Metric Dimension of Different Chemical Structures

By

AQSA FAROOQ

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A Post Graduate Thesis submitted to the Department of Mathematics as partial fulfillment of the requirement for the award of Degree of M.S in Mathematics.

AQSA FAROOQ	CIIT/FA17-RMT-029/LHR

Supervisor

Dr. Muhammad Hussain

Associate Professor, Department Mathematics

Lahore Campus.

COMSATS University Islamabad, Lahore Campus

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Final Approval

This thesis titled

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By

AQSA FAROOQ

CIIT/FA17-RMT-029/LHR

Has been approved

For the COMSATS University Islamabad, Lahore Campus

External Examiner: ____

Supervisor: ____

Dr. Muhammad Hussain

Department of Mathematics /CUI, Lahore Campus

HoD: ____

Dr. Sarfraz Ahmad

Department of Mathematics /CUI, Lahore Campus

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I AQSA FAROOQ (CIIT/FA17-RMT-029/LHR) hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis. I shall be liable to punishable action under the plagiarism rules of the HEC.

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It is certified that AQSA FAROOQ (CIIT/FA17-RMT-029/LHR) has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus and the work fulfills the requirement for award of MS degree.

Date: ______________________

Supervisor:

____

Dr. Muhammad Hussain
Associate Professor

Head of Department:

____

Dr. Sarfraz Ahmad
Associate Professor
Department of Mathematics

DEDICATION

To
My Parents

ACKNOWLEDGEMENTS

All praises to ALLAH Almighty who created the universe and knows whatever is in it hidden or evident, who bestowed upon me the intellectual ability and wisdom to search for it's secrets and whose blessing flourished my thoughts and thrived my ambitions and enable me to contribute sincerely to the scared wealth of knowledge. Peace and blessing of Allah be upon the Holy Prophet (PBUH) who exhort his followers to seek for knowledge from cradle to grave.

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COMSATS University Islamabad, Lahore Campus.

AQSA FAROOQ

CIIT/FA17-RMT029/LHR

ABSTRACT

Metric Dimension of Different Chemical Structures

Let $G = (V, E)$ be a connected graph and distance between any two vertices m and n in G is $(m - n)$ geodesic and is denoted by $d(m, n)$. A set of vertices S resolves a graph G if each vertex is uniquely determined by its vector of distances to the vertices in S. A metric dimension of G is the minimum cardinality of a resolving set of G. In this thesis line graph of honeycomb network , azteic diamond,polycyclic aromatic hydrocarbon,triangualr benzenoid,honeycomb cup network has been derived and than calculate the metric dimension on line graph of Honeycomb network, azteic diamond ,polycyclic aromatic hydrocarbon, triangualr benzenoid and honeycomb cup network .

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Chapter 1

Introduction

The graph theory was originated from the problem of Konigsberg bridge [31]. Konigsberg town was located on both sides of the River Pregel and based on two large islands connected by 7 bridges to each other or to the town. A question arose in the mind of Konigsberg residents, whether how they could travel around the city, crossing each of the seven bridges once and once. In 1736, Leonhard Euler$(1707-1783)$ was attracted by Konigsberg Bridge problem and he found that it was not possible to do so, however, how much a man was walking around the earth or exactly where bridges were. Thus, the concept of the Eulerian graph was brought about. In order to solve this problem Euler introduced the new mathematical branch, the graphic theory [31].

A.F. Mobius,in 1940, introduced the idea of a bipartite graph and a complete k_n graph. The concept of the tree was introduced by Gustav Kirchov, In 1845, to calculate electricity currents using the concept of a tree. In 1852 the well-known four-color problem was introduced by Thomas Gutherie. In the year 1856, Kirkman. P and R. Hamilton studied polyhydral cycles and discover the Hamilton graph concept.

Regardless the four color problem had been discover but it hadnt been solved at that time. After about a century in 1976, Kenneth Appel and Wolfgang Haken [22] resolved this problem. It was the first major computer theorem to be proven. In 1878, Sylvester found the term Graph.This was a brief history about the evolution of graph theory.

The ideas and notions of graph theory are already have been used in various fields of science. In general, these are also used in day-to-day life without noticing the graphic concepts. For example, if we have to reach to a certain place which connects with our point of departure by different ways, then we will follow the shortest road to get to the desired location soon.In the graph theory point of view, however, the two places can be considered as vertices and the roads as edges [31].

