Some Degree-Based Topological Indices of Wheel Related Graphs

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I Muhammad Umer Butt, **CIIT/FA17-RMT-042/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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It is certified that **Muhammad Umer Butt (CIIT/FA17-RMT-042/LHR)** has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus, and the work fulfills the requirement for award of MS degree.

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DEDICATION

To

My parents and My family

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In the name of Allah, the Most Gracious and Most Merciful, all the praises to Allah for the strengths and His blessing in completing this thesis. I feel much pleasure in expressing my heartiest gratitude to my supervisor Dr. Hafiz Mohammad Afzal Siddiqui for his dynamic supervision and whose inspiring attitude made it very easy to undertake this project. A very special thanks to my respectable teacher Dr. Faisal Nadeem, without his effort I was not able to complete this task. It was only through their guidance that my efforts were fruitful. I express my sincerest gratitude to my institution and a special thanks to the Head of Department of Mathematics for providing us with very good research facilities, required for the accomplishment of our goals. My deepest appreciations are extended to my beloved Parents and Family for my success, is really the fruit of their devoted prayers and encouragement. And a special thanks for the person who motivated me to achieve my goals.

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ABSTRACT

Some Degree-Based Topological Indices of Wheel Related Graphs

A topological index is a numeric value associated with the graph. In this thesis, we study the topological indices of certain wheel related graphs, namely m-level gear graph, m-level antiweb-wheel graph, and m-level antiweb-gear graph represented by $J_{2n,m}$, $AWW_{n,m}$ and $AWJ_{2n,m}$, respectively. We compute geometric arithmetic index, atom bond connectivity index, first and second multiple Zagreb indices, first and second Zagreb indices, third Zagreb index, first and second Zagreb polynomial indices, F-index, augmented Zagreb index, harmonic index, Shigehalli and Kanabur indices and Randi$\hat{c}$ index.

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Chapter 1

Introduction and Basic Definitions

1 Introduction and Basic Definitions

In this chapter, we discuss the brief history of graphs, some basic concepts related to graphs. Also some basic notions, definitions and terminologies that will be used throughout in this thesis.

1.1 Historical Background of Graphs

The Königsberg Bridge Problem was the main paradigm shift that made the basis of the graph theory. 1736 was the time when Euler presented the solution to the Königsberg bridge problem. In Russia Königsberg was set on either sides of the Pregel river, and it had two big islands that were joined together, connected with seven bridges that makes it a part of the major portions of the city. The main conflict to be resolved was to make a gateway to have this problem solved, that one cross each bridge only once. The following map will clarify the exact figure of the map relating Königsberg in the time frame of Euler that portrays the main picture of the seven bridges of Königsberg bridge problem.

Figure 1: Map of Königsberg

First, Euler pointed out that the choice of route inside each land mass is irrelevant. The only important feature of a route is the sequence of bridges crossed. This allowed him to reformulate the problem in abstract terms (laying the foundations of graph theory), eliminating all features except the list of land masses and the bridges connecting them. In modern terms, one replaces each land mass with an abstract *vertex/node*, and each bridge with an abstract connection, an *edge*, which only serves to record which pair of vertices (land masses) is connected by that bridge. The resulting mathematical structure is called a *graph*. The first graph he formed of this problem was:

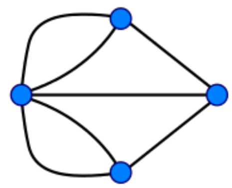

Figure 2: Graph of Königsberg

1.2 Graph

“ A **Graph** (G) consists of a non-empty finite set $V(G)$ of elements called vertices and finite set $E(G)$ of distinct unordered pairs of distinct elements $V(G)$ called edges. We call $V(G)$ is called vertex set and $E(G)$ the set of edges of G ” [15]

1.3 Size and Order of Graph

“The size of a graph G is the number of edges $n(E)$ in G and the number of vertices $n(V)$ of G is its order” [15].

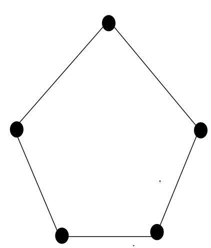

Figure 3: The size and order of graph is 5.

1.4 Simple Graph

A graph having no loops and no multiple edges known as **simple graph.**

1.5 Multiple Graph

A graph having more then one edges between two vertices is called **multiple graph.**

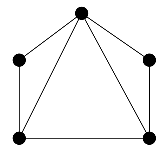

Figure 4: Simple graph

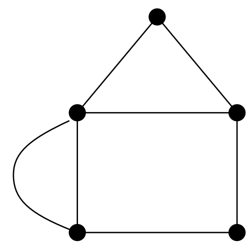

Figure 5: Multiple graph

1.6 Degree of a vertex

“If v is any vertex of a graph G than the number of edges incident with v and wtitten as $\deg(v)$” [15]

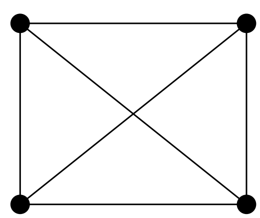

Figure 6: In this graph degree of each vertex is 3.

1.7 Regular Graph

“If the degree of all vertices is same in a graph, such graph is known as **Regular graph**” [9].

1.8 Path Graph

"A **path graph** is alternate sequence of vertices and edges is called path graph. It is denoted by p_n" [15].

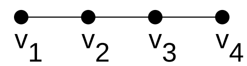

Figure 7: The path graph of P_4.

1.9 Distance in Graph

"The length of shortest path between two vertices is called the **distance in graph**" [15].

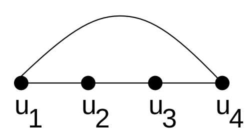

Figure 8: The distance between u_1, u_4 is 1.

1.10 Cycle Graph

A simple graph with n vertices (n$\geq$ 3) and n edges is called a cycle graph if all its edges form a cycle of length n. Number of vertices and edges in C_n are same and degree of each vertex is 2.

1.11 Join of Graph

The **join** of two graphs K_1 and K_2 is represented as $\{K_1+K_2\}$, having a nodes set $V(K_1+K_2) = \{V_1(K_1) \cup V_2(K_2)\}$ and the edge set $E(K_1+K_2) = \{E_1(K_1) \cup E_2(K_2) \cup \{ij : i \in V_1(K_1), j \in V_2(K_2)\}\}$.

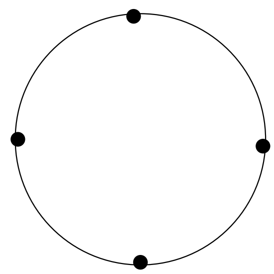

Figure 9: The cyclic graph of C_4.

1.12 Wheel Graph

A wheel graph $W_n \cong C_n + k_1$ is a join of C_n and k_1 with vertex set $V(W_n) = V(C_n) \cup V(k_1)$ and $E(W_n) = E(C_n) \cup \{x_i y : x_i \in V(C_n)\}$ and y is the fixed center vertex. It has an order $n+1$ that has the size $2n$. A wheel graph W_8 is shown in the following figure[10]:

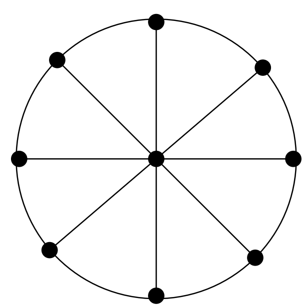

Figure 10: A wheel graph W_8.

