac{L\sqrt{-\Delta}}{2(1-M)}\left(\frac{ic_1\cos(\frac{\sqrt{-\Delta}}{2}\xi) - c_2\sin(\frac{\sqrt{-\Delta}}{2})\xi}{ic_1\sin(\frac{\sqrt{-\Delta}}{2}\xi) - c_2\cos(\frac{\sqrt{-\Delta}}{2})\xi}\right) \tag{2.9.14}$$

Case 2: If $L \neq 0$ and $\Delta = L^2 + 4K - 4KM \geq 0$

$$\frac{G'(\xi)}{G(\xi)} = \frac{L}{2(1-M)} + \frac{L\sqrt{\Delta}}{2(1-M)}\left(\frac{ic_1\exp(\frac{\sqrt{\Delta}}{2}\xi) - c_2\exp(\frac{-\sqrt{\Delta}}{2})\xi}{ic_1\exp(\frac{\sqrt{\Delta}}{2}\xi) - c_2\exp(\frac{-\sqrt{\Delta}}{2})\xi}\right) \tag{2.9.15}$$

Case 3: If $L = 0$ and $\Delta = K(1-M) < 0$

$$\frac{G'(\xi)}{G(\xi)} = \frac{\sqrt{-\Delta}}{2(1-M)}\left(\frac{ic_1\cosh\sqrt{-\Delta}\xi - c_2\sinh\sqrt{-\Delta}\xi}{-c_1\sinh\sqrt{-\Delta}\xi - c_2\cosh\sqrt{-\Delta}\xi}\right) \tag{2.9.16}$$

Case 4: If $L = 0$ and $\Delta = K(1-M) \geq 0$

$$\frac{G'(\xi)}{G(\xi)} = \frac{\sqrt{\Delta}}{2(1-M)}\left(\frac{ic_1\cos\sqrt{\Delta}\xi + c_2\sin\sqrt{\Delta}\xi}{c_1\sin\sqrt{\Delta}\xi - c_2\cos\sqrt{\Delta}\xi}\right) \tag{2.9.17}$$

where $\xi = s - vt$ and K,L,M,c_1,and c_2 are constants.

Now by putting Eq. (2.9.11) into Eq. (2.9.12) along with Eq. (2.9.10) we have coefficients of $\left(\frac{G'(\xi)}{G(\xi)}\right)^i$ where $(i = 0, 1, ...)$ then we get a set of algebraic equations by setting each coefficients to zero, for K,L,M and c. After solving these algebraic equations and then by putting into Eq. (2.9.11) we get general solution.

Chapter 3

New Solitary Wave Solutions for LPD Model

3.1 Governing Equation:

One of the famous NLSE is Lakshmanan-Porsezian-Daniel (LPD) model [21]. It was first observed in 1988, studied in a variety of context including fiber optics. The dimensionless form of LPD model with higher order dimension is given as [23]:

$$iq_t + aq_{xx} + bq_{xt} + cF(|q|^2)q = \sigma q_{xxxx} + \alpha(q_x)^2 q* + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta|q|^4 q = 0 \tag{3.1.1}$$

where $q(x,t)$ represents the complex valued wave function. The term q_t represents the temporal evolution of the nonlinear wave, the coefficient a represents GVD, coefficient b represents STD and F is the real-valued algebraic function which is the source of nonlinearity. Fourth order dispersion is represented by σ and two photon absorption by δ.

We start with the assumption:

$$q(x,t) = p(\xi)e^{i(\phi)} \tag{3.1.2}$$

where $p(\xi)$ represents the shape of wave profile, and the phase constant $\phi(x,t)$ is defined as

$$\xi = x - \upsilon t$$
$$\phi(x,t) = -\kappa x + \omega t + \theta \tag{3.1.3}$$

with κ represents the soliton frequency, ω is wave number and θ is the phase constant. Now,

$$q_x = (-i\kappa p + p')e^{i(\phi)} \tag{3.1.4}$$

$$q_t = (i\omega p - \nu p')e^{i(\phi)} \tag{3.1.5}$$

$$q_{xx} = (-\kappa^2 p + p'' - 2i\kappa p')e^{i(\phi)} \tag{3.1.6}$$

$$q_{xxx} = (p''' - 3i\kappa p'' - 3\kappa^2 p' + i\kappa^3 p)e^{i(\phi)} \tag{3.1.7}$$

$$q_{xxxx} = (p'''' - 4i\kappa p''' - 6\kappa^2 p'' + 4i\kappa^3 p' - \kappa^4 p)e^{i(\phi)} \tag{3.1.8}$$

$$q_{xt} = (i\kappa\upsilon p' - \upsilon p'' + \kappa\omega p + i\omega p')e^{i(\phi)} \tag{3.1.9}$$

Now,

$$q*(x,t) = p(\xi)e^{-i(\phi)} \tag{3.1.10}$$

where

$$q*_x = (i\kappa p + p') + e^{-i(\phi)} \tag{3.1.11}$$

$$q*_{xx} = (-\kappa^2 p + 2i\kappa p' + p'')e^{-i(\phi)} \tag{3.1.12}$$

$$|q|^2 q = p^3 e^{i(\phi)} \tag{3.1.13}$$

$$(q_x)^2 = (-\kappa^2 p^2 + p'^2 - 2i\kappa pp')e^{2i(\phi)} \tag{3.1.14}$$

$$|q|^2 q_{xx} = (-\kappa^2 p^3 - 2i\kappa p^2 p' + p^2 p'')e^{i(\phi)} \tag{3.1.15}$$

$$|q_x|^2 q = (pp'^2 + \kappa^2 p^3) * e^{i(\phi)} \tag{3.1.16}$$

$$q^2 q^*_{xx} = (-\kappa^2 p^3 + 2i\kappa p' p^2 + p^2 p)e^{i(\phi)} \tag{3.1.17}$$

$$|q|^4 q = p^5 e^{i(\phi)} \tag{3.1.18}$$

Using Eq.(3.1.2)- Eq.(3.1.8) into Eq. (3.1.1), we get real and imaginary parts,

By equating real part,we get

$$\sigma p'''' - (a + 6\sigma\kappa^2)p'' - b\upsilon p'' - (b\kappa\omega - \omega - a\kappa^2 - \sigma\kappa^4)p \tag{3.1.19}$$

$$-(\alpha + \gamma + \lambda - \beta)\kappa^2 p^3 + \delta p^5 - cF(p^2)p + (\alpha + \beta)pp'^2 + (\lambda + \gamma)p^2 p'' = 0$$

By equating imaginary parts, we have

$$4\sigma\kappa p''' + 2(\alpha + \gamma - \lambda)\kappa p^2 p' - (2\alpha\kappa + 4\kappa^3\sigma - b\omega)p' + (-1 + b\kappa)\upsilon p' = 0 \tag{3.1.20}$$

By taking,

$$\sigma = 0$$

$$\lambda = \alpha + \gamma$$

Eq. (3.1.20) reduces to,

$$\frac{(b\omega - 2\alpha\kappa)}{(1 - b\kappa)} = \upsilon \tag{3.1.21}$$

Eq. (3.1.19) reduces to,

$$\begin{aligned} -ap'' - b\upsilon p'' - (b\kappa\omega - \omega - a\kappa^2)p - (2\lambda - \beta)\kappa^2 p^3 + \\ \delta p^5 - cF(p^2)p + (\alpha + \beta)pp'^2 + (\lambda + \gamma)p^2 p'' = 0 \end{aligned} \tag{3.1.22}$$

3.2 Application of the Modified Trial Equation to the LPD model by using Kerr law of nonlinearity

We will apply modified trial equation method to LPD model along with Kerr law.
For Kerr law we have

$$G(p) = p$$

By applying Kerr law Eq. (3.1.1) becomes,

$$\begin{aligned} iq_t + aq_{xx} + bq_{xt} + c(|q|^2)q = \sigma q_{xxxx} + \alpha(q_x)^2 q* + \beta|q_x|^2 q + \\ \gamma|q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta|q|^4 q = 0, \end{aligned} \tag{3.2.1}$$

Eq. (3.1.2) becomes,

$$-ap'' - b\upsilon p'' - (b\kappa\omega - \omega - a\kappa^2)p - (c + (2\lambda - \beta)\kappa^2)p^3 +$$
$$\delta p^5 + (\alpha + \beta)pp'^2 + (\lambda + \gamma)p^2 p'' = 0 \qquad (3.2.2)$$

We will use modified trial equation method to Eq. (3.2.2) to get solitary wave solutions. By considering,

$$p' = \frac{R(p)}{T(p)} = \frac{\sum\limits_{i=0}^{n} a_i p^i}{\sum\limits_{j=0}^{l} b_j p^j} \qquad (3.2.3)$$

$$p'' = \frac{R(p)(R'(p)T(p) - R(p)T'(p))}{T^3(p)} \qquad (3.2.4)$$

Reduce Eq. (3.2.3) to the elementary integral form:

$$\pm(\mu - \mu_o) = \int \frac{R(p)}{T(p)} dp \qquad (3.2.5)$$

where $R(p)$ and $T(p)$ are polynomials. Now by using balance principle yields,

$$n = l + 2$$

Case 1:

If we take $l = 0$ then $n = 2$, Eq. (3.2.3) and Eq. (3.2.4) takes the form,

$$p' = \frac{a_o + a_1 p + a_2 p^2}{b_o} \qquad (3.2.6)$$

and

$$p'' = \frac{(a_1 + 2a_2 p)(a_o + a_1 p + a_2 p^2)}{b_o^2} \tag{3.2.7}$$

where $a_2 \neq 0, b_o \neq 0$. By putting values of p' and p'' in Eq. (3.2.2) we get,

$$\begin{aligned}(-a - b\upsilon + (\lambda+\gamma)p^2)(\frac{(a_1+2a_2p)(a_o+a_1p+a_2p^2)}{b_o^2}) + \\ (\alpha+\beta)(\frac{a_o+a_1p+a_2p^2}{b_o})^2 p - (b\kappa\omega - \omega - a\kappa^2)p \\ -(c + (2\lambda-\beta)\kappa^2)p^3 + \delta p^5\end{aligned}$$

Let,

$$(\lambda+\gamma) = A, \ \ (\alpha+\beta) = B, \ \ (-a - b\upsilon) = C,$$

$$((2\lambda-\beta)\kappa^2 + c) = D, \ \ (b\kappa\omega - \omega - a\kappa^2) = E$$

$$\begin{aligned}(C+Ap^2)(\frac{a_oa_1 + a_1^2p + a_1a_2p^2 + 2a_2a_op + 2a_1a_2p^2 + 2a_2^2p^3}{b_o^2}) \\ +Bp(\frac{a_o^2 + a_1^2p^2 + a_2^2p^4 + 2a_oa_1p + 2a_1a_2p^3 + 2a_oa_2p^2}{b_o^2}) \\ -Ep - Dp^3 + \delta p^5\end{aligned} \tag{3.2.8}$$

After simplifying Eq. (3.2.8), we get different powers of p,

$$\frac{2Aa_2^2}{b_o^2} + \frac{Ba_2^2}{b_o^2} + \delta = 0 \tag{3.2.9}$$

$$\frac{3Aa_1a_2}{b_o^2} + \frac{2Ba_1a_2}{b_o^2} = 0 \tag{3.2.10}$$

$$\frac{2Ca_2^2}{b_o^2}+\frac{Aa_1^2}{b_o^2}+\frac{2Aa_oa_2}{b_o^2}+\frac{Ba_1^2}{b_o^2}+\frac{2Ba_oa_2}{b_o^2}-D=0 \tag{3.2.11}$$

$$\frac{3Ca_1a_2}{b_o^2}+\frac{Aa_oa_1}{b_o^2}+\frac{2Ba_oa_1}{b_o^2}=0 \tag{3.2.12}$$

$$\frac{Ca_1^2}{b_o^2}+\frac{2Ca_oa_2u}{b_o^2}+\frac{Ba_o^2}{b_o^2}-E=0 \tag{3.2.13}$$

$$\frac{Ca_oa_1}{b_o^2}=0 \tag{3.2.14}$$

After solving Eq. (3.2.9) - Eq. (3.2.14), we get following solutions: $a_o=0, b_o=b_o$, $a_1=\pm b_o\sqrt{\frac{(b\kappa\omega-\omega-a\kappa^2)}{(-a-b\upsilon)}}$, $v=\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{-a_1^2b}$, $a_2=\pm\sqrt{\frac{(((2\lambda-\beta)\kappa^2+c)(-a-b\upsilon)-(b\kappa\omega-\omega-a\kappa^2)((\lambda+\gamma)(\alpha+\beta))(b_o)}{2(-a-b\upsilon)}}$
After putting coefficients in Eq. (3.2.5) we get,

$$\pm(\mu-\mu_o)=\int\frac{b_o}{a_2p^2+a_1p}dp \tag{3.2.15}$$

Integrating Eq. (3.2.15), we have solution of Eq. (3.2.2) as,

$$q(x,t)=\frac{a_1}{\exp[\frac{a_1}{2b_o}(x+(\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{a_1^2b}t)-\xi_o)-a_2]}e^{i(\phi)} \tag{3.2.16}$$

Where $\xi=x-vt$ where $v=\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{-a_1^2b}$. If we take $\xi_o=0, a_1=1, a_2=-1$ then solution (3.2.16) can reduce to hyperbolic function solution.

$$q(x,t)=\left(\frac{1}{2}-\frac{1}{2}\tanh\left(B\left(x+\left(\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{a_1^2b}t\right)\right)\right)\right)e^{i(\phi)} \tag{3.2.17}$$

Where $B=\frac{1}{2b_o}$. Here B is inverse widths of the soliton.

Case No.2.1:

If we take $l=1$ then $n=3$, by using Eq. (3.2.3) and Eq. (3.2.4) we get values of p' and p'' as follows:

$$p' = \frac{a_o + a_1 p + a_2 p^2 + a_3 p^3}{b_o + b_1 p} \tag{3.2.18}$$

$$p'' = \frac{(a_o + a_1 p + a_2 p^2 + a_3 p^3)((a_1 + 2a_2 p + 3a_3 p^2)(b_o + b_1 p)) - b_1(a_o + a_1 p + a_2 p^2 + a_3 p^3)}{(b_o + b_1 p)^3} \tag{3.2.19}$$

Where $a_3 \neq 0$, $b_1 \neq 0$. By putting values of p' and p'' in Eq. (3.2.2) we have,

$$-(c + (2\lambda - \beta)\kappa^2)p^3 + (-a - b\upsilon + (\lambda + \gamma)p^2)$$

$$\frac{(a_o + a_1 p + a_2 p^2 + a_3 p^3)((a_1 + 2a_2 p + 3a_3 p^2)(b_o + b_1 p) - b_1(a_o + a_1 p + a_2 p^2 + a_3 p^3))}{(b_o + b_1 p)^3} + \delta p^5$$

$$+(\alpha + \beta)(\frac{a_o + a_1 p + a_2 p^2 + a_3 p^3}{b_o + b_1 p})^2 p - (b\kappa\omega - \omega - a\kappa^2)p \tag{3.2.20}$$

Let

$$(\lambda + \gamma) = A, \;\; (\alpha + \beta) = B, \;\; (-a - b\upsilon) = C,$$

$$((2\lambda - \beta)\kappa^2 + c) = F, \;\; (b\kappa\omega - \omega - a\kappa^2) = E$$

$$(C + Ap^2)\frac{(a_o + a_1 p + a_2 p^2 + a_3 p^3)((a_1 + 2a_2 p + 3a_3 p^2)(b_o + b_1 p) - b_1(a_o + a_1 p + a_2 p^2 + a_3 p^3))}{(b_o + b_1 p)^3} \tag{3.2.21}$$

$$B(\frac{a_o+a_1p+a_2p^2+a_3p^3}{b_o+b_1p})^2p-Ep-Fp^3+\delta p^5$$

After simplifying Eq. (3.2.21), we get different powers of p,

$$\delta b_1^3+3Aa_3^2b_1+Ba_3^2b_1=0 \tag{3.2.22}$$

$$Ba_3^2b_o+3Aa_3^2b_o+5Aa_2a_3b_1+2Ba_2a_3b_1=0 \tag{3.2.23}$$

$$3\delta b_o^2b_1+3\delta b_ob_1^2-Fb_1^3+Ba_2^2b_1+3Ca_3^2b_1+5Aa_2a_3b_o \tag{3.2.24}$$

$$+2Aa_2^2b_1+4Aa_1a_3b_1+2Ba_2a_3b_o+2Ba_1a_3b_1$$

$$\delta b_o^3+2Aa_2^2b_o+Ba_2^2b_o+4Aa_1a_3b_o+3Ca_3^2b_o+2Ba_1a_2b_1+5Ca_2a_3b_1 \tag{3.2.25}$$

$$+3Aa_1a_2b_1-Ab_1a_3+2Ba_oa_3b_1+2Ba_1a_3b_o+3Aa_oa_3b_1-3Fb_ob_1^2=0$$

$$-3Fb_o^2b_1-3Fb_1^2b_o-Eb_1^3+Ba_1^2b_1+3Aa_oa_3b_o+2Aa_oa_2b_1+3Aa_1a_2b_o \tag{3.2.26}$$

$$+2Ba_1a_2b_o+2Ca_2^2b_1+5Ca_2a_3b_o-Ab_1a_2+Aa_1^2b_1+2Ba_oa_2b_1+2Ba_oa_3b_o+4Ca_1a_3b_1=0$$

$$-Fb_o^3+Ba_1^2b_o+Aa_oa_1b_1+3Ca_1a_2b_1+Aa_1^2b_o+2Ca_2^2b_o-Ab_1a_1-Cb_1a_3 \tag{3.2.27}$$

$$+2Ba_oa_2b_o+2Ba_oa_1b_1+3Ca_oa_3b_1+4Ca_1a_3b_1+2Aa_oa_2b_o-3Eb_ob_1^2=0$$

$$-3Eb_o^2b_1+Ba_o^2b_1+2Ca_oa_2b_1+Ca_1^2b_1-Cb_1a_2 \tag{3.2.28}$$

$$+3Ca_1a_2b_o+3Ca_oa_3b_o+2Ba_oa_1b_o+Aa_oa_1b_o-Ab_1a_o=0$$

$$-Eb_o^3+Ba_0^2b_o-Cb_1a_1+2Ca_oa_2b_o+Ca_oa_1b_1+Ca_1^2b_o=0 \tag{3.2.29}$$

$$-Cb_1a_o+Ca_oa_1b_o=0 \tag{3.2.30}$$

After solving Eq. (3.2.22) - Eq. (3.2.30), we get following solutions: $a_o=a_o$, $a_1=\frac{b_1}{b_o}$,$a_2=\frac{b_1(3(b\kappa\omega-\omega-a\kappa^2)b_o^2-(-a-b\upsilon)a_1^2)}{(-a-b\upsilon)(3a_1b_o-b_1)}$, $a_3=\sqrt{\frac{-\delta b_1^2}{3(\lambda+\gamma)+(\alpha+\beta)}}$, $b_o=b_o$,$b_1=b_1$, $\nu=\frac{b_o^3(b\kappa\omega-\omega-a\kappa^2)+aa_1^2b_o+aa_1b_1}{a_1^2bb_o-a_1bb_1}$