Now-a-days Google Map is a useful tool in order to go around the world for a couple of days. By using Google map, we can find all the possible routes connecting the point of departure to the point of destination. Hence, the shortest route between the two locations can be found easily. In the case of the Google Map, the places can be considered as vertices and the paths as edges. Then Google map software searches out all the possible edges between these two places or vertices when it finds the routes between two places and also indicates the shortest edge as the shortest path.

The operation of traffic lights, for instance, turning Green or Red and timing between them, uses the technique of coloring vertex in order to solve time and space conflicts by identifying the chromatic numbers for the required cycles.

Graph theory, for operational research, is a very useful tool. With the help of graphs some OR problems can be solved. In the case of transport problems, the graphic-theoretical approach is useful when we have to minimize costs of transport or maximize profit. It is also used for various tasks, such as assignment of various people to different jobs, administration of school time table, assignment of offices etc.

Graph theory is used in chemistry to model chemical phenomena in mathematics. By using graph theory we can make a molecule's natural model in which vertices represent atoms while edges represent bond. Chemical Graph Theory(CGT),a mathematical chemistry branch, deals with non-trivial graph theory applications to solve molecular problems. Balaban Alexander, Ante Graovac,Milan Randic, Ivan Gutman, Nenad Trinajstic Haruo Hosoya, among others, are pioneers in the chemical graph theory. Graph theory is also used for biochemistry computational [31]. Graph theory has its unparallel and idiomatic sway in various fields of life, which is on its way to grow with every passing day.

1.1 Metric Dimension

In graph theory, the metric dimension has many applications in the real world. It has been applied to the problems of optimization in complex networks, analyzing electrical networks; showing the business relationships, robotics, control of production processes, etc . A graph-structured framework can be used to study navigation in which the navigating agent (which we assume is a point robot) moves from node to node of a "graph space." The presence of "landmark" nodes that are clearly labeled helps the robot to locate itself in the graph space.[18]

Visual detection of a distinctive landmark informs the robot that navigates in Euclidean space about the orientation of the landmark and allows the robot to define its position by triangulation. However, the concept of direction as well as the concept of visibility do not exist on a graph. Rather, it is assumed that the distances to a set of landmarks can be sensed by a robot navigating a graph. Indeed, the position of the robot on the graph is uniquely determined if it knows its distance from an adequate set of landmarks.

In the graph, how many landmarks are necessary and where should they be located so that the distances between the landmark positions are determined in a unique way by the robot on the graph? The following problem is suggested by this: In fact, it is a classic problem of metric space. A minimal set of points determining the position of the robot in a unique way is known as the ”metric basis” and the the minimum no of landmarks is known as metric dimension.

Chapter 2

Basic Concepts and Terminologies

In running chapter, there are basic definitions of graph theory and chemical Graphs. Their importance to give clear concept that is based on the present study.

2.1 Basic Definitions

2.1.1 Graphs

" Let G be a Graph which consist of an ordered pair (V,E), where V is the set of vertices of G and E is the set of edges in G i.e

$V(G) = \{p_1, p_2, p_3, ..., p_n\}$

$E(G) = \{p_1p_2, p_2p_3, p_3p_4, ..., p_{n-1}p_n\}$ is known as *finite graph* when its set of vertices and set of edges are finite."

"If the cardinality of vertex set and edge set become infinite i.e

$V(G) = \{p_1, p_2, v_3, ...\}$

$E(G) = \{p_1p_2, p_2p_3, p_3p_4, ...\}$

then the $graph(G)$ is known as *Infinite graph*."

" The vertices having degree one are called **Pendant vertices**.

An edge of a graph is known as **Pendant edge** if one of its vertices is a pendant vertex."

" The cardinality set of vertices in a graph G is called the **order** of g. It is labeled by p."

" The cardinality set of edges in a graph G is known as the **size** of a graph.it is labeled by q." For example the order and size of C_4 is 4 i.e $p = q = 4$ in fig 2.1.

2.1.2 Line Graphs

"The line graph denoted by $L(G)$ of a graph G is the graph whose vertices are the edges of existing graph G; two vertices f and g are incident if and only if they have a common end vertex in graph G."