1.12.1 Antiweb Wheel Graph

The *antiweb wheel graph* can be obtained by joining square shape cycle C_n^2 and K_1 i.e, $AWW_n \cong C_n^2 + K_1$. The order and size of antiweb wheel graph is n+1, 3n respectively. The antiweb-wheel graph AWW_8 is shown in the following figure[13]:

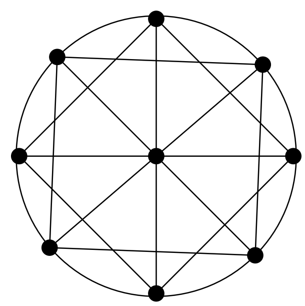

Figure 11: An antiweb-wheel graph AWW_8

1.13 Double-Wheel Graph

A **double-wheel graph** $W_{n,2} \cong 2C_n + K_1$ is a join of K_1 and 2 copies of C_n is known as double-wheel graph, and represented by $W_{n,2}$.

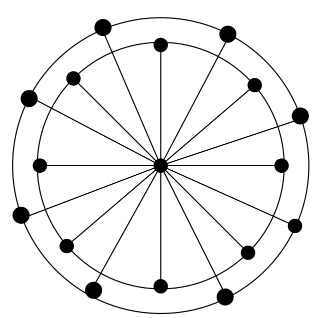

Figure 12: A double-wheel $W_{8,2}$

1.14 Jahangir/Gear Graph

A *Jahangir/Gear Graph* is defined as, $G+H$ that is the joining of two graphs, a gear graph is represented by J_{2n} that defines as $J_{2n} \cong C_{2n} + K_1$, where $C_{2n} : v_1, v_2, \cdots, v_2n, v_1$ for $n \geq 3$ is a cycle of length $2n$. J_{2n} has order $2n+1$ and size $3n$. Its can be obtained by wheel W_{2n} by alternately deleting n spokes.

A Jahangir graph J_{16} is shown in the following figure[11]:

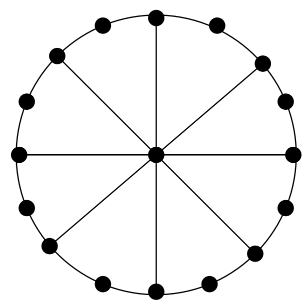

Figure 13: A Jahangir graph J_{16}

1.14.1 Antiweb Gear Graph

We obtain antiweb gear graph by replacing C_{2n} with $(C_{2n})^2$ in a gear graph i.e., J_{2n}, it is denoted by AWJ_{2n}. The size and order of antiweb gear graph is $2n$+1, $5n$ respectively. An antiweb-gear graph AWJ_{16} is shown in the following figure[14]: .

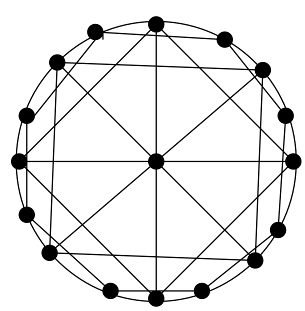

Figure 14: An antiweb-gear graph AWJ_{16}

Chapter 2

Degree Base Topological Indices

2 Topological Indices

A **topological index** is infact the numerical index that determines the topological properties of graphs. These indices characterize topology and are usually hold mathematical properties of graph invariant. The molecular descriptors are extensively used in the study of *quantitative structure-activity relationships* (*QSARs*) which give insight to the biological properties based upon chemical structures. We investigate *Rãndic* index, first and second Zagreb indices, first and second Zagreb polynomials, first and second Zagreb multiple indices, atom bond connectivity index *ABC*, geometric-arithmetic index *GA*, harmonic index, Shigehalli and Kanabur indices and augmented index for wheel related graphs. In graph theory there are three important different classes of topological indices which are as follows:

(a) First distance based topological index.

(b) Second degree based topological index.

(c) Third Spectrum based topological index.

From all these topological indices but the most important and commonly used is *degree based topological index*.

In 1947, Harold Wiener [14] introduced the ***Wiener index***. This index is also called path number by *Weiner*. The wiener index is the oldest index of topological index in molecular field. There are many other topological indices for chemical graphs.

Later on In 1972, Zagreb indices have been introduced by **Zagreb**. There are many other different names are used for Zagreb index in the graph theory, As usually we like to call the Zagreb group indices [2] , the Zagreb group parameters but the most common is Zagreb indices [1]. Some basic information of an important topological index are given by Ivan Gutm*án* and Trin*ájstiȁ* [11].

Definition 1. " The *first Zagreb index* and *second Zagreb index* for a graph G is:

$$M_1(G) = \sum_{kl \in E(G)} (d_k + d_l) \tag{2.1}$$

$$M_2(G) = \sum_{kl \in E(G)} (d_k \cdot d_l) \tag{2.2}$$

In 1975, [13] *Randić* index have been defined by *Milan Randić*.

Definition 2. The *Randić index* for a graph G is defined as:

$$R_{1/2}(G) = \sum_{kl \in E(G)} 1/\sqrt{(d_k d_l)} \tag{2.3}$$

where d_k and d_l are the degrees of the vertices.

In 1987, Fajtlowicz et al. [5] introduced the degree based *harmonic* index.

Definition 3. $Harmonic$ index for a graph G is defined as:

$$H(G) = \sum_{kl \in E(G)} \left(\frac{2}{d_k + d_l} \right) \tag{2.4}$$

where d_k and d_l are the degrees of the vertices.

A new and commonly used index established in 1998, Estrada et al. [4] which was named as atom-bond connectivity *ABC* index.

Definition 4. The atom-bond connectivity *ABC* index for a graph G is defined as:

$$ABC(G) = \sum_{kl \in E(G)} \sqrt{\frac{d_k + d_l - 2}{d_k d_l}} \tag{2.5}$$

In 2008, it played an important role for the formation of alkanes [4],[3]. Where $E(G)$ represents the set of edges and d_k, d_l represents the degrees of vertices k and l in graph G, respectively.
In 2009, Vukicevi$\tilde{c}$ and Furtul$\tilde{a}$ [12] established first *GA* index.

Definition 5. The geometric-arithmetic *GA* index for a graph G is defined as:

$$GA(G) = \sum_{kl \in E(G)} \frac{2\sqrt{d_k d_l}}{d_k + d_l} \tag{2.6}$$

In 2009 , Fath-Tabar [6] described first and the second Zagreb polynomials.

Definition 6. The first and second Zagreb polynomials for a graph G defined as:

$$Z_1(G,x) = \sum_{kl \in E(G)} x^{[d_k + d_l]} \tag{2.7}$$

$$Z_2(G,x) = \sum_{kl \in E(G)} x^{[d(k).d(l)]} \tag{2.8}$$

In 2010, Furtul*á* et al. established augmented Zagreb index [7].

Definition 7. The *augmented Zagreb index* for a graph G is defined as:

$$AZM(G) = \sum_{kl \in E(G)} \left(\frac{d_k d_l}{d_k + d_l - 2} \right)^3 \tag{2.9}$$

In 2011, after first and second Zagreb indices, the third Zagreb index which are degree based are discussed in [8].

Definition 8. The *third Zagreb index* for a graph G is defined as:

$$M_3(G) = \sum_{kl \in E(G)} (d_k + d_l)^2 \tag{2.10}$$

where d_k and d_l are the degrees of the vertices.

Ghorbani and Azimi described two new versions of Zagreb indices of a graph G in 2012 [10]. These indices represnted as the first multiple Zagreb index $PM_1(G)$ and the second multiple Zagreb index $PM_2(G)$.

Definition 9. The first and second multiple Zagreb indices for a graph G are defined as:

$$PM_1(G) = \Pi_{kl \in E(G)} (d_k + d_l) \tag{2.11}$$

$$PM_2(G) = \Pi_{kl \in E(G)} (d_k \cdot d_l) \tag{2.12}$$

Definition 10. The shigehalli and kanabur indices for a graph G defined as;

$$SK(G) = \sum_{kl \in E(G)} \left(\frac{d_k + d_l}{2} \right) \tag{2.13}$$

$$SK_1(G) = \sum_{kl \in E(G)} \left(\frac{d_k d_l}{2} \right) \tag{2.14}$$

$$SK_2(G) = \sum_{kl \in E(G)} \left(\frac{d_k d_l}{2} \right)^2 \tag{2.15}$$

where d_k and d_l are the degrees of the vertices.