Now by putting coefficients in Eq. (3.2.5),we have

$$\pm(\mu-\mu_o)=\int\frac{b_o+b_1p}{a_3p^3+a_2p^2+a_1p+a_o}dp \tag{3.2.31}$$

After integrating Eq. (3.2.31), we get,

$$\pm(\mu-\mu_o)=-\frac{b_o-b_1\alpha_1+2b_1p}{2(p-\alpha_1)^2} \tag{3.2.32}$$

$$\pm(\mu-\mu_o)=\frac{(b_o+b_1\alpha_2)\ln|\frac{p-\alpha_2}{p-\alpha_1}|-\frac{(b_o+b_1\alpha_1)(\alpha_1-\alpha_2)}{p-\alpha_1}}{(\alpha_1-\alpha_2)^2} \tag{3.2.33}$$

$$\pm(\mu-\mu_o)=\frac{(b_o+b_1\alpha_1)\ln|p-\alpha_1|}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}-\frac{(b_o+b_1\alpha_2)\ln|p-\alpha_2|}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)}+\frac{(b_o+b_1\alpha_3)\ln|p-\alpha_3|}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)} \tag{3.2.34}$$

where α_1,α_2, and α_3 are the roots of the polynomial equation

$$p_3+\frac{a_2}{a_3}p^2+\frac{a_1}{a_3}p+\frac{a_o}{a_3}=0$$

By putting Eq. (3.2.32) into Eq. (3.2.5), we obtain

$$q(x,t)=\left[\frac{b_1+2\alpha_1(\xi-\xi_o)\pm\sqrt{b_1^2-(2b_o2b_1\alpha_1)\xi+2(b_o+b_1\alpha_1)\xi_o}}{2(\xi-\xi_o)}\right]e^{i\phi} \quad (3.2.35)$$

By considering $\xi_o=0$ in Eq. (3.2.35), we get rational function solutions:

$$q(x,t)=\frac{b_1+2\alpha_1(x-Et)\pm\sqrt{b_1^2-(2b_o2b_1\alpha_1)(x-Et)}}{2(x-Et)}e^{i\phi} \quad (3.2.36)$$

where $E=\frac{b_o^3D+aa_1^2b_o-aa_1b_1}{a_1^2bb_o-a_1bb_1}$ and $D=b\kappa\omega-\omega-a\kappa^2$

Case No.2.2:

Now considering $a_o=0$ in Eq. (3.2.18) and Eq. (3.2.19), we have

$$p'=\frac{a_1p+a_2p^2+a_3p^3}{b_o+b_1p} \quad (3.2.37)$$

$$p''=\frac{(a_1p+a_2p^2+a_3p^3)((a_1+2a_2p+3a_3p^2)(b_o+b_1p))-b_1(a_1p+a_2p^2+a_3p^3)}{(b_o+b_1p)^3} \quad (3.2.38)$$

Where $a_3\neq 0$, $b_1\neq 0$. By putting values of p' and p'' in Eq. (3.2.2) we have,

$$(-a-b\upsilon+(\lambda+\gamma)p^2)$$

$$\frac{(a_o+a_1p+a_2p^2+a_3p^3)((a_1+2a_2p+3a_3p^2)(b_o+b_1p)-b_1(a_o+a_1p+a_2p^2+a_3p^3))}{(b_o+b_1p)^3} \quad (3.2.39)$$

$$(\alpha+\beta)(\frac{a_o+a_1p+a_2p^2+a_3p^3}{b_o+b_1p})^2p-(b\kappa\omega-\omega-a\kappa^2)p-(c+(2\lambda-\beta)\kappa^2)p^3+\delta p^5$$

Let,

$$(\lambda+\gamma)=A,\ \ (\alpha+\beta)=B,\ \ (-a-b\upsilon)=C,$$

$$((2\lambda-\beta)\kappa^2+c)=F,\ \ (b\kappa\omega-\omega-a\kappa^2)=E$$

$$(C+Ap^2)\frac{(a_o+a_1p+a_2p^2+a_3p^3)((a_1+2a_2p+3a_3p^2)(b_o+b_1p)-b_1(a_o+a_1p+a_2p^2+a_3p^3))}{(b_o+b_1p)^3} \tag{3.2.40}$$

$$B(\frac{a_o+a_1p+a_2p^2+a_3p^3}{b_o+b_1p})^2p-Ep-Fp^3+\delta p^5$$

After simplifying Eq. (3.2.40), we get different powers of p.

$$\delta b_1^3+3Aa_3^2b_1+Ba_3^2b_1=0 \tag{3.2.41}$$

$$5Aa_2a_3b_1+Ba_3^2b_o+3Aa_3^2b_o+2Ba_2a_3b_1+3\delta b_ob_1^2=0 \tag{3.2.42}$$

$$-Fb_1^3+5Aa_2a_3b_o+4a_1a_3b_1+3Ca_3^2b_1+2Aa_2^2b_1+3\delta b_o^2b_1+2Ba_2a_3b_o \tag{3.2.43}$$

$$+2Ba_1a_3b_1+Ba_2^2b_1=0$$

$$\delta b_o^3+2Aa_2^2b_o+Ba_2^2b_o+4Aa_1a_3b_o+3Ca_3^2b_o+2Ba_1a_2b_1+5Ca_2a_3b_1 \tag{3.2.44}$$

$$+3Aa_1a_2b_1 - Ab_1a_3 + 2Ba_1a_3b_o - 3Fb_ob_1^2 = 0$$

$$-3Fb_o^2b_1 - Eb_1^3 + Ba_1^2b_1 + 3Aa_1a_2b_o + 2Ba_1a_2b_o + 2Ca_2^2b_1 \tag{3.2.45}$$

$$+5Ca_2a_3b_o - Ab_1a_2 + Aa_1^2b_1 + 4Ca_1a_3b_1 = 0$$

$$-Fb_o^3 + Ba_1^2b_o + 3Ca_1a_2b_1 + Aa_1^2b_o + 2Ca_2^2b_o - Ab_1a_1 - Cb_1a_3 \tag{3.2.46}$$

$$+4Ca_1a_3b_1 - 3Eb_ob_1^2 = 0$$

$$-3Eb_o^2b_1 + Ca_1^2b_1 - Cb_1a_2 + 3Ca_1a_2b_o = 0 \tag{3.2.47}$$

$$-Cb_1a_1 + Ca_1^2b_o = 0 \tag{3.2.48}$$

After solving Eq. (3.2.41) - Eq. (3.2.48), we get following solutions:
$a_o = a_o, a_1 = \frac{b_1}{b_o}, a_2 = \frac{b_1(3(b\kappa\omega-\omega-a\kappa^2)b_o^2-(-a-b\upsilon)a_1^2)}{(-a-b\upsilon)(3a_1b_o-b_1)}$ $a_3 = \sqrt{\frac{-\delta b_1^2}{3(\lambda+\gamma)+(\alpha+\beta)}}$ $b_o = b_o$,
$b_1 = b_1, \nu = \frac{b_o^3(b\kappa\omega-\omega-a\kappa^2)+aa_1^2b_o+aa_1b_1}{a_1^2bb_o-a_1bb_1}$
Now by putting coefficients in Eq. (3.2.5),we have

$$\pm(\mu-\mu_o) = \int \frac{b_o+b_1p}{(a_3p^3+a_2p^2+a_1p)}dp \tag{3.2.49}$$

After integrating Eq. (3.2.49) we get solution of Eq. (3.2.2) as follows:

$$\pm(\mu-\mu_o) = \frac{2(a_2b_o-a_1b_1)\arctan(\frac{a_2+2a_3p}{M}) + b_oM\ln|\frac{a_1+p(a_2+a_3p)}{p^2}|}{2a_1N} \tag{3.2.50}$$

where,

$$M = \sqrt{4a_1a_3-a_2^2}$$

Case No.2.3:

Now take $a_o = a_2 = 0$, in Eq. (3.2.18) and Eq. (3.2.19), we have

$$p' = \frac{a_1 p + a_3 p^3}{b_o + b_1 p} \tag{3.2.51}$$

$$p'' = \frac{(a_1 p + a_3 p^3)((a_1 + 3a_3 p^2)(b_o + b_1 p)) - b_1(a_1 p + a_3 p^3)}{(b_o + b_1 p)^3} \tag{3.2.52}$$

Where $a_3 \neq 0$, $b_1 \neq 0$. By putting values of p' and p'' in Eq. (3.2.2) we have,

$$(-a - b\upsilon + (\lambda + \gamma)p^2)\frac{(a_1 p + a_3 p^3)((a_1 + 3a_3 p^2)(b_o + b_1 p) - b_1(a_1 p + a_3 p^3))}{(b_o + b_1 p)^3} \tag{3.2.53}$$

$$+(\alpha + \beta)(\frac{a_1 p + a_3 p^3}{b_o + b_1 p})^2 p - (b\kappa\omega - \omega - a\kappa^2)p - (c + (2\lambda - \beta)\kappa^2)p^3 + \delta p^5$$