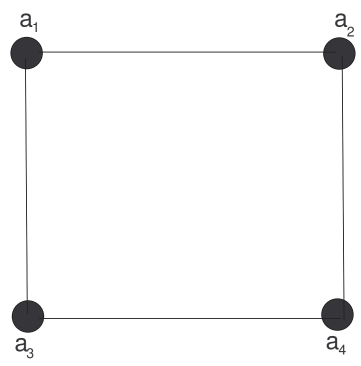

Figure 2.1: C_4

2.1.3 Simple and Multiple Graph

" A graph in which no loop, no parallel edges or no multiple edges is known as **simple graph**." fig 2.2 shows the *simple graph* where $V(G)$ is represented by vertex set i.e $V(G) = \{a_1, a_2, a_3, a_4\}$ and $E(G)$ is represented by edge set i.e $E(G) = \{a_1a_2, a_2a_3, a_3a_4, a_2a_4, a_1a_4, a_1a_3\}$.

" If there is more than one edges among the two adjacent vertices then that graph is

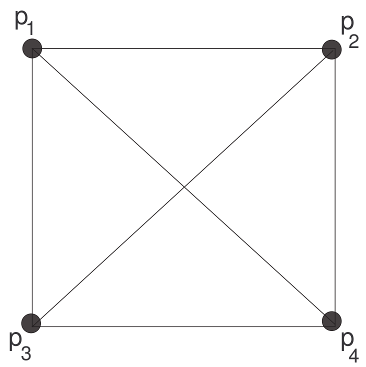

Figure 2.2: Simple Graph

called **Multiple graph**." as shown in fig 2.3

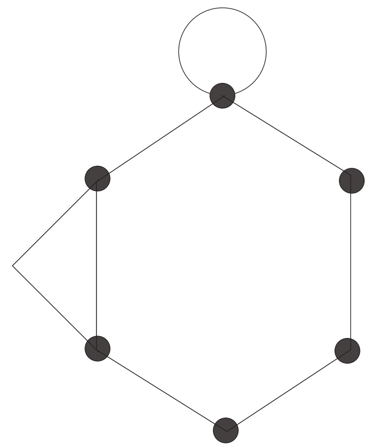

Figure 2.3: Multiple Graph

2.1.4 Complete Graph

" Complete graph on n vertices where every two vertices are joining each other exactly one time, as shown in fig 2.4 and it is denoted by K_n."

Remark 1:

"Every complete graph is necessary to simple."

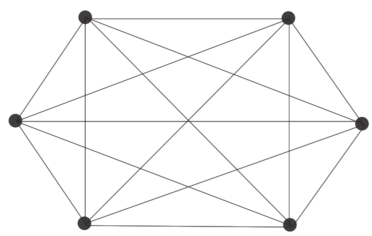

Figure 2.4: Complete Graph

2.1.5 Regular Graph

" A graph will be regular if its all vertices carry equal degree, as show in fig 2.5."

Remark 2:

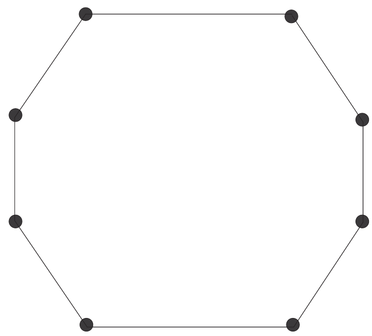

Figure 2.5: Regular Graph

“ K_n is $(n-1)$ regular graph.”

2.1.6 Bipartite and Complete bipartite graphs

“ A graph G will be a **bipartite graph** if its vertex set is split into two vertex set i.e $V = A \cup B$ and there is no edges between vertices of A and vertices of B and vertices of A are bordering to some vertices of B.”

“ A graph G will be a **complete bipartite graph** if its vertex set is split into two vertex set i.e $V = A \cup B$ and there is no edges between vertices of A and vertices of B and vertices of A are bordering to all vertices of B.” as shown in following fig 2.6 and 2.7.