Definition 11. The F-index for a graph G defined as;

$$F(G) = M_3(G) - 2M_2(G) \qquad (2.16)\text{”}$$

Chapter 3

Main Results

3 Degree Based Topological Indices for Wheel Related Graphs

This section, is devoted to analyze the topological indices for wheel related graphs, dubbed as m-level gear graph, m-level wheel graph, m-level antiweb-wheel graph and m-level antiweb-gear graph denoted by $J_{2n,m}$, $W_{n,m}$, $AWW_{n,m}$ and $AWJ_{2n,m}$ respectively. We also study degree based polnomials of said graphs.

3.0.2 Topological Indices of m-level Wheel Graph

"Now we will find the results for m**-level wheel graph** $W_{m,n}$ where $n \geq 3$ by using harmonic index, *Randić* index, atom bond connectivity index ABC, geometric arithmetic index GA, first and second Zagreb indices, third Zagreb index, Shigehalli and Kanabur indices, augmented Zagreb index, first and second multiple Zagreb indices, F-index and first and second Zagreb polynomial for $W_{m,n}$."

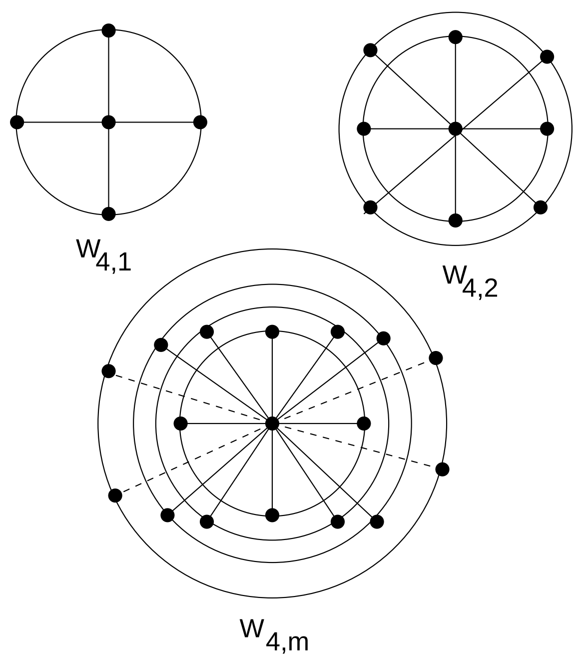

Figure 15: First level wheel graph $W_{4,1}$, second level wheel graph $W_{4,2}$, m-level wheel graph $W_{4,m}$.

The following table gives us different types of edges.

Table 1: Edge partitioned of a m-level wheel graph based on degrees of end vertices of each edge.

(d_k, d_l),where $kl \in E(G)$	No. of edges E(G)
$(mn, 3)$	mn
$(3, 3)$	mn

Theorem 3.1. *The geometric arithmetic index for $G \cong W_{n,m}$ is determined as*

$$GA(G) = \frac{2mn\sqrt{3mn}}{mn+3} + \frac{2mn\sqrt{9}}{6}$$

Proof. Using Equation 2.6 and Table 1 we obtain

$$GA(G) = \frac{2mn\sqrt{3mn}}{mn+3} + \frac{6mn}{6}$$

$$= \frac{2mn\sqrt{3mn}}{mn+3} + \frac{mn}{1}$$

After an easy calculations, we obtain

$$GA(G) = \frac{2mn\sqrt{3mn}}{mn+3} + \frac{2mn\sqrt{9}}{6}$$

Theorem 3.2. *The atom bond connectivity index for $G \cong W_{n,m}$ is determined as*

$$ABC(G) = \frac{3mn\sqrt{mn+1} + 2mn\sqrt{3mn}}{3\sqrt{3mn}}$$

Proof. Using Equation 2.5 and Table 1 we obtain

$$ABC(G) = mn\sqrt{\frac{mn+1}{3mn}} + mn\sqrt{\frac{4}{9}}$$

$$= mn\sqrt{\frac{mn+1}{3mn}} + mn\frac{2}{3}$$

After an easy calculation, we obtain

$$ABC(G) = \frac{3mn\sqrt{mn+1} + 2mn\sqrt{3mn}}{3\sqrt{3mn}}$$

Theorem 3.3. *Let $G \cong W_{n,m}$. Then the first and second multiple Zagreb indices are as follows*

$$PM_1(G) = 9m^3n^3 \text{ and } PM_2(G) = 27m^3n^3$$

Proof. Using Equation 2.11 and Table 1 we obtain

$$PM_1(G) = (1)(mn)^2 \times mn(3)^2$$

$$= (mn)^2 \times 9(mn)$$

$$PM_1(G) = 9m^3n^3$$

Also using Equation 2.12 and Table 1 we obtain

$$PM_2(G) = mn(3mn) \times (9)mn$$

$$= 3m^2n^2 \times 9mn$$

$$PM_2(G) = 27m^3n^3$$

Theorem 3.4. *Let $G \cong W_{n,m}$. Then the first and second Zagreb indices are as determined*

$$M_1(G) = m^2n^2 + 9mn \text{ and } M_2(G) = 3m^2n^2 + 9mn$$

Proof. Using Equation 2.11 and Table 1 we obtain

$$M_1(G) = mn(mn+3) + mn(3+3)$$

$$= 3mn + m^2n^2 + 6mn$$

$$= m^2n^2 + 9mn$$

Also using Equation 2.12 and Table 1 we obtain

$$M_2(G) = mn(mn \times 3) + mn(3 \times 3)$$

$$M_2(G) = 3m^2n^2 + 9mn$$

Theorem 3.5. *Let $G \cong W_{n,m}$. Then The third zagreb index for is as follows*

$$M_3(G) = m^3n^3 + 6m^2n^2 + 45mn$$

Proof. Using Equation 2.10 and Table 1 we obtain

$$M_3(G) = mn(mn+3)^2 + 2mn(3+3)^2$$

$$= mn(m^2n^2 + 6mn + 9) + mn(6)^2$$

After easy caculation we obtain

$$M_3(G) = m^3n^3 + 6m^2n^2 + 45mn$$

Theorem 3.6. *The F-index for $G \cong W_{n,m}$ is determined as*

$$F(G) = m^3n^3 + 27mn$$

Proof. Using Equation 2.16 and Table 1 we obtain

$$F(G) = m^3n^3 + 6m^2n^2 + 45mn - 2(3m^2n^2 + 9mn)$$

$$= m^3n^3 + 6m^2n^2 + 45mn - 6m^2n^2 - 18mn$$

$$F(G) = m^3n^3 + 27mn$$

Theorem 3.7. *Let $G \cong W_{n,m}$. Then the augmented Zagreb index is follows as*

$$AZI(G) = \frac{2457m^4n^4 + 2187m^3n^3 + 2187m^2n^2 + 729mn}{64m^3n^3 + 192m^2n^2 + 192mn + 64}$$

Proof. Using Equation 2.9 and Table 1 we obtain

$$AZI(G) = mn\left(\frac{3mn}{3+mn-2}\right)^3 + mn\left(\frac{9}{4}\right)^3$$

$$= \frac{27m^4n^4}{m^3n^3 + 3m^2n^2 + 3mn + 1} + \frac{729mn}{64}$$

$$= \frac{1728m^4n^4 + 729m^4n^4 + 2187m^3n^2 187m^2n^2 + 729mn}{64m^3n^3 + 192m^2n^2 + 192mn + 64}$$