Let,

$$(\lambda + \gamma) = A, \ \ (\alpha + \beta) = B, \ \ (-a - b\upsilon) = C,$$

$$((2\lambda - \beta)\kappa^2 + c) = F, \ \ (b\kappa\omega - \omega - a\kappa^2) = E$$

$$(C + Ap^2)\frac{(a_1 p + a_3 p^3)((a_1 + 3a_3 p^2)(b_o + b_1 p) - b_1(a_1 p + a_3 p^3))}{(b_o + b_1 p)^3} \tag{3.2.54}$$

$$+B(\frac{a_1 p + a_3 p^3}{b_o + b_1 p})^2 p - Ep - Fp^3 + \delta p^5$$

After simplifying Eq. (3.2.54), we get different powers of p.

$$\delta b_1^3 + 3Aa_3^2 b_1 + Ba_3^2 b_1 = 0 \tag{3.2.55}$$

$$5Aa_2a_3b_1+Ba_3^2b_o+3Aa_3^2b_o+2Ba_2a_3b_1+3\delta b_ob_1^2=0 \tag{3.2.56}$$

$$-Fb_1^3+5Aa_2a_3b_o+4a_1a_3b_1+3Ca_3^2b_1+2Aa_2^2b_1+3\delta b_o^2b_1 \tag{3.2.57}$$

$$+2Ba_2a_3b_o+2Ba_1a_3b_1+Ba_2^2b_1=0$$

$$\delta b_o^3+2Aa_2^2b_o+Ba_2^2b_o+4Aa_1a_3b_o+3Ca_3^2b_o+2Ba_1a_2b_1+5Ca_2a_3b_1 \tag{3.2.58}$$

$$+3Aa_1a_2b_1-Ab_1a_3+2Ba_1a_3b_o-3Fb_ob_1^2=0$$

$$-3Fb_o^2b_1-Eb_1^3+Ba_1^2b_1+3Aa_1a_2b_o+2Ba_1a_2b_o+2Ca_2^2b_1 \tag{3.2.59}$$

$$+5Ca_2a_3b_o-Ab_1a_2+Aa_1^2b_1+4Ca_1a_3b_1=0$$

$$-Fb_o^3+Ba_1^2b_o+3Ca_1a_2b_1+Aa_1^2b_o+2Ca_2^2b_o-Ab_1a_1-Cb_1a_3 \tag{3.2.60}$$

$$+4Ca_1a_3b_1-3Eb_ob_1^2=0$$

$$-3Eb_o^2b_1+Ca_1^2b_1-Cb_1a_2+3Ca_1a_2b_o=0 \tag{3.2.61}$$

$$-Cb_1a_1+Ca_1^2b_o=0 \tag{3.2.62}$$

After solving Eq. (3.2.55) - Eq. (3.2.62), we get following solutions:

$a_o=a_o, a_1=\frac{b_1}{b_o}, a_3=\sqrt{\frac{-\delta b_1^2}{3(\lambda+\gamma)+(\alpha+\beta)}}$ $b_o=b_o, b_1=b_1$, $\nu=\frac{b_o^3(b\kappa\omega-\omega-a\kappa^2)+aa_1^2b_o+aa_1b_1}{a_1^2bb_o-a_1bb_1}$

Again by putting coefficients in Eq. (3.2.5) it becomes,

$$\pm(\mu-\mu_o)=\int\frac{b_o+b_1p}{a_3p^3+a_1p}dp \tag{3.2.63}$$

After integrating Eq.(3.2.63), we get solution of Eq. (3.2.2) as follows:

$$\pm(\mu-\mu_o)=\frac{2b_1\sqrt{a_1}\arctan(\sqrt{\frac{a_3}{a_1}p})+b_o\sqrt{a_3}\ln|\frac{p^2}{a_1+a_3p^2}|}{2a_1\sqrt{a_3}} \tag{3.2.64}$$

3.3 Application of the Modified Trial Equation to the LPD model by using Parabolic law of nonlinearity

We consider the following nonlinearity as:

$$F(p)=p+\nu p^2$$

Now in this section we will use parabolic law of nonlinearity [47] in Eq. (3.1.24) to obtain dark soliton solutions by using modified trail equation method.
Now we have,

$$F(p^2)=p+\nu p^4$$

By putting value in Eq. (3.1.24)it becomes,

$$-ap''-b\upsilon p''-(b\kappa\omega-\omega-a\kappa^2)p-(2\lambda-\beta)\kappa^2p^3+\delta p^5 \tag{3.3.1}$$

$$-c(p^3+\nu p^5)p+(\alpha+\beta)pp'^2+(\lambda+\gamma)p^2p''=0$$

Now we will apply modified trial equation method in system (3.3.1).

Case No.1

After putting values of p' and p'' from Eq. (3.2.6) and Eq. (3.2.7) in system (3.3.1).

we get,

$$((-a-b\upsilon)+(\lambda+\gamma)p^2)\frac{(a_1+2a_2p)(a_o+a_1p+a_2p^2)}{b_o^2} \tag{3.3.2}$$

$$-(b\kappa\omega-\omega-a\kappa^2)p-(2\lambda-\beta)\kappa^2p^3+\delta p^5-c(p^3+\nu p^5)+(\alpha+\beta)p(\frac{a_o+a_1p+a_2p^2}{b_o})^2$$

where $b_o \neq 0$.

Let,

$$(-a-b\upsilon)=A,\ \ (\lambda+\gamma)=B,\ \ (b\kappa\omega-\omega-a\kappa^2)=D$$

$$((2\lambda-\beta)\kappa^2)=E,\ \ (\alpha+\beta)=F$$

$$(A+Bp^2)\frac{(a_1+2a_2p)(a_o+a_1p+a_2p^2)}{b_o^2}-Dp \tag{3.3.3}$$

$$-Ep^3+\delta p^5-c(p^3+\nu p^5)+Fp(\frac{a_o+a_1p+a_2p^2}{b_o})^2$$

where $b_o \neq 0$. Thus we have system of algebraic equations from the coefficients of polynomial of p.

After simplifying Eq. (3.3.3), we have different powers of p.

$$\frac{2Ba_2^2}{b_o^2}-C\nu+\frac{Fa_2^2}{b_o^2}+\delta=0 \tag{3.3.4}$$

$$\frac{3Ba_1a_2}{b_o^2}+\frac{2Fa_1a_2}{b_o^2}=0 \tag{3.3.5}$$

$$\frac{2Aa_2^2}{b_o^2}+\frac{Ba_1^2}{b_o^2}+\frac{2Ba_2a_o}{b_o^2}-E-C+\frac{2Fa_oa_2}{b_o^2}+\frac{Fa_1^2}{b_o^2}=0 \tag{3.3.6}$$

$$\frac{3Aa_1a_2}{b_o^2}+\frac{Ba_1a_o}{b_o^2}+\frac{2Fa_oa_1}{b_o^2}=0 \tag{3.3.7}$$

$$\frac{Aa_1^2}{b_o^2}+\frac{2Aa_2a_o}{b_o^2}-D+\frac{Fa_o^2}{b_o^2}=0 \tag{3.3.8}$$

$$\frac{Aa_1a_o}{b_o^2}=0 \tag{3.3.9}$$

After solving Eq.(3.3.4)- Eq. (3.3.9), we get following solutions:

$a_o=0, b_o=b_o,\ \nu=\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{-a_1^2b}, a_1=\sqrt{\frac{-2(-a-b\nu)a_2^2+b_o^2(((2\lambda-\beta)\kappa^2)+c)}{(\lambda+\gamma)(\alpha+\beta)}},$
$a_2=b_o\sqrt{\frac{C\nu+\delta}{2(\lambda+\gamma)(\alpha+\beta)}},$

Now by putting coefficients in Eq. (3.2.5) we get,

$$\pm(\mu-\mu_o)=\int\frac{b_o}{a_2p^2+a_1p}dp \tag{3.3.10}$$

By integrating Eq. (3.3.10) we get solution as follows:

$$q(x,t)=\frac{a_1}{\exp[\frac{a_1}{2b_o}(x+(\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{a_1^2b}t)-\xi_o)-a_2]}e^{i(\phi)} \tag{3.3.11}$$

Where $\xi=x-\nu t$ where $\nu=\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{-a_1^2b}$. If we take $\xi_o=0, a_1=1, a_2=-1$ then solution (3.3.11) can reduce to hyperbolic function solution.

$$q(x,t)=\left(\frac{1}{2}-\frac{1}{2}\tanh\left(B\left(x+\left(\frac{b_o^2(b\kappa\omega-\omega-a\kappa^2)+aa_1^2}{a_1^2b}t\right)\right)e^{i(\phi)} \tag{3.3.12}$$

Where $B = \frac{1}{2b_o}$.