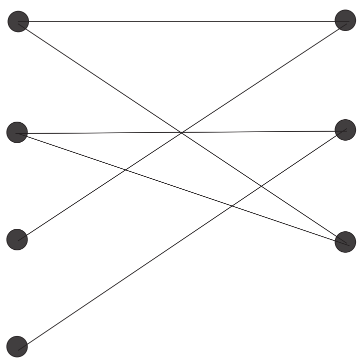

Figure 2.6: $B_{4,3}$ Bipartite Graph

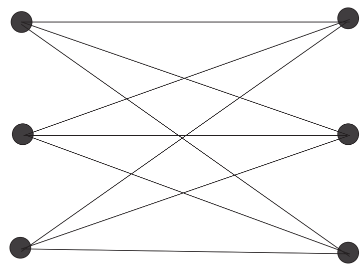

Figure 2.7: $K_{3,3}$ Complete Bipartite Graph

2.1.7 Operations on graphs

“ Assume that $G_1(p_1,q_1)$ and $G_2(p_2,q_2)$ be two graphs then $G = G_1 \cup G_2$ is the **union** of graphs G_1 and G_2 with,

$V(G) = V(G_1) \cup V(G_2)$

$E(G) = E(G_1) \cup E(G_2)$.”

“ Suppose that $G_1(p_1,q_1)$, $G_2(p_2,q_2)$ be two graphs then **Product** of G_1 and G_2 is denoted by $G = G_1 \times G_2$,

$V(G) = \{(u,v); u\varepsilon V(G_1)$ and $v\varepsilon V(G_2)\}$

$E(G) = \{(u,v)(u^{'},v^{'})$ such that $u = u^{'}$ and $vv^{'}\varepsilon E(G_2)$, $v = v^{'}$ and $uu^{'}\varepsilon E(G_1)$.”

“ Let $G_1(p_1,q_1)$, $G_2(p_2,q_2)$ be two graphs then their **Corona** of G_1 and G_2 is denoted by $G_1 \circ G_2 = G$, as shown in fig 2.8 and is defined as the following in three steps,

1. Take one copy of G_1.

2. Take p_1 times copies of G_2.

3. join i^{th} copies of G_2 with i^{th} vertex of G_1."

Remark 3:

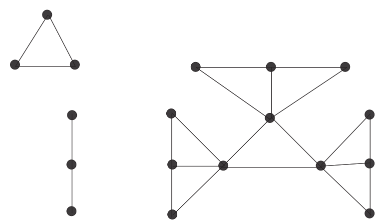

Figure 2.8: $G_1 \circ G_2$

" $G_1 \circ G_2 \neq G_2 \circ G_1$."

"Assume that G be a graph then **Complement** of G is labeled by $\overline{G}$ and defined as $uv\varepsilon\ E(\overline{G})$ iff uv does not belongs to $E(G)$."

Remark 4:

" if $G \cong \overline{G}$ then G is known as **Self Complement** $e.g$ C_5 is self complement."

2.1.8 Sub-graphs

" Suppose that G and K be two graphs the vertex set and the edge set of G and K are $V(G)$, $E(G)$ and $V(K)$, $E(K)$ respectively,then K will be **subgraph** of G if the vertex set of K is the subset of the vertex set of G and the edge set of K are also subset of the edge set of G. "

" Consider G and K be two graphs the vertex set and the edge set of G and K are $V(G)$, $E(G)$ and $V(K)$, $E(K)$ respectively,then K will be **Spanning sub-graph** of G if cardinality of vertex set of G and the vertex set of K is equal but the edge set of K is the subset of edge set of G."

2.1.9 Cyclic graph

" A graph G is said to be cyclic graph if first vertex of G is connected to last its vertex, in other words we can say all vertices are connected in a closed chain. A cyclic graph

G with n vertices then graph G is known as C_n. cyclic graph is shown in fig.2.9."

Remarks 5:

" The order and the size of a graph G will be same if it is cycle, which is labeled by C_n. The degree of each vertex is 2 $i.e$ two edges are incident with every vertex."

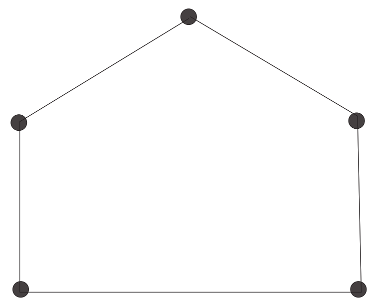

Figure 2.9: Cyclic Graph

2.1.10 Degree of a vertex

" In any graph G the number of edges which are incident to a vertex is known as degree of a vertex and labeled by $deg(V)$. If there is any **loop** on the vertex then the degree of that vertex will be 2. loop as shown in fig.2.10."