After easy calculation we obtain

$$AZI(G) = \frac{2457m^4n^4 + 2187m^3n^3 + 2187m^2n^2 + 729mn}{64m^3n^3 + 192m^2n^2 + 192mn + 64}$$

Theorem 3.8. *The First Zagreb polynomial for $G \cong W_{n,m}$ is determined as*

$$ZG_1(G,x) = mn\left((x)^{mn+3} + 2x^6\right)$$

Proof. Using Equation 2.7 and Table 1 we obtain

$$ZG_1(G,x) = mn(x)^{mn+3} + 2mn(x)^6$$

$$ZG_1(G,x) = mn\left((x)^{mn+3} + 2(x)^5\right)$$

Theorem 3.9. *The second Zagreb polynomial for $G \cong W_{n,m}$ is determined as*

$$ZG_2(G,x) = mn\left((x)^{3m}) + 2(x)^6\right)$$

Proof. Using Equation 2.8 and Table 1 we obtain

$$ZG_2(G,x) = mn(x)^{3m} + 2mn(x)^9$$

$$ZG_2(G,x) = mn\left((x)^{3mn} + 2(x)^9\right)$$

Theorem 3.10. *Let $G \cong W_{n,m}$. Then Shigehalli and Kanabur indices are as determined*

$$SK(G) = \frac{m^2n^2 + 6mn}{2},\ S_1K(G) = \frac{3m^2n^2 + 9mn}{2}$$

and

$$S_2K(G) = \frac{m^3n^3 + 6m^2n^2 + 45mn}{4}$$

Proof. Using Equation 2.13 and Table 1 we obtain

$$SK(G) = mn\left(\frac{mn+3}{2}\right) + mn\left(\frac{6}{2}\right)$$

$$= \frac{3mn + m^2n^2 + 6mn}{2}$$

$$SK(G) = \frac{m^2n^2 + 6mn}{2}$$

Also using Equation 2.14 and Table 1 we obtain

$$S_1K(G) = mn\left(\frac{3mn}{2}\right) + mn\frac{9}{2}$$

$$= \frac{3m^2n^2}{2} + \frac{9mn}{2}$$

$$S_1K(G) = \frac{3m^2n^2 + 9mn}{2}$$

And also using Equation 2.15 and Table 1 we obtain

$$S_2K(G) = mn\left(\frac{mn+3}{2}\right)^2 + mn\left(\frac{6}{2}\right)^2$$

After easy calculation we obtain

$$S_2K(G) = \frac{m^3n^3 + 6m^2n^2 + 45mn}{4}$$

Theorem 3.11. *The harmonic index for $G \cong W_{n,m}$ is determined as*

$$H(G) = \frac{m^2n^2 + 9mn}{3mn+9}$$

Proof. Using Equation 2.4 and Table 1 we obtain

$$H(G) = mn\left(\frac{2}{mn+3}\right) + 2mn\left(\frac{2}{3+3}\right)$$

$$= \frac{2mn}{mn+3} + \frac{mn}{3}$$

After easy calculation we obtain

$$= \frac{6mn + m^2n^2 + 3mn}{5mn+15}$$

$$H(G) = \frac{m^2n^2 + 9mn}{3mn+9}$$

Theorem 3.12. *Let $G \cong W_{n,m}$. Then Randiĉ index is determined as*

$$R_{\frac{1}{2}}(G) = \frac{3mn + mn\sqrt{3mn}}{3\sqrt{3mn}}$$

Proof. Using Equation 2.3 and Table 1 we obtain

$$R_{\frac{1}{2}}(G) = \frac{mn}{\sqrt{3mn}} + \frac{mn}{\sqrt{9}}$$

After easy calculation we obtain

$$R_{\frac{1}{2}}(G) = \frac{3mn + mn\sqrt{3mn}}{3\sqrt{3mn}}$$

3.0.3 Topological Indices of m-level Antiweb Wheel Graph

“ We will find the results for **m-level antiweb Gear graph** $AWW_{2n,m}$ by using harmonic index, *Randić* index, atom bond connectivity index ABC, geometric arithmetic index GA, first and second Zagreb indices, third Zagreb index, Shigehalli and Kanabur indices, augmented Zagreb index, F-index, first and second multiple Zagreb indices, first and second Zagreb polynomials for AWW_n, m.”

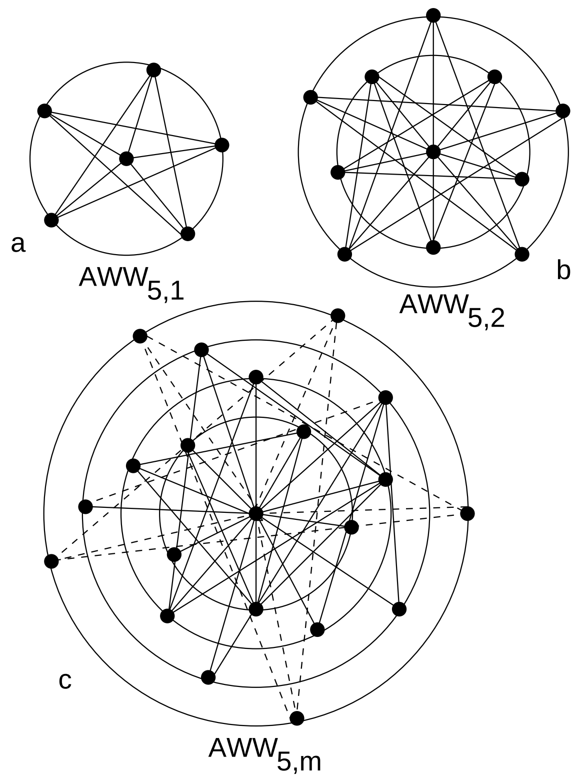

Figure 16: First level antiweb wheel graph $AWW_{5,1}$, second level antiwheel graph $AWW_{5,2}$, m-level antiwheel graph $AWW_{5,m}$

The following table gives us different types of edges.

Table 2: Edge partitioned of a m-level antiweb wheel graph based on degrees of end vertices of each edge.

(d_k, d_l),where $kl \in E(G)$	No. of edges E(G)
$(mn, 5)$	mn
$(5, 5)$	$2mn$

Theorem 3.13. *Let $G \cong AWW_{n,m}$. Then geometric arithmetic index is as determined*

$$GA(G) = \frac{2mn\sqrt{5mn} + 2m^2n^2 + 10mn}{mn+5}$$

Proof. Using Equation 2.6 and Table 2 we obtain

$$GA(G) = \frac{2mn\sqrt{5mn}}{mn+5} + \frac{4mn\sqrt{25}}{10}$$

$$= \frac{2mn\sqrt{5mn}}{mn+5} + \frac{20mn}{10}$$

After an easy calculations, we obtain

$$GA(G) = \frac{2mn\sqrt{5mn} + 2m^2n^2 + 10mn}{mn+5}$$

Theorem 3.14. *The atom bond connectivity index for $G \cong AWW_{n,m}$ is determined as*

$$ABC(G) = \frac{5mn\sqrt{mn+3} + 4mn\sqrt{10mn}}{5\sqrt{5mn}}$$

Proof. Using Equation 2.5 and Table 2 we obtain

$$ABC(G) = mn\sqrt{\frac{mn+3}{5mn}} + 2mn\sqrt{\frac{8}{25}}$$

$$= mn\frac{\sqrt{mn+3}}{\sqrt{5mn}} + 4mn\frac{\sqrt{2}}{5}$$

After an easy calculation, we obtain

$$ABC(G) = \frac{5mn\sqrt{mn+3} + 4mn\sqrt{10mn}}{5\sqrt{5mn}}$$

Theorem 3.15. *The first and second multiple Zagreb indices for $G \cong AWW_{n,m}$ is determined as*