Case No.2.1:

Now for $l = 1$ and $n = 3$

Now by putting values of p' and p'' from Eq. (3.2.18) and Eq. (3.2.19) into Eq. (3.3.1).

$$((-a-b\nu)+(\lambda+\gamma))$$

$$(\frac{(a_o+a_1p+a_2p^2+a_3p^3)((a_1+2a_2p+3a_3p^2)(b_o+b_1p))-b_1(a_o+a_1p+a_2p^2+a_3p^3)}{(b_o+b_1p)^3}) \tag{3.3.13}$$

$$-(b\kappa\omega-\omega-a\kappa^2)p-(2\lambda-\beta)\kappa^2p^3+\delta p^5-c(p^3+\nu p^5)p+(\alpha+\beta)p(\frac{a_o+a_1p+a_2p^2+a_3p^3}{b_o+b_1p})^2$$

where $a_3 \neq 0, b_1 \neq 0$, Now let,

$$(-a-b\upsilon) = A, (\lambda+\gamma) = B, (b\kappa\omega-\omega-a\kappa^2) = D$$

$$((2\lambda-\beta)\kappa^2) = E, \ \ (\alpha+\beta) = F$$

$$(A+Bp^2)\frac{(a_o+a_1p+a_2p^2+a_3p^3)((a_1+2a_2p+3a_3p^2)(b_o+b_1p))-b_1(a_o+a_1p+a_2p^2+a_3p^3)}{(b_o+b_1p)^3} \tag{3.3.14}$$

$$-Dp-Ep^3+\delta p^5-c(p^3+\nu p^5)+Fp(\frac{a_o+a_1p+a_2p^2}{b_o+b_1p})^2$$

where $a_3 \neq 0, b_1 \neq 0$. Thus we have system of algebraic equations from the coefficients of polynomial of p.

After simplifying Eq. (3.3.14), we get different powers of p.

$$2Fa_2a_3b_1-3C\nu b_ob_1^2+3\delta b_ob_1^2+Fa_3^2b_o+3Ba_3^2b_1 = 0 \tag{3.3.15}$$

$$-Eb_1^3+2Fa_2a_3b_o-3C\nu b_o^2b_1+Fa_2^2b_1+3\delta b_o^2b_1 \tag{3.3.16}$$

$$+5Ba_2a_3b_1+2Fa_1a_3b_1+3Ba_3^2b_o+3Aa_3^2-Cb_1=0$$

$$\delta b_o^3+4Ba_1a_3b_1+2Fa_oa_3b_1+2Fa_1a_2b_1+5Ba_2a_3b_o+5Ba_2a_3b_1 \tag{3.3.17}$$

$$-C\nu b_o^3+Fa_2^2b_o-3Cb_ob_1^2-3Eb_ob_1^2+2Fa_1a_3b_o+3Aa_3^2b_o+2Ba_2^2b_1=0$$

$$2Ba_2^2b_o+2Fa_oa_3b_o+2Fa_oa_2b_1+2Fa_1a_2b_o-Db_1^3+Fa_1^2b_1+4Aa_1a_3b_1 \tag{3.3.18}$$

$$+5Aa_2a_3b_o-Bb_1a_3-3Cb_o^2b_1-3Eb_o^2b_1+3Ba_1a_2b_1+3Ba_oa_3b_1$$

$$-4Ba_1a_3b_o+2Aa_2^2b_1=0$$

$$2Fa_oa_1b_1-Eb_o^3+2Fa_oa_2b_o+Fa_1^2b_o+3Aa_oa_3b_1+3Aa_1a_2b_1+4Aa_1a_3b_o \tag{3.3.19}$$

$$-Bb_1a_2+2Ba_oa_2b_1-3Db_ob_1^2+3Ba_1a_2b_o+3Ba_oa_3b_o+Ba_1^2b_1$$

$$+2Aa_2^2b_o-Ab_1a_3-Cb_o^3=0$$

$$2Fa_oa_1b_o+Ba_oa_1b_1+3Aa_oa_3b_o+3Aa_1a_2b_o+2Aa_oa_2b_1-3Db_o^2b_1 \tag{3.3.20}$$

$$+Fa_o^2b_1+Ba_1^2b_o+Aa_1^2+2Ba_oa_2b_o-Ab_1a_1-Bb_1a_1=0$$

$$Aa_oa_1b_1+Ba_oa_1b_o-Db_o^3+2Aa_oa_2b_o+Fa_o^2b_o \tag{3.3.21}$$

$$-Bb_1a_o-Ab_1a_1+Aa_1^2b_o=0$$

$$Ab_1a_o+Aa_oa_1b_o=0 \tag{3.3.22}$$

After solving Eq. (3.3.15) - Eq. (3.3.22), we get following solutions:

$a_o = a_o,$

$a_1 = \frac{b_o^3(C\nu+\delta)+(\lambda+\gamma)(5a_2a_3b_o-2a_2^2b_1)+(-a-b\nu)(5a_2a_3b_1-3a_3^2b_o)-a_2^2b_o(\alpha+\beta)+3b_ob_1^2(c+(2\lambda-\beta)\kappa^2)}{4(\lambda+\gamma)a_3b_1+2(\alpha+\beta)(a_2b_1+a_3b_o)},$

$$a_2 = \frac{3(b\kappa\omega-\omega-a\kappa^2)b_o^2b_1+(\lambda+\gamma)(b_oa_1^2-b_1a_1)+(-a-b\nu)(a_1^2b_1-b_1a_1)}{3(-a-b\nu)a_1b_o}$$

$a_3 = \frac{(((2\lambda-\beta)\kappa^2)+c)b_o^3-(\alpha+\beta)a_1^2b_o(\lambda+\gamma)(b_1a_2-a_1^2b_1+a_1a_2b_o)-(-a-b\nu)(3a_1a_2b_1+2a_2^2b_o)+3(b\kappa\omega-\omega-a\kappa^2)b_ob_1^2}{(-a-b\nu)(4a_1b_o-b_1)},$

$b_o = b_o$, $b_1 = b_1$, $\nu = \frac{b_o^3(b\kappa\omega-\omega-a\kappa^2)+aa_1^2b_o-aa_1b_1}{a_1^2bb_o-a_1bb_1}$

By putting coefficients in Eq. (3.2.5), we have

$$\pm(\mu-\mu_o) = \int \frac{b_o+b_1p}{a_3p^3+a_2p^2+a_1p+a_o}dp \tag{3.3.23}$$

By integrating Eq. (3.3.23) we get,

$$\pm(\mu-\mu_o) = -\frac{b_o-b_1\alpha_1+2b_1p}{2(p-\alpha_1)^2} \tag{3.3.24}$$

$$\pm(\mu-\mu_o) = \frac{(b_o+b_1\alpha_2)\ln\left|\frac{p-\alpha_2}{p-\alpha_1}\right| - \frac{(b_o+b_1\alpha_1)(\alpha_1-\alpha_2)}{p-\alpha_1}}{(\alpha_1-\alpha_2)^2} \tag{3.3.25}$$

$$\pm(\mu-\mu_o) = \frac{(b_o+b_1\alpha_1)\ln|p-\alpha_1|}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)} - \frac{(b_o+b_1\alpha_2)\ln|p-\alpha_2|}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)} + \frac{(b_o+b_1\alpha_3)\ln|p-\alpha_3|}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)} \tag{3.3.26}$$

Where α_1, α_2, and α_3 are the roots of the polynomial equation

$$p_3 + \frac{a_2}{a_3}p^2 + \frac{a_1}{a_3}p + \frac{a_o}{a_3} = 0$$

Substituting the Eq. (3.3.24) into Eq. (3.2.5), then we find the solution as,

$$q(x,t)=\frac{b_1+2\alpha_1(\xi-\xi_o)\pm\sqrt{b_1^2-(2b_o2b_1\alpha_1)\xi+2(b_o+b_1\alpha_1)\xi_o}}{2(\xi-\xi_o)} \tag{3.3.27}$$

Where $\xi=x-\nu t$ and $\nu=\frac{b_o^3(b\kappa\omega-\omega-a\kappa^2)+aa_1^2b_o-aa_1b_1}{a_1^2bb_o-a_1bb_1}$.If we take $\xi_o=0$ then the Eq.(3.3.27) can reduces as,

$$q(x,t)=\frac{b_1+2\alpha_1(x-Et)\pm\sqrt{b_1^2-(2b_o2b_1\alpha_1)(x-Et)}}{2(x-Et)}e^{i\phi} \tag{3.3.28}$$

where $E=\frac{b_o^3D+aa_1^2b_o-aa_1b_1}{a_1^2bb_o-a_1bb_1}$ and $D=b\kappa\omega-\omega-a\kappa^2$

Case No.2.2:

Now considering $a_o=0$ by putting values of p' and p'' from Eq. (3.2.37) and Eq. (3.2.38) into Eq. (3.3.1), we have

$$((-a-b\nu)+(\lambda+\gamma))(\frac{(a_1p+a_2p^2+a_3p^3)((a_1+2a_2p+3a_3p^2)(b_o+b_1p))-b_1(a_1p+a_2p^2+a_3p^3)}{(b_o+b_1p)^3})$$

$$-(b\kappa\omega-\omega-a\kappa^2)p-(2\lambda-\beta)\kappa^2p^3+\delta p^5-c(p^3+\nu p^5)p+(\alpha+\beta)p(\frac{a_1p+a_2p^2+a_3p^3}{b_o+b_1p})^2 \tag{3.3.29}$$

Let

$$(-a-b\upsilon)=A,(\lambda+\gamma)=B,(b\kappa\omega-\omega-a\kappa^2)=D$$

$$((2\lambda-\beta)\kappa^2)=E,\ \ (\alpha+\beta)=F$$

$$(A+Bp^2)\frac{(a_1p+a_2p^2+a_3p^3)((a_1+2a_2p+3a_3p^2)(b_o+b_1p))-b_1(a_1p+a_2p^2+a_3p^3)}{(b_o+b_1p)^3} \tag{3.3.30}$$

$$-Dp - Ep^3 + \delta p^5 - c(p^3 + \nu p^5) + Fp(\frac{a_1 p + a_2 p^2}{b_o + b_1 p})^2$$

where $a_3 \neq 0, b_1 \neq 0.$ Thus we have system of algebraic equations from the coefficients of polynomial of p.