2.1.11 Sum of degree of vertex

"Assume that G be a graph, $u \in V(G)$ is any vertex of graph G, $v_1, v_2, v_3, ..., v_n$ are the neighbouring vertices of u. The sum of degree of vertex u is obtained by adding the degrees of all neighbouring vertices v_i which is denoted by S_u and defined as:

$$S_u = \sum_{v_i \in N_G(u)} d(v_i)$$

where,

$$N_G(u) = v \in V(G) | uv \in E(G)$$

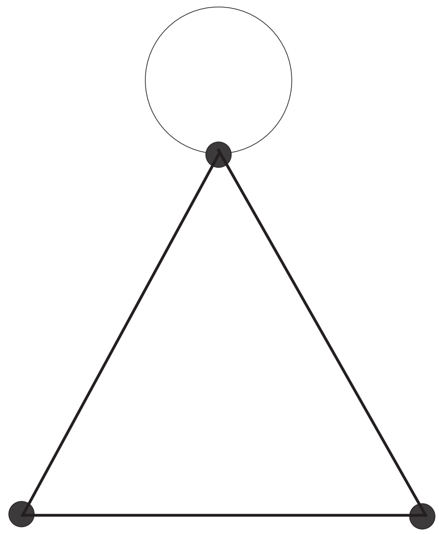

Figure 2.10: Degree of Graph

."

"Suppose that G be a finite Graph, e is any vertex of G,then the Sum of degree of vertex e is sum of degree of all vertices bordering to vertex e. Vertices a ,b, c and d are bordering to e , as we know that if two vertices have an edge among them, we call them adjacent vertices. Sum of degree of vertex e is 12 which is shown in fig 2.11."

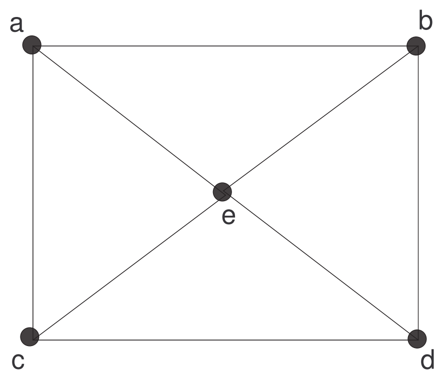

Figure 2.11: Sum of Degree of Graph

2.1.12 Chemical Compounds

“ The Chemical Compound that contain at least one oxygen atom and one other atom in its chemical formula is known as oxide. An oxide is an ion of oxygen i.e O^{2-} where -2 is the oxidation state. The compound that contain anionic silicon compound is known as silicate. Most of the silicates are oxides. SiO_4 contain 4 oxygen ions and 1 silicon ion. These compound consist of silicate anions that is balanced by various cations and minerals are also included in it.”

2.1.13 Sub-divided Graphs

“ Let G be a Graph then by inserting a new vertex of degree 2 in each edge of the graph G we get a new graph and that Graph is called subdivided graphs.”
following $fig2.12$ shows the subdivided graph. By subdivision of graphs or networks we can get different types of new graphs. For example from the subdivision of honeycomb network we can get Domination oxide network.

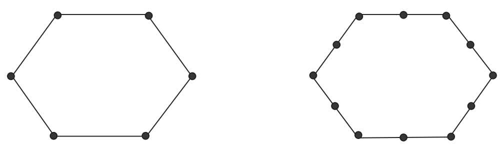

Figure 2.12: Honeycomb and Subdivided Honeycomb Network

2.1.14 Molecular or Chemical Graphs

“ Cheminformatics is a new subject that combines chemistry, mathematics, and science of information. A lot of research has been done in this area in the last decades. Graph theory has provided a variety of useful tools to theoretical chemists, such as topology matrices, topology polynomials, and topology indices. In terms of graph theory, a structural formula of a chemical compound is repre-sent by a molecular graph.

Molecular graphs ’ vertices and edges correspond to the compound atoms and chemical bonds, respectively.”