$$PM_1(G) = 25m^3n^3 \text{ and } PM_2(G) = 250m^3n^3$$

Proof. Using Equation 2.11 and Table 2 we obtain

$$PM_1(G) = (1)(mn)^2 \times mn(5)^2$$

$$= m^2n^2 \times 25mn$$

$$PM_1(G) = 25m^3n^3$$

Also using Equation 2.12 and Table 2 we obtain

$$PM_2(G) = mn(5mn) \times 2mn(25)$$

$$= 5m^2n^2 \times 50mn$$

$$PM_2(G) = 250m^3n^3$$

Theorem 3.16. *The first and second Zagreb indices for $G \cong AWW_{n,m}$ is determined as*

$$M_1(G) = m^2n^2 + 25mn \; M_2(G) = 5m^2n^2 + 50mn$$

Proof. Using Equation 2.1 and Table 2 we obtain

$$M_1(G) = mn(mn+5) + 2mn(5+5)$$

$$= 5mn + m^2n^2 + 20mn$$

$$M_1(G) = m^2n^2 + 25mn$$

Also using Equation 2.2 and Table 2 we obtain

$$M_2(G) = mn(mn \times 5) + mn(5 \times 5)$$

$$M_2(G) = 5m^2n^2 + 50mn$$

Theorem 3.17. *Let $G \cong AWW_{n,m}$. Then the third zagreb Index is as follows*

$$M_3(G) = m^3n^3 + 10m^2n^2 + 225mn$$

Proof. Using Equation 2.10 and Table 2 we obtain

$$M_3(G) = mn(mn+5)^2 + 2mn(5+5)^2$$

$$= mn(m^2n^2 + 10mn + 25) + 2mn(10)^2$$

$$= m^3n^3 + 10m^2n^2 + 200mn + 25mn$$

$$M_3(G) = m^3n^3 + 10m^2n^2 + 225mn$$

Theorem 3.18. *The F-index for $G \cong AWW_{n,m}$ is determined as*

$$F(G) = m^3n^3 + 125mn$$

Proof. Using Equation 2.16 and Table 2 we obtain

$$F(G) = m^3n^3 + 10m^2n^2 + 225mn - 2(5m^2n^2 + 50mn)$$

$$= m^3n^3 + 10m^2n^2 + 225mn - 10m^2n^2 - 100mn$$

$$F(G) = m^3n^3 + 125mn$$

Theorem 3.19. *Let $G \cong AWW_{n,m}$. Then augmented Zagreb index is follows as*

$$AZI(G) = \frac{9250m^4n^4 + 21825m^3n^3 + 843750m^2n^2 + 843750mn}{512(mn+3)^3}$$

Proof. Using Equation 2.9 and Table 2 we obtain

$$AZI(G) = mn\left(\frac{5mn}{mn+3}\right)^3 + 2mn\left(\frac{25}{8}\right)^3$$

$$= \frac{mn(5mn)^3}{(mn+3)^3} + \frac{2mn(25)^3}{8^3}$$

After easy calculation we obtain

$$AZI(G) = \frac{9250m^4n^4 + 21825m^3n^3 + 843750m^2n^2 + 843750mn}{512(mn+3)^3}$$

Theorem 3.20. *The first Zagreb polynomial for $G \cong AWW_{n,m}$ is determined as*

$$ZG_1(G,x) = mn\left(x^{mn+5} + 2x^{10}\right)$$

Proof. Using Equation 2.7 and Table 2 we obtain

$$ZG_1(G,x) = mn(x)^{mn+5} + 2mn(x)^{10}$$

$$ZG_1(G,x) = mn\left((x)^{mn+5} + 2(x)^{10}\right)$$

Theorem 3.21. *The second Zagreb polynomial for $G \cong AWW_{n,m}$ is determined as*

$$ZG_2(G,x) = mn\left((x)^{5mn} + 2(x)^{25}\right)$$

Proof. Using Equation 2.8 and Table 2 we obtain

$$ZG_2(G,x) = mn(x)^{5mn} + 2mn(x)^{25}$$

$$ZG_2(G,x) = mn\left((x)^{5mn} + 2(x)^{25}\right)$$

Theorem 3.22. *The Shigehalli and Kanabur indices for G=$AWW_{n,m}$ is determined as*

$$SK(G) = \frac{m^2n^2 + 25mn}{2} \quad , \quad S_1K(G) = \frac{5m^2n^2 + 50mn}{2}$$

and

$$S_2K(G) = \frac{m^3n^3 + 10m^2n^2 + 225mn}{4}$$

Proof. Using Equation 2.13 and Table 2 we obtain

$$SK(G) = mn\left(\frac{mn+5}{2}\right) + 2mn\left(\frac{10}{2}\right)$$

$$= \frac{mn + m^2n^2 + 5mn}{2} + \frac{10mn}{1}$$

$$= \frac{m^2n^2 + 5mn + 20mn}{2}$$

$$SK(G) = \frac{m^2n^2 + 6mn}{2}$$

Also using Equation 2.14 and Table 2 we obtain

$$S_1K(G) = mn\left(\frac{5mn}{2}\right) + 2mn\left(\frac{25}{2}\right)$$

$$= \frac{5m^2n^2}{2} + \frac{25mn}{1}$$

$$S_1K(G) = \frac{5m^2n^2 + 50mn}{2}$$

And also using Equation 2.15 and Table 2 we obtain

$$S_2K(G) = mn\left(\frac{mn+5}{2}\right)^2 + 2mn\left(\frac{10}{2}\right)^2$$

After easy calculation we obtain

$$S_2K(G) = \frac{m^3n^3 + 10m^2n^2 + 225mn}{4}$$

Theorem 3.23. *The harmonic index for $G \cong AWW_{n,m}$ is determined as*

$$H(G) = \frac{2m^2n^2 + 20mn}{5mn + 25}$$

Proof. Using Equation 2.4 and Table 2 we obtain

$$H(G) = mn\left(\frac{2}{mn+5}\right) + 2mn\left(\frac{2}{5+5}\right)$$

$$= \frac{2mn}{mn+5} + \frac{2mn}{5}$$

After easy calculation we obtain

$$H(G) = \frac{2m^2n^2 + 20mn}{5mn + 25}$$

Theorem 3.24. *The Randiĉ index for G=$AWW_{n,m}$ is determined as*

$$R_{\frac{1}{2}}(G) = \frac{5mn + 2mn\sqrt{5mn}}{5\sqrt{5mn}}$$

Proof. Using Equation 2.3 and Table 2 we obtain

$$R_{\frac{1}{2}}(G) = \frac{mn}{\sqrt{5mn}} + \frac{2mn}{\sqrt{25}}$$

$$= \frac{mn}{\sqrt{5mn}} + \frac{2mn}{5}$$

$$R_{\frac{1}{2}}(G) = \frac{5mn + 2mn\sqrt{5mn}}{5\sqrt{5mn}}$$

3.0.4 Topological Indices of m-level Gear Graph

"We find the results for m**-level gear graph** $J_{2n,m}$ by using definitions of geometric arithmetic index GA, atom bond connectivity index ABC, first and second multiple Zagreb indices, third Zagreb index, harmonic index, *Randić* index, first and second Zagreb indices, F-index, Shigehalli and Kanabur indices , augmented Zagreb index, first and second Zagreb polynomials for $J_2 n, m$."