After simplifying Eq. (3.3.30), we get different powers of p.

$$2Fa_2a_3b_1 - 3C\nu b_o b_1^2 + 3\delta b_o b_1^2 + Fa_3^2 b_o + 3Ba_3^2 b_1 = 0 \tag{3.3.31}$$

$$-Eb_1^3 + 2Fa_2a_3b_o - 3C\nu b_o^2 b_1 + Fa_2^2 b_1 + 3\delta b_o^2 b_1 + 5Ba_2a_3b_1 \tag{3.3.32}$$

$$+2Fa_1a_3b_1 + 3Ba_3^2 b_o + 3Aa_3^2 - Cb_1 = 0$$

$$\delta b_o^3 + 4Ba_1a_3b_1 + 5Ba_2a_3b_o - C\nu b_o^3 + Fa_2^2 b_o - 3Cb_o b_1^2 \tag{3.3.33}$$

$$-3Eb_o b_1^2 + 2Fa_1a_3b_o + 3Aa_3^2 b_o + 2Ba_2^2 b_1 + 5Aa_2a_3b_1 = 0$$

$$Ba_2^2 b_o + 2Fa_1a_2b_o - Db_1^3 + Fa_1^2 b_1 + 4Aa_1a_3b_1 + 5Aa_2a_3b_o + 2Aa_2^2 b_1 \tag{3.3.34}$$

$$-Bb_1a_3 - 3Cb_o^2 b_1 - 3Eb_o^2 b_1 + 3Ba_1a_2b_1 - 4Ba_1a_3b_o + 3Aa_2a_1b_1^2 = 0$$

$$-Eb_o^3 + Fa_1^2 b_o + 3Aa_1a_2b_1 + 4Aa_1a_3b_o - Bb_1a_2 - 3Db_o b_1^2 \tag{3.3.35}$$

$$+3Ba_1a_2b_o + Ba_1^2 b_1 + 2Aa_2^2 b_o - Ab_1a_3 - Cb_o^3 = 0$$

$$3Aa_1a_2b_o - 3Db_o^2 b_1 + Ba_1^2 b_o + Aa_1^2 b_1 - Ab_1a_1 - Bb_1a_1 = 0 \tag{3.3.36}$$

$$-Ab_1a_1 - Db_o^3 + Aa_1^2b_o = 0 \tag{3.3.37}$$

After solving Eq. (3.3.31)- Eq. (3.3.37), we get following solutions: $a_o = 0$, $a_1 = \frac{b_o^3(c\nu+\delta)+(\lambda+\gamma)(5a_2a_3b_o-2a_2^2b_1)+(-a-b\nu)(5a_2a_3b_1-3a_3^2b_o)-a_2^2b_o(\alpha+\beta)+3b_ob_1^2(c+(2\lambda-\beta)\kappa^2)}{4(\lambda+\gamma)a_3b_1+2(\alpha+\beta)(a_2b_1+a_3b_o)}$,
$a_2 = \frac{3(b\kappa\omega-\omega-a\kappa^2)b_o^2b_1+(\lambda+\gamma)(b_oa_1^2-b_1a_1)+(-a-b\nu)(a_1^2b_1-b_1a_1)}{3(-a-b\nu)a_1b_o}$, $b_o = b_o$, $b_1 = b_1$, $\nu = \frac{b_o^3(b\kappa\omega-\omega-a\kappa^2)+aa_1^2b_o-aa_1b_1}{a_1^2bb_o-a_1bb_1}$
$a_3 = \frac{(((2\lambda-\beta)\kappa^2)+c)b_o^3-(\alpha+\beta)a_1^2b_o+(\lambda+\gamma)(b_1a_2-a_1^2b_1+a_1a_2b_o)-(-a-b\nu)(3a_1a_2b_1+2a_2^2b_o)+3(b\kappa\omega-\omega-a\kappa^2)b_ob_1^2}{(-a-b\nu)(4a_1b_o-b_1)}$,
Again by putting coefficients in Eq. (3.2.5),we have

$$\pm(\mu - \mu_o) = \int \frac{b_o + b_1p}{(a_3p^3 + a_2p^2 + a_1p)} dp \tag{3.3.38}$$

After integrating Eq. (3.3.38) we get,

$$\pm(\mu - \mu_o) = \frac{2(a_2b_o - a_1b_1)\arctan(\frac{a_2+2a_3p}{N}) + b_oN\ln|\frac{a_1+p(a_2+a_3p)}{p^2}|}{2a_1N} \tag{3.3.39}$$

where,

$$N = \sqrt{4a_1a_3 - a_2^2}$$

Case No.2.3:

Now for $a_o = a_2 = 0$,by putting values of p' and p'' from Eq. (3.2.51) and Eq. (3.2.52) into Eq. (3.3.1), we have

$$((-a-b\nu)+(\lambda+\gamma))(\frac{(a_1p+a_3p^3)((a_1+3a_3p^2)(b_o+b_1p))-b_1(a_1p+a_3p^3)}{(b_o+b_1p)^3}) \tag{3.3.40}$$

$$-(b\kappa\omega-\omega-a\kappa^2)p-(2\lambda-\beta)\kappa^2p^3+\delta p^5-c(p^3+\nu p^5)p+(\alpha+\beta)p(\frac{a_1p+a_3p^3}{b_o+b_1p})^2$$

Let,

$$(-a-b\upsilon) = A, (\lambda+\gamma) = B, (b\kappa\omega-\omega-a\kappa^2) = D$$

$$((2\lambda-\beta)\kappa^2)=E,\ \ (\alpha+\beta)=F$$

$$(A+Bp^2)\frac{(a_1p+a_3p^3)((a_1+3a_3p^2)(b_o+b_1p))-b_1(a_1p+a_3p^3)}{(b_o+b_1p)^3}-Dp \tag{3.3.41}$$

$$-Ep^3+\delta p^5-c(p^3+\nu p^5)+Fp(\frac{a_1p+a_3p^3}{b_o+b_1p})^2$$

where $a_3\neq 0, b_1\neq 0.$ Thus we have system of algebraic equations from the coefficients of polynomial of p.

After simplifying Eq. (3.3.41), we get different powers of p.

$$-3C\nu b_ob_1^2+3\delta b_ob_1^2+Fa_3^2b_o+3Ba_3^2b_1=0 \tag{3.3.42}$$

$$-Eb_1^3-3C\nu b_o^2b_1+3\delta b_o^2b_1+2Fa_1a_3b_1+3Ba_3^2b_o+3Aa_3^2-Cb_1=0 \tag{3.3.43}$$

$$\delta b_o^3+4Ba_1a_3b_1-C\nu b_o^3-3Cb_ob_1^2-3Eb_ob_1^2+2Fa_1a_3b_o+3Aa_3^2b_o=0 \tag{3.3.44}$$

$$-Db_1^3+Fa_1^2b_1+4Aa_1a_3b_1+Bb_1a_3-3Cb_o^2b_1-3Eb_o^2b_1-4Ba_1a_3b_o=0 \tag{3.3.45}$$

$$-Eb_o^3+Fa_1^2b_o+4Aa_1a_3b_o-3Db_ob_1^2+Ba_1^2b_1-Ab_1a_3-Cb_o^3=0 \tag{3.3.46}$$

$$-3Db_o^2b_1+Ba_1^2b_o+Aa_1^2b_1-Bb_1a_1=0 \tag{3.3.47}$$

$$-Ab_1a_1-Db_o^3+Aa_1^2b_o=0 \tag{3.3.48}$$

After solving Eq. (3.3.42)- Eq. (3.3.48), we get following solutions:

$a_o=a_2=0,\, a_1=\frac{b_o^3(c\nu+\delta)+(\lambda+\gamma)(5a_2a_3b_o-2a_2^2b_1)+(-a-b\nu)(5a_2a_3b_1-3a_3^2b_o)-a_2^2b_o(\alpha+\beta)+3b_ob_1^2(c+(2\lambda-\beta)\kappa^2)}{4(\lambda+\gamma)a_3b_1+2(\alpha+\beta)(a_2b_1+a_3b_o)}$,

$a_3=\frac{(((2\lambda-\beta)\kappa^2)+c)b_o^3-(\alpha+\beta)a_1^2b_o+(\lambda+\gamma)(b_1a_2-a_1^2b_1+a_1a_2b_o)-(-a-b\nu)(3a_1a_2b_1+2a_2^2b_o)+3(b\kappa\omega-\omega-a\kappa^2)b_ob_1^2}{(-a-b\nu)(4a_1b_o-b_1)}$,

$b_o=b_o,\, b_1=b_1,\, \nu=\frac{b_o^3(b\kappa\omega-\omega-a\kappa^2)+aa_1^2b_o-aa_1b_1}{a_1^2bb_o-a_1bb_1}$