Chapter 3

Metric Dimension of Different Chemical Structures

3.1 Introduction

Metric dimension is one of the most significant field of graph theory. It various applications in numerous field of life, for instance coin weighting , network theory, pattern recognition, optimization, robot navigation,Network discovery and verification [4],geometrical routing protocols,join in graphs [21] and coin weighting problems [27] accentuate its significant. A point move along a graph might be traced by calculating the distance from a point to a conglomeration of sonar stations positioned clearly in the graph. Throughout consider G as connected graph and $d(m,n)$ denote m-n geodesic. Assume set $P = \{p_1, p_2, ..., p_t\}$ is being an ordered, subset of $V(G)$.Representation of $r(m|P)$ of a m w.r.t P is the $t-tuple$ $(d(m, p_1), d(m, p_2), d(m, p_3)$ $, ..., d(m, p_t))\}$ where P is called a resolving or locating set [5],[26], if each graph vertex is uniquely identify by its distances from the vertices of B. The minimum set of vertices in a resolving set is commonly known as the basis of graph G and cardinality of set of basis element is considered as the metric dimension of G, which is represented as $dim(G)$ [19]. A fairish literature related to metric basis is discussed in [1, 12, 20]. We have an ordered set of vertices $P = \{p_1, p_2, ..., p_t\}$ for a graph G. The $d(m, p_i)$ is zero iff $u = m_i$. If $r(s|P) \neq r(t|P)$ of every pair of unique vertices s,t belongs to $V(G)|P$ then P is known as a resolving set. For the first time slater, a mathematician gave the metric dimension concept in [26]. Later on the same concept was also studied by Melter et al. also investigated the same concept independently in [12]. F. Simon Raj et al. studied metric dimensions of various chemical networks as well as star of David network $SD(n)$ in [23, 24] . The metric dimension of a connected graph changes when the number of vertices is altered in the graph and turns infinite when numbers of vertices are infinite and is known as unbounded metric dimension. likewise metric dimension remains finite when altering in number of vertices is finite and is called bounded metric dimension. Finally,if the metric dimension stick around same for all number of vertices connected in a graph G, then it is called a constant metric dimension [26]. path graph has 1 metric dimension in [7] for every $n \geq 3$ cycle have metric dimension 2.Baca calculated the metric dimension of regular bipartile graphs in [19]. After getting the notion of Manuel,metric dimension of line graph of honeycomb,aztec diamond, honeycomb cup , triangular benzenoid, polycyclic aromatic hydrocarbon networks would be determined in this thesis.

3.1.1 Honeycomb Network

Number of designs in which hexagons offer to build HC network. If HC_1 is a hexagonic honeycomb network, we will add 6 hexagons to the exterior edges of HC_1 in order to obtain the honeycomb network HC_2. Consequently, when we add a layer of hexagon on the boundary of HC_{n-1}, we get honeycomb network HC_n. The hexagons between the centre and boundary of HC_n decides the parameter n of HC_n. Honeycomb network is very useful network. This network is useful in navigation, computer graphics, image processing and cell phone.

Theorem 3.1.1. For $G \cong [LHC_n]$; $n = 1$ then G has metric dimension 2.

Proof: $L[HC(1)]$ is a cycle with 6 vertices as it is not path so its metric dimension is not 1 [4] and as it is C_6 so have metric dimension 2.

Theorem 3.1.2. For $G \cong [LHC_n]$; $n \geq 2$ then G has metric dimension greater than 2.

Proof:

Here, we have to show that,there doesn't exist any set L with two vertices i.e a resolving set of G. Let on contrary G has metric dimension equal to 2

Let $L = \{c_{m+1}, a^m_{f+1}\}$ be a resolving set then

$$r(v^m_f|L) = r(u^m_{f+3}|L)$$

L is not resolving set for the graph.

Let $L = \{u_{f+3}^{p+1}, u_{f+4}^{m+2}\}$ a resolving set then

$$r(u_{f+2}^{m+2}|L) = r(u_{f+5}^{m+1}|L)$$

L is not resolving set.

Let $L = \{c_{m+2}, u_{f}^{m+2}\}$ a resolving set then

$$r(u_{f+2}^{m+1}|L) = r(u_{f+3}^{m+1}|L)$$

L is not resolving set.

Let $L = \{v_{f+1}^{m+1}, u_{f+5}^{m+2}\}$ a resolving set then

$$r(b_{f+3}^{m}|L) = r(b_{f+4}^{m}|L)$$

L is not resolving set.

Let $L = \{b_{f+3}^{m}, u_{f+1}^{m+2}\}$ a resolving set then

$$r(b_{f+2}^{m}|L) = r(u_{f+4}^{m+2}|L)$$

L is not resolving set.