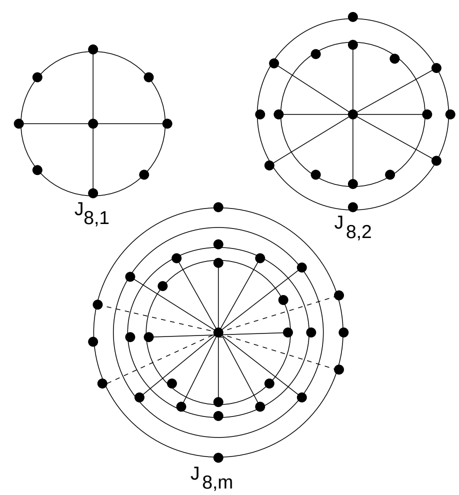

Figure 17: First level gear graph $J_{8,1}$, second level gear graph $J_{8,2}$, m-level gear graph $J_{8,m}$

The following table gives us different types of edges.

Table 3: Edge partitioned of a m-level gear graph based on degrees of end vertices of each edge.

(d_k, d_l),where $kl \in E(G)$	No. of edges E(G)
$(mn, 3)$	mn
$(2, 3)$	$2mn$

Theorem 3.25. *The geometric arithmetic index for $G \cong J_{2n,m}$ is determined as*

$$GA(G) = \frac{10mn\sqrt{3mn} + 4m^2n^2\sqrt{6} + 12mn\sqrt{6}}{5mn + 15}$$

Proof. Using Equation 2.6 and Table 3 we obtain

$$GA(G) = \frac{2mn\sqrt{3mn}}{mn+3} + \frac{4mn\sqrt{6}}{5}$$

$$= \frac{10mn\sqrt{3mn} + (mn+3)4mn\sqrt{6}}{5mn+15}$$

After an easy calculation, we obtain

$$GA(G) = \frac{10mn\sqrt{3mn} + 4m^2n^2\sqrt{6} + 12mn\sqrt{6}}{5mn + 15}$$

Theorem 3.26. *The atom bond connectivity Index for $G \cong J_{2n,m}$ is determined as*

$$ABC(G) = \frac{mn\sqrt{2mn+2} + 2mn\sqrt{3mn}}{\sqrt{6mn}}$$

Proof. Using Equation 2.5 and Table 3 we obtain

$$ABC(G) = mn\sqrt{\frac{mn+3-2}{3mn}} + 2mn\sqrt{\frac{3+2-2}{6}}$$

$$= mn\sqrt{\frac{mn+1}{3mn}} + 2mn\sqrt{\frac{1}{2}}$$

After an easy calculation, we obtain

$$ABC(G) = \frac{mn\sqrt{2mn+2}+2mn\sqrt{3mn}}{\sqrt{6mn}}$$

Theorem 3.27. *Let $G \cong J_{2n,m}$. Then first and second multiple Zagreb Indices are as follows*

$$PM_1(G) = 36m^4n^4 \text{ and } PM_2(G) = 36m^3n^3$$

Proof. Using Equation 2.11 and Table 3 we obtain

$$PM_1(G) = (1)(mn)^2 \times (2)^2(mn) \times (3)^2(mn)$$

$$= (mn)^2 \times 4(mn) \times 9(mn)$$

$$PM_1(G) = 36m^4n^4$$

Also using Equation 2.12 and Table 3 we obtain

$$PM_2(G) = mn(3mn) \times (6)2mn$$

$$= 3m^2n^2 \times 12mn$$

$$PM_2(G) = 36m^3n^3$$

Theorem 3.28. *The first and second Zagreb indices for $G \cong J_{2n,m}$ is determined as* $M_1(G) = m^2n^2 + 13mn$ *and* $M_2(G) = 3m^2n^2 + 6mn$

Proof. Using Equation 2.1 and Table 3 we obtain

$$M_1(G) = mn(mn+3) + 2mn(2+3)$$

$$= 3mn + m^2n^2 + 10mn$$

$$M_1(G) = m^2n^2 + 13mn$$

Also using equation 2.2 and table 3 we obtain

$$M_2(G) = mn(mn \times 3) + 2mn(2 \times 3)$$

$$M_2(G) = 3m^2n^2 + 6mn$$

Theorem 3.29. *The third Zagreb Index for $G \cong J_{2n,m}$ is determined as*

$$M_3(G) = m^3n^3 + 6m^2n^2 + 14mn$$

Proof. Using Equation 2.10 and Table 3 we obtain

$$M_3(G) = mn(mn+3)^2 + 2mn(2+3)^2$$

$$= mn(m^2n^2 + 6mn + 9) + 2mn(5)^2$$

After easy caculation we obtain

$$M_3(G) = m^3n^3 + 6m^2n^2 + 14mn$$

Theorem 3.30. *Let $G \cong J_{2n,m}$. Then F-index is determined as*

$$F(G) = m^3n^3 + 26mn$$

Proof. Using Equation 2.16 and Table 3 we obtain

$$F(G) = m^3n^3 + 6m^2n^2 + 14mn - 2(3m^2n^2 + 6mn)$$

$$= m^3n^3 + 6m^2n^2 + 14mn - 6m^2n^2 + 12mn$$

$$F(G) = m^3n^3 + 26mn$$

Theorem 3.31. *Let $G \cong J_{2n,m}$. Then augmented Zagreb index is determined as*

$$AZI(G) = \frac{44m^4n^4 + 48m^3n^3 + 64mn}{m^3n^3 + 3m^2n^2 + 3mn + 1}$$

Proof. Using Equation 2.9 and Table 3 we obtain

$$AZI(G) = mn\left(\frac{3mn}{3+mn-2}\right)^3 + 2mn\left(\frac{6}{3}\right)^3$$

$$= mn\left(\frac{3mn}{mn+1}\right)^3 + 2mn(2)^3$$

$$= mn\left(\frac{3mn}{mn+1}\right)^3 + 16mn$$

After easy calculation we obtain

$$AZI(G) = \frac{44m^4n^4 + 48m^3n^3 + 64mn}{m^3n^3 + 3m^2n^2 + 3mn + 1}$$

Theorem 3.32. *The first Zagreb polynomial for $G \cong J_{2n,m}$ is determined as*

$$ZG_1(G,x) = mn\left(x^{mn+3} + 2x^5\right)$$

Proof. Using Equation 2.7 and Table 3 we obtain

$$ZG_1(G,x) = mn(x)^{mn+3} + 2mn(x)^5$$

$$ZG_1(G,x) = mn\left(x^{mn+3} + 2(x)^5\right)$$

Theorem 3.33. *The second Zagreb polynomial for $G \cong J_{2n,m}$ is determined as*

$$ZG_2(G,x) = mn\left((x)^{3mn} + 2(x)^6\right)$$

Also using Equation 2.8 and Table 3 we obtain

$$ZG_2(G,x) = mn(x)^{3mn} + 2mn(x)^6$$

$$ZG_2(G,x) = mn\left((x)^{3mn} + 2(x)^6\right)$$

Theorem 3.34. *The Shigehalli and Kanabur indices for $G \cong J_{2n,m}$ is determined as*