Now Eq. (3.2.5) becomes,

$$\pm(\mu-\mu_o)=\int\frac{b_o+b_1p}{a_3p^3+a_1p}dp \tag{3.3.49}$$

After integrating Eq. (3.3.49) we get, solution as follow:

$$\pm(\mu-\mu_o)=\frac{2b_1\sqrt{a_1}\arctan(\sqrt{\frac{a_3}{a_1}p})+b_o\sqrt{a_3}\ln|\frac{p^2}{a_1+a_3p^2}|}{2a_1\sqrt{a_3}} \tag{3.3.50}$$

Chapter 4

New Traveling Wave Solutions for LPD Model

4.1 Application of Improved G'/G expansion method to the LPD model by using Kerr law of nonlinearity

Now we will apply $(\frac{G'}{G})$ -expansion method in system (3.2.2)to generate new and more general exact traveling wave solutions [24].
As we have the transformation,

$$q(x,t) = p(\xi)e^{i(\phi)} \tag{4.1.1}$$

$\xi = x - \upsilon t$. The solution $p(\xi)$ is considered as:

$$p(\xi) = \Sigma a_i \left(\frac{G'(\xi)}{G(\xi)}\right)^i \tag{4.1.2}$$

By balancing Eq. (3.2.2), we get $n = 1$. Now by using improved$(\frac{G'}{G})$ expansion method, solution takes the form:
Eq. (4.1.2) becomes,

$$p(\xi) = a_o + a_1 \left(\frac{G'(\xi)}{G(\xi)}\right) \tag{4.1.3}$$

Also from [46]

$$GG'' = DG^2 + EGG' + F(G')^2 \tag{4.1.4}$$

Now by putting Eq. (4.1.3)and Eq. (4.1.4) into Eq. (3.2.2) and collecting the coefficients of $\left(\frac{G'(\xi)}{G(\xi)}\right)^i$ where$(i = 0,1,2,3)$ then we get set of algebraic equations after putting each coefficient to zero , Now let,

$$(-a - b\upsilon) = C, (\lambda + \gamma) = A, (b\kappa\omega - \omega - a\kappa^2) = Z$$

$$(c+(2\lambda-\beta)\kappa^2)=I, \ \ (\alpha+\beta)=B$$

$$(C+Ap^2)p''-zp-Ip^3+\delta p^5+Bpp'^2=0 \tag{4.1.5}$$

By solving these algebraic equations with the aid of Mapple, we get following equations.

$$2aAa_1^3+\delta a_1^5+Ba_1^3+2Aa_1^3F^2-4Aa_1^3+Ba_1^3F^2-2Ba_1^3F=0 \tag{4.1.6}$$

$$2Ba_1^3EF+Ba_1^2F^2-2Ba_1^2Fa_o+4Aa_oa_1^2+4Aa_oa_1^2F^2- \tag{4.1.7}$$

$$8Aa_oa_1^2F+3Aa_1^3EF-3Aa_1^3E-2Ba_1^3E+Ba_1^2a_o+5\delta a_oa_1^4=0$$

$$2Ba_1^3DF+2Ba_1^2EFa_o-2Ba_1^2Ea_o+2Ca_1-Ia_1^3+Aa_1^3E^2+2Aa_o^2a_1-2Aa_1^3D \tag{4.1.8}$$

$$-4Ca_1F+2Aa_o^2a_1F^2-4Aa_o^2a_1F+6Aa_oa_1^2EF-6Aa_oa_1^2E-2Ba_1^3+Ba_1^3E^2$$

$$+10\delta a_o^2a_1^3+2Ca_1F^2+2Aa_1^3FD=0$$

$$2Ba_1^2DFa_o-2Ba_1^2Da_o+Ba_1^2E^2a_o+3Ca_1EF+3Aa_o^2a_1EF-3Aa_o^2a_1E \tag{4.1.9}$$

$$+2Aa_oa_1^2E^2+4Aa_oa_1^2FD-4Aa_oa_1^2D+Aa_1^3ED-3Ia_oa_1^2$$

$$+10\delta a_oa_1^2+2Ba_1^3DE-3Ca_1E=0$$

$$Ca_1E^2+2Ca_1FD-za_1-2Aa_o^2a_1D+Aa_o^2a_1E^2+2Aa_o^2a_1FD+2Aa_oa_1^2ED \tag{4.1.10}$$

$$+Ba_1^3D^2-3Ia_o^2a_1+5\delta a_o^4a_1+2Ba_1^2DEa_o-2Ca_1D=0$$

$$Aa_o^2a_1ED+Ca_1ED+Ba_1^2D^2a_o-za_o-Ia_o^3+\delta a_o^5=0 \tag{4.1.11}$$

After solving Eq.(4.1.6)-Eq. (4.1.11), we get following solutions:
$a_o=0, a_1=\sqrt{\frac{3(-a-b\upsilon)E(1-F)}{((\lambda+\gamma)+2(\alpha+\beta))DE}}, D=D, E=E, F=\frac{za_1+Ba_1^3D+C(2a_1D-a_1E^2)}{2Ca_1D}$,
$v=\frac{a_1^2DE((\lambda+\gamma)+2(\alpha+\beta))+3aE(1-F)}{3bE(F-1)}$.
If we put these values in Eq. (4.1.4), we find the following cases,
Case 1: when $E\neq 0$ and $\Delta=E^2+4D+4DF<0$

$$p_1(\xi)=\sqrt{\frac{3(-a-b\upsilon)E(1-F)}{((\lambda+\gamma)+2(\alpha+\beta))DE}}\left(\frac{E}{2(1-F)}\right) \tag{4.1.12}$$

$$+\frac{E\sqrt{-\Delta}}{2}\left(\frac{ic_1\cos(\frac{\sqrt{-\Delta}}{2}(x-\upsilon t))-c_2\sin(\frac{\sqrt{-\Delta}}{2}(x-\upsilon t))}{ic_1\sin(\frac{\sqrt{-\Delta}}{2}(x-\upsilon t))+c_2\cos(\frac{\sqrt{-\Delta}}{2}(x-\upsilon t))}\right)$$

Case 2: when $E\neq 0$ and $\Delta=E^2+4D+4DF\geq 0$

$$p_2(\xi)=\sqrt{\frac{3(-a-b\upsilon)E(1-F)}{((\lambda+\gamma)+2(\alpha+\beta))DE}}\left(\frac{E}{2(1-F)}\right) \tag{4.1.13}$$

$$+\frac{E\sqrt{\Delta}}{2}\left(\frac{ic_1\exp(\frac{\sqrt{\Delta}}{2}(x-\upsilon t))+c_2\exp(\frac{-\sqrt{\Delta}}{2}(x-\upsilon t))}{ic_1\exp(\frac{\sqrt{\Delta}}{2}(x-\upsilon t))-c_2\exp(\frac{-\sqrt{\Delta}}{2}(x-\upsilon t))}\right)$$

Case 3: when $E=0$ and $\Delta=D(1-F)<0$

$$p_3(\xi)=\sqrt{\frac{3(-a-b\upsilon)E(1-F)}{((\lambda+\gamma)+2(\alpha+\beta))DE}} \tag{4.1.14}$$

$$\left(\frac{\sqrt{-\Delta}}{(1-F)}\right)\left(\frac{ic_1\cosh(\sqrt{-\Delta}(x-\upsilon t))-c_2\sinh(\sqrt{-\Delta}(x-\upsilon t))}{-c_1\sinh(\sqrt{-\Delta}(x-\upsilon t))-c_2\cosh(\sqrt{-\Delta}(x-\upsilon t))}\right)$$

Case 4: when $E = 0$ and $\Delta = D(1-F) \geq 0$

$$p_4(\xi) = \sqrt{\frac{3(-a-b\upsilon)E(1-F)}{((\lambda+\gamma)+2(\alpha+\beta))DE}}\left(\frac{\sqrt{\Delta}}{(1-F)}\right) \tag{4.1.15}$$

$$\left(\frac{ic_1\cos(\sqrt{\Delta}(x-\upsilon t))+c_2\sin(\sqrt{\Delta}(x-\upsilon t))}{c_1\sin(\sqrt{\Delta}(x-\upsilon t))-c_2\cos(\sqrt{\Delta}(x-\upsilon t))}\right)$$

Eq. (4.1.12) can be rewritten at $c_1 = -c_2$,as follow:

$$p_1(\xi) = \sqrt{\frac{3(-a-b\upsilon)E(1-F)}{((\lambda+\gamma)+2(\alpha+\beta))DE}}\left(\frac{E}{2(1-F)}\right) \tag{4.1.16}$$

$$+\frac{E\sqrt{-\Delta}}{2}\tanh\frac{\sqrt{-\Delta}}{2}(x-\nu t)$$

Again Eq. (4.1.12) can be rewritten at $c_1 = c_2$,as follow:

$$p_1(\xi) = \sqrt{\frac{3(-a-b\upsilon)E(1-F)}{((\lambda+\gamma)+2(\alpha+\beta))DE}}\left(\frac{E}{2(1-F)}\right) \tag{4.1.17}$$

$$+\frac{E\sqrt{-\Delta}}{2}\coth\frac{\sqrt{-\Delta}}{2}(x-\nu t)$$

4.2 Application of Improved G'/G expansion method to the LPD model by using Parabolic law of nonlinearity

Now we will apply $(\frac{G'}{G})$ -expansion method in system (3.3.1) to generate new and more general exact traveling wave solutions.