$$SK(G) = \frac{m^2n^2 + 13mn}{2}, S_1K(G) = \frac{3m^2n^2 + 6mn}{2}$$

and

$$S_2K(G) = \frac{m^3n^3 + 6m^2n^2 + 59mn}{4}$$

Proof. Using Equation 2.13 and Table 3 we obtain

$$SK(G) = mn\left(\frac{mn+3}{2}\right) + 2mn\left(\frac{3+2}{2}\right)$$

$$= \frac{3mn + m^2n^2}{2} + \frac{10mn}{2}$$

$$SK(G) = \frac{m^2n^2 + 13mn}{2}$$

Also, using Equation 2.14and Table 3 we obtain

$$S_1K(G) = mn\left(\frac{3mn}{2}\right) + 2mn(6mn)$$

$$S_1K(G) = \frac{3m^2n^2 + 6mn}{2}$$

And also using Equation 2.15 and Table 3 we obtain

$$S_2K(G) = mn\left(\frac{mn+3}{2}\right)^2 + 2mn\left(\frac{5}{2}\right)^2$$

After easy calculation we attain

$$S_2K(G) = \frac{m^3n^3 + 6m^2n^2 + 59mn}{4}$$

Theorem 3.35. *The harmonic index for $G \cong J_{2n,m}$ is determined as*

$$H(G) = \frac{4m^2n^2 + 22mn}{5mn + 15}$$

Proof. Using Equation 2.4 and Table 3 we obtain

$$H(G) = mn\left(\frac{2}{mn+3}\right) + 2mn\left(\frac{2}{2+3}\right)$$

$$= \frac{2mn}{mn+3} + \frac{4mn}{5}$$

After easy calculation we obtain

$$= \frac{10mn + 4m^2n^2 + 12mn}{5mn + 15}$$

$$H(G) = \frac{4m^2n^2 + 22mn}{5mn + 15}$$

Theorem 3.36. *The Randiĉ index for $G \cong J_{2n,m}$ is determined as*

$$R_{\frac{1}{2}}(G) = \frac{mn\sqrt{6} + 2mn\sqrt{3mn}}{\sqrt{18mn}}$$

Proof. Using Equation 2.3 and Table 3 we get

$$R_{\frac{1}{2}}(G) = \frac{mn}{\sqrt{3mn}} + \frac{2mn}{\sqrt{6}}$$

After easy calculation we obtain

$$R_{\frac{1}{2}}(G) = \frac{mn\sqrt{6} + 2mn\sqrt{3mn}}{\sqrt{18mn}}$$

3.0.5 Topological Indices of *m*-level Antiweb Gear Graph

“We will find the results for **m-level antiweb gear graph** $AWJ_{2n,m}$ by using harmonic index, *Randić* index, atom bond connectivity index ABC, geometric arithmetic index GA, first and second Zagreb indices, third Zagreb index, Shigehalli and Kanabur indices, augmented

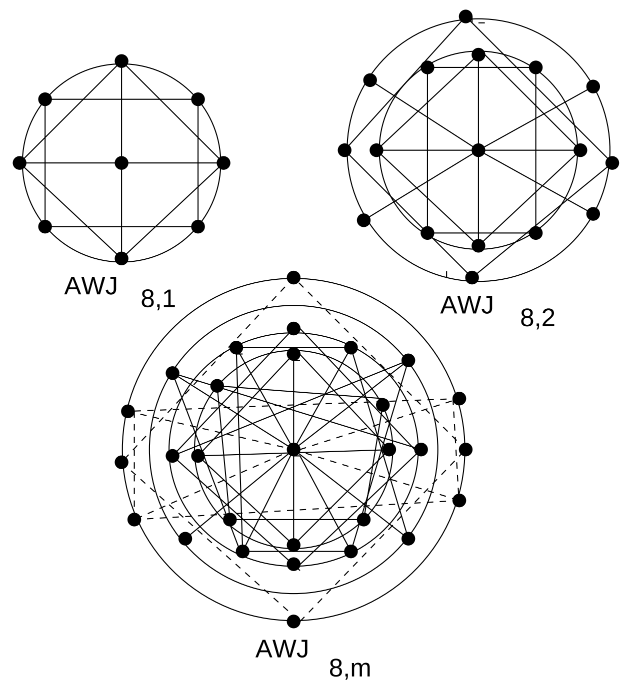

Figure 18: First level antiweb gear graph $AWJ_{8,1}$, second level antiweb gear graph $AWJ_{8,2}$, m-level antiweb gear graph $AWJ_{8,m}$

The following table gives us different types of edges.

Table 4: Edge partitioned of a m-level antiweb gear graph based on degrees of end vertices of each edge.

(d_k,d_l),where $kl \in E(G)$	No. of edges E(G)
$(mn,5)$	mn
$(5,5)$	mn
$(4,4)$	mn
$(4,5)$	$2mn$

Zagreb index, F-index, first and second multiple Zagreb indices, first and second Zagreb polynomials for AWJ_2n,m."

Theorem 3.37. *The geometric arithmetic index for $G \cong AWJ_{2n,m}$ is determined as*

$$AWJ_{2n,m}(G) = \frac{18mn\sqrt{5mn} + 18m^2n^2 + 90mn + 4m^2n^2\sqrt{20} + 10\sqrt{20}}{9mn+5}$$

Proof. Using Equation 2.9 and Table 4 we obtain

$$= \frac{2mn\sqrt{5mn}}{5+mn} + \frac{2mn\sqrt{5\times5}}{5+5} + \frac{2mn\sqrt{4\times4}}{4+4} + \frac{4mn\sqrt{4\times5}}{4+5}$$

$$= \frac{2mn\sqrt{5mn}}{mn+5} + \frac{2mn\sqrt{25}}{10} + \frac{2mn\sqrt{16}}{8} + \frac{4mn\sqrt{20}}{9}$$

$$AWJ_{2n,m}(G) = \frac{2mn\sqrt{5mn}}{mn+5} + \frac{10mn}{10} + \frac{8mn}{8} + \frac{4mn\sqrt{20}}{9}$$

After easy caculation we obtain

$$AWJ_{2n,m}(G) = \frac{18mn\sqrt{5mn} + 18m^2n^2 + 90mn + 4m^2n^2\sqrt{20} + 10\sqrt{20}}{9mn+5}$$

Theorem 3.38. *The atom bond connectivity index for $G \cong AWJ_{2n,m}$ is determined as*

$$ABC(G) = \frac{20\sqrt{20mn+60} + 40\sqrt{8mn} + 50\sqrt{6mn} - 20\sqrt{35mn}}{200\sqrt{mn}}$$

Proof. Using Equation 2.5 and Table 4 we obtain

$$ABC(G) = \sqrt{\frac{5+mn-2}{5mn}} + \sqrt{\frac{5+5-2}{5\times5}} + \sqrt{\frac{4+4-2}{4\times4}} + \sqrt{\frac{4+5-2}{4\times5}}$$

$$= \sqrt{\frac{5+mn-2}{5mn}} + \sqrt{\frac{5+5-2}{25}} + \sqrt{\frac{4+4-2}{16}} + \sqrt{\frac{4+5-2}{20}}$$

After an easy calculation, we obtain

$$ABC(G) = \frac{20\sqrt{20mn+60} + 40\sqrt{8mn} + 50\sqrt{6mn} - 20\sqrt{35mn}}{200\sqrt{mn}}$$

Theorem 3.39. *The first and second multiple Zagreb indices for $G \cong AWJ_{2n,m}$ is determined as*

$$PM_1(G) = 400m^4n^4 \text{ and } PM_2(G) = 80000m^5n^5$$

Proof. Using Equation 2.11 and Table 4 we obtain

$$PM_1(G) = (1)(mn)^2 \times (4)^2(mn) \times (5)^2(mn)$$

$$= m^2n^2 \times 16mn \times 25mn$$

$$PM_1(G) = 400m^4n^4$$

Also using Equation 2.12 and Table 4 we obtain

$$PM_2(G) = mn(5mn) \times mn(25) \times 16mn \times 20(2mn)$$

$$= 5m^2n^2 \times 25mn \times 16mn \times 40mn$$

$$PM_2(G) = 80000m^3n^3$$

Theorem 3.40. *The first and second Zagreb indices for $G \cong AWJ_{2n,m}$ is determined as*

$$M_1(G) = m^2n^2 + 41mn \text{ and } M_2(G) = 5m^2n^2 + 81mn$$

Proof. Using Equation 2.1 and Table 4 we obtain

$$M_1(G) = mn(mn+5) + mn(5+5) + mn(4+4) + 2mn(5+4)$$

$$= 5mn + m^2n^2 + 10mn + 8mn + 18mn$$

$$M_1(G) = m^2n^2 + 41mn$$

Also using Equation 2.2 and Table 4 we obtain

$$M_2(G) = mn(5mn) + mn(25) + m(16)n + 2mn(40)$$

$$= 5m^2n^2 + 25mn + 16mn + 40mn$$

$$M_2(G) = 5m^2n^2 + 81mn$$

Theorem 3.41. *The third zagreb Index for $G \cong AWJ_{2n,m}$ is determined as*

$$M_3(G) = m^2n^2 + 41mn$$

Proof. Using Equation 2.10 and Table 4 we obtain

$$M_3(G) = mn(5+mn) + mn(5+5) + mn(4+4) + 2mn(4+5)$$

$$= 5mn + m^2n^2 + 10mn + 8mn + 18mn$$

$$M_3(G) = m^2n^2 + 41mn$$

Theorem 3.42. *The F-index for $G \cong AWJ_{2n,m}$ is determined as*

$$F(G) = m^2n^2 - 131mn$$

Proof. Using Equation 2.16 and Table 4 we obtain

$$F(G) = m^2n^2 + 41mn - 2(5m^2n^2 + 81mn)$$

$$= m^2n^2 + 41mn - 10m^2n^2 - 162m^2n^2$$

$$F(G) = m^2n^2 - 131mn$$

Theorem 3.43. *The augmented Zagreb index for for $G \cong AWJ_{2n,m}$ is determined as*

$$AZI(G) = \frac{623917m^3n^3 + 2777677m^2n^2 + 45285031mn + 27131591}{592704m^3n^3 + 277677m^2n^2 + 16003008mn + 2133734}$$

Proof. Using Equation 2.9 and Table 4 we obtain

$$AZI(G) = mn\left(\frac{mn+5}{5+mn-2}\right)^3 + mn\left(\frac{5+5}{5+5-2}\right)^3 + mn\left(\frac{4+4}{4+4-2}\right)^3 + 2mn\left(\frac{5+4}{5+4-2}\right)^3$$

$$AGI(G) = mn\left(\frac{mn+5}{mn+3}\right)^3 + mn\left(\frac{5}{4}\right)^3 + mn\left(\frac{4}{3}\right)^3 + 2mn\left(\frac{9}{7}\right)^3$$

After easy calculation we obtain

$$AZI(G) = \frac{623917m^3n^3 + 2777677m^2n^2 + 45285031mn + 27131591}{592704m^3n^3 + 277677m^2n^2 + 16003008mn + 2133734}$$

Theorem 3.44. *The first Zagreb polynomial for for $G \cong AWJ_{2n,m}$ is determined as*

$$ZG_1(G,x) = mn\left(x^{mn+5} + x^{10} + x^8 + 2(x)^9\right)$$

Proof. Using Equation 2.7 and Table 4 we obtain

$$ZG_1(G,x) = mn(x)^{mn+5} + mn(x)^{10} + mn(x)^8 + 2mn(x)^9$$

$$ZG_1(G,x) = mn\left(x^{mn+5} + x^{10} + x^8 + 2x^9\right)$$

Theorem 3.45. *The second Zagreb polynomial for $G \cong AWJ_{2n,m}$ is determined as*

$$ZG_2(G,x) = mn\left((x)^{5mn} + (x)^{25} + (x)^{16} + 2(x)^{20}\right)$$

Proof. Using Equation 2.8 and Table 4 we obtain

$$ZG_2(G,x) = mn(x)^{5mn} + mn(x)^{25} + mn(x)^{16} + 2mn(x)^{20}$$

$$ZG_2(G,x) = mn\left((x)^{5mn} + mn(x)^{25} + mn(x)^{16} + 2(x)^{20}\right)$$

Theorem 3.46. *The Shigehalli and Kanabur indices for for $G \cong AWJ_{2n,m}$ is determined as*

$$SK(G) = \frac{m^3n^3 + 41mn}{2}, \quad S_1K(G) = \frac{5m^2n^2 + 81mn}{2}$$

and

$$S_2K(G) = \frac{m^3n^3 + 10m^2n^2 + 228mn}{4}$$

Proof. Using Equation 2.11 and Table 4 we obtain

$$SK(G) = mn\left(\frac{mn+5}{2}\right) + mn\left(\frac{5+5}{2}\right) + mn\left(\frac{4+4}{2}\right) + 2mn\left(\frac{9}{2}\right)$$

$$= \frac{5mn+m^2n^2}{2} + 5mn + 4mn + 9mn$$

After easy calculation we obtain

$$SK(G) = \frac{m^3n^3 + 41mn}{2}$$

Also using Equation 2.12 and Table 4 we obtain

$$S_1K(G) = mn\left(\frac{5mn}{2}\right) + mn\left(\frac{25}{2}\right) + mn\left(\frac{16}{2}\right) + 2mn\left(\frac{20}{2}\right)$$

$$= \frac{5m^2n^2}{2} + \frac{25mn}{2} + \frac{28mn}{1}$$

After easy calculation we obtain

$$S_1K(G) = \frac{5m^2n^2 + 81mn}{2}$$

And also using Equation 2.14 and Table 4 we obtain

$$= mn\left(\frac{mn+5}{2}\right)^2 + mn\left(\frac{5+5}{2}\right)^2 + mn\left(\frac{4+4}{2}\right)^2 + 2mn\left(\frac{9}{2}\right)^2$$

After easy calculation we obtain

$$S_2K(G) = \frac{m^3n^3 + 10m^2n^2 + 228mn}{4}$$

Theorem 3.47. *The harmonic index for for $G \cong AWJ_{2n,m}$ is determined as*

$$H(G) = \frac{16m^2n^2 + 1165mn}{180mn + 900}$$

Proof. Using Equation 2.4 and Table 4 we obtain

$$H(G) = mn\left(\frac{2}{mn+5}\right) + mn\left(\frac{2}{5+5}\right) + mn\left(\frac{2}{4+4}\right) + 2mn\left(\frac{2}{4+5}\right)$$

$$= \frac{2mn}{mn+5} + \frac{2mn}{10} + \frac{2mn}{8}\frac{4mn}{9}$$

After easy calculation we obtain

$$H(G) = \frac{16m^2n^2 + 1165mn}{180mn + 900}$$

Theorem 3.48. *The Randiĉ indicex for for $G \cong AWJ_{2n,m}$ is determined as*

$$R_{\frac{1}{2}}(G) = \frac{20mn\sqrt{20} + 90mn\sqrt{mn} + 40mn\sqrt{5mn}}{200\sqrt{mn}}$$

Proof. Using Equation 2.3 and Table 4 we obtain

$$R_{\frac{1}{2}}(G) = \frac{mn}{\sqrt{5mn}} + \frac{mn}{\sqrt{25}} + \frac{mn}{\sqrt{25}} + \frac{2mn}{\sqrt{20}}$$

After easy calculation we obtain

$$R_{\frac{1}{2}}(G) = \frac{20mn\sqrt{20} + 90mn\sqrt{mn} + 40mn\sqrt{5mn}}{200\sqrt{mn}}$$

Chapter 5

4 Conclusion

We investigated the topological indices for wheel related graphs, namely m-level gear graph, m-level wheel graph, m-level antiweb-wheel graph, m-level antiweb-gear denoted by $J_{2n,m}$, $W_{n,m}$, $AWW_{n,m}$ and $AWJ_{2n,m}$, respectively. We have find the results by using harmonic index, *Randić* index$R(G)$, atom bond connectivity index ABC, geometric arithmetic index GA, first and second Zagreb indices, third Zagreb index, Shigehalli and Kanabur indices, augmented Zagreb index, first and second multiple Zagreb indices, first and second Zagreb polynomials for wheel related graphs.

Chapter 6

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