By balancing Eq. (3.3.1), we get $n = 1$. The solution for Eq. (4.1.3) can be written as,

$$p(\xi) = a_o + a_1\frac{G'(\xi)}{G(\xi)} \tag{4.2.1}$$

Now by putting Eq.(4.1.4) and Eq. (4.2.1) into Eq. (3.3.1) and collecting the coefficients of $\left(\frac{G'(\xi)}{G(\xi)}\right)^i$,(i=0, 1, 2, 3,...), we get set of algebraic equations

$$(-a-b\upsilon)=,(\lambda+\gamma)=B,(b\kappa\omega-\omega-a\kappa^2)=Z$$

$$(2\lambda-\beta)\kappa^2=K,\ \ (\alpha+\beta)=I$$

$$(A+Bp^2)p''-zp-Kp^3+\delta p^5-Cp^3-C\nu p^5+Ipp'^2=0 \tag{4.2.2}$$

After solving above system (4.2.2) we get set of algebraic equations as follows:

$$Ia_1^3+\delta a_1^5+2Ba_1^3+2Ba_1^3F^2-4Ba_1^3F-C\nu a_1^5+Ia_1^3F^2-2Ia_1^3F=0 \tag{4.2.3}$$

$$4Ba_oa_1^2+4Ba_oa_1^2F^2-8Ba_oa_1^2F+3Ba_1^3EF-3Ba_1^3E+5\delta a_oa_1^4+Ia_1^2a_o \tag{4.2.4}$$

$$-2Ia_1^3E-5C\nu a_oa_1^4+Ia_1^2F^2a_o+2Ia_1^3EF-2IFa_oa_1^2=0$$

$$-Ba_1^3E^2+2Aa_1-Ka_1^3-Ca_1^3-4Ba_o^2a_1F-4Aa_1F+2Ba_o^2a_1F^2+6Ba_oa_1^2EF \tag{4.2.5}$$

$$6Ba_oa_1^2E-2Ba_1^3D+10\delta a_o^2a_1^3-2Ia_1^2Ea_o-2Ia_1^3D+2Ba_1^3FD$$

$$-10C\nu a_o^2a_1^3+Ia_1^3E^2+2Ia_1^3DF+2Ia_1^2EFa_o+2Ba_o^2a_1+2Aa_1F^2=0$$

$$Ba_1^3ED+3Aa_1EF+3Ba_o^2a_1EF-3Ba_o^2a_1E+2Ba_oa_1^2E^2+4Ba_oa_1^2FD \tag{4.2.6}$$

$$-4Ba_oa_1^2D-3Ka_oa_1^2+10\delta a_o^3a_1^2-3Ca_oa_1^2-10C\nu a_o^3a_1^2$$

$$+2Ia_1^2DFa_o+2Ia_1^3DE-2IDa_1^2a_o+Ia_1^2E^2a_o-3Aa_1E=0$$

$$-za_1 - 2Aa_1D + Aa_1E^2 + 2Aa_1FD + Ba_o^2a_1E^2 + 2Ba_o^2a_1FD - 2Ba_o^2a_1D \quad (4.2.7)$$

$$+2Ba_oa_1^2ED - 3Ka_o^2a_1 + 5\delta a_o^4a_1 - 3Ca_o^2a_1 + Ia_1^3D^2 - 5C\nu a_o^4a_1 + 2Ia_1^2EDa_o = 0$$

$$-C\nu a_o^5 + Ba_o^2a_1ED - za_o + Ia_1^2D^2a_o - Ka_o^3 + \delta a_o^5 - Ca_o^3 + Aa_oa_1ED = 0 \quad (4.2.8)$$

After solving Eq.(4.2.3)-Eq. (4.2.8), we get following solutions:

$a_o = 0,\ a_1 = \sqrt{\frac{(b\kappa\omega-\omega-a\kappa^2)+(-a-b\upsilon)(2D-E^2+2FD)}{(\alpha+\beta)D^2}}$

$D = D,\ E = E, F = \frac{(b\kappa\omega-\omega-a\kappa^2)a_1+(-a-b\nu)(2a_1D+a_1E^2)+(\alpha+\beta)a_1^3D^2}{2(-a-b\nu)a_1D}$

$\nu = \frac{aa_1E^2+2aa_1(D-FD)+a_1(b\kappa\omega-\omega-a\kappa^2)-(\alpha+\beta)a_1^3D^2}{2a_1bD(1-FD)-a_1bE^2}$

If we put these values in Eq. (4.4.1),we find the following cases:

Case 1: when $E \neq 0$ and $\Delta = E^2 + 4D + 4DF < 0$

$$p_1(\xi) = \sqrt{\frac{(b\kappa\omega - \omega - a\kappa^2) + (-a - b\upsilon)(2D - E^2 + 2FD)}{(\alpha+\beta)D^2}} \quad (4.2.9)$$

$$\left(\frac{E}{2(1-F)}\right) + \frac{E\sqrt{-\Delta}}{2}\left(\frac{ic_1\cos(\frac{\sqrt{-\Delta}}{2}(x-\nu t)) - c_2\sin(\frac{\sqrt{-\Delta}}{2}(x-\nu t))}{ic_1\sin(\frac{\sqrt{-\Delta}}{2}(x-\nu t)) + c_2\cos(\frac{\sqrt{-\Delta}}{2}(x-\nu t))}\right)$$

Case 2:when $E \neq 0$ and $\Delta = E^2 + 4D + 4DF \geq 0$

$$p_2(\xi) = \sqrt{\frac{(b\kappa\omega - \omega - a\kappa^2) + (-a - b\upsilon)(2D - E^2 + 2FD)}{(\alpha+\beta)D^2}} \quad (4.2.10)$$

$$\left(\frac{E}{2(1-F)}\right) + \frac{E\sqrt{\Delta}}{2}\left(\frac{ic_1\exp(\frac{\sqrt{\Delta}}{2}(x-\nu t)) + c_2\exp(\frac{-\sqrt{\Delta}}{2}(x-\nu t))}{ic_1\exp(\frac{\sqrt{\Delta}}{2}(x-\nu t)) - c_2\exp(\frac{-\sqrt{\Delta}}{2}(x-\nu t))}\right)$$

Case 3:when $E = 0$ and $\Delta = D(1-F) < 0$

$$p_3(\xi) = \sqrt{\frac{(b\kappa\omega - \omega - a\kappa^2) + (-a - b\upsilon)(2D - E^2 + 2FD)}{(\alpha+\beta)D^2}} \quad (4.2.11)$$

$$\left(\frac{\sqrt{-\Delta}}{(1-F)}\right)\left(\frac{ic_1\cosh(\sqrt{-\Delta}(x-\nu t))-c_2\sinh(\sqrt{-\Delta}(x-\nu t))}{-c_1\sinh(\sqrt{-\Delta}(x-\nu t))-c_2\cosh(\sqrt{-\Delta}(x-\nu t))}\right)$$

Case 4: when $E=0$ and $\Delta = D(1-F)\geq 0$

$$p_4(\xi)=\sqrt{\frac{(b\kappa\omega-\omega-a\kappa^2)+(-a-b\upsilon)(2D-E^2+2FD)}{(\alpha+\beta)D^2}} \tag{4.2.12}$$

$$\left(\frac{\sqrt{\Delta}}{(1-F)}\right)\left(\frac{ic_1\cos(\sqrt{\Delta}(x-\nu t))+c_2\sin(\sqrt{\Delta}(x-\nu t))}{c_1\sin(\sqrt{\Delta}(x-\nu t))-c_2\cos(\sqrt{\Delta}(x-\nu t))}\right)$$

Eq. (4.2.11)can be rewritten at $c_1=-c_2$ as follow:

$$p_1(\xi)=\sqrt{\frac{(b\kappa\omega-\omega-a\kappa^2)+(-a-b\upsilon)(2D-E^2+2FD)}{(\alpha+\beta)D^2}} \tag{4.2.13}$$

$$\left(\frac{E}{2(1-F)}\right)+\frac{E\sqrt{-\Delta}}{2}\tanh\frac{\sqrt{-\Delta}}{2}(x-\nu t)$$

Again Eq. (4.2.9) can be rewritten at $c_1=c_2$,as follow:

$$p_1(\xi)=\sqrt{\frac{(b\kappa\omega-\omega-a\kappa^2)+(-a-b\upsilon)(2D-E^2+2FD)}{(\alpha+\beta)D^2}} \tag{4.2.14}$$

$$\left(\frac{E}{2(1-F)}\right)+\frac{E\sqrt{-\Delta}}{2}\coth\frac{\sqrt{-\Delta}}{2}(x-\nu t)$$

The results of the chapter 3 and 4 are published in [30].

Chapter 5

Conclusions

5.1 Conclusions

In this thesis Lakshmanan-Porsezian-Daniel model is studied. By using Kerr and parabolic nonlinearity we use modified trail equation method and improved $(\frac{G'}{G})$ expansion method. By using modified trial equation method we obtain some soliton solutions, kink solutions, hyperbolic solutions and rational function solutions to LPD model. This method is very reliable and effective,it gives several new solutions. For obtaining traveling wave solutions improved $(\frac{G'}{G})$expansion method has been applied to LPD model successfully. This method is very helpful for finding the exact solutions, dark and singular soliton solution of NLPDE's. All results presented in this thesis are new and will be beneficial in field of telecommunications. In future we will solve many nonlinear fractional PDE's by this approach.

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