Let $L = \{a_{f+3}^{m}, v_{f+1}^{m+2}\}$ a resolving set then

$$r(a_{f+2}^{m}|L) = r(v_{f+4}^{m+2}|L)$$

L is not resolving set.

Let $L = \{v_{f+3}^{m+1}, v_{f+4}^{m+2}\}$ a resolving set then

$$r(v_{f+2}^{m+2}|L) = r(v_{f+5}^{m+2}|L)$$

L is not resolving set.

Let $L = \{a_{f+1}^{m}, b_{f+3}^{m}\}$ a resolving set then

$$r(v_{f+5}^{m}|L) = r(u_{f+2}^{m}|L)$$

so L is not resolving set.

Let $L = \{a_{f+6}^{m+1}, a_{f+5}^{m+3}\}$ a resolving set then

$$r(v_{f+10}^{m+2}|L) = r(v_{f+13}|L)$$

L is not resolving set.

Let $L = \{c_{f+7}, b_{f+8}^{m}\}$ a resolving set then

$$r(u_{f+14}^{m}|L) = r(u_{f+10}^{m}|L)$$

so L is not resolving set.

Let $L = \{c_{f+5}, v_{f+9}^{m+1}\}$ a resolving set then

$$r(v_{f+8}^{m+1}|L) = r(v_{f+10}^{m+1}|L)$$

L is not resolving set.

Similarly,

therefore there no set L with two vertices be a resolving set for $L[HC_n]$ network so its metric dimension is greater than 2.

Theorem 3.1.3. For $G \cong L[HC(n)]$; $n \geq 1$ then G has metric dimension greater than 2.

Proof: The $L[HC(n)]$ with one vertex between every two vertices has vertex set,

$$V[L(HC(n))] = \{v_q^m : 1 \leq m \leq n; 1 \leq q \leq k, k = 4n - 2m\}$$

$$\cup \{a_q^m : 1 \leq m \leq n-1; 1 \leq q \leq k, k = 2n - m\}$$

$$\cup \{u_q^m : 1 \leq m \leq n; 1 \leq q \leq k, k = 4n - 2m\} \cup \{b_q^m : 1 \leq m \leq n-1; 1 \leq q \leq k, k = 2n - m\}$$

$$\cup \{c_q : 1 \leq q \leq 2n\}$$

Now let $L = \{a_{q+14}^m, b_{q+14}^m, c_q\}$ be the resolving set for the above graph.

$$\lambda(v_q^m) = \begin{cases} q, 4n-m-q, 4n-m-q-1 \\ for \qquad m \leq 1, 1 \leq q = 4n-6 \\ \\ 2m+q-2, 4n-q-1, 4n-q-3 \\ for \qquad 2 \leq m \leq n, 1 \leq q \leq 4n-2m-2 \\ \\ 2m+q-2, 4n-q, 4n-q-3 \\ for \qquad 2 \leq m \leq n, q \leq 4n-2m-1 \\ \\ 2m+q-2, 4n-q+1, 4n-q-3 \\ for \qquad 2 \leq m \leq n, q \leq 4n-2m \\ \\ q, 4n-q, 4n-q-2 \\ for \qquad q \leq 4n-2m-1, 1 \leq m \leq 2, \\ \\ q, 4n-q-1, 4n-q-2 \\ for \qquad m \leq 1, q \leq 4n-2, \\ \\ q, 4n-q+1, 4n-q-2 \\ for \qquad m \leq 1, q \leq 4n-4, \end{cases}$$

$$\lambda(a_q^m) = \begin{cases} 2q, 4n-2q, 4n-2q-m \\ for \qquad m \leq 1, 1 \leq q \leq 2n-2, \\ \\ 2m+2q-2, 4n-2q, 4n-2q+2 \\ for \qquad 2 \leq m \leq n-1, 1 \leq q \leq 2n-m+1, \\ \\ 2m+2q-2, 4n-2q+2, 4n-2q-2 \\ for \qquad 1 \leq m \leq n-1, q \leq 2n-m+2, \\ \end{cases}$$

$$\lambda(u_q^m) = \begin{cases} q, 4n-m-q-1, 4n-m-q \\ for \qquad m \leq 1, 1 \leq q \leq 4n-6, \\ \\ 2m+q-2, 4n-q-3, 4n-q-1 \\ for \qquad 2 \leq m \leq n, 1 \leq q \leq 4n-2m-2, \\ \\ 2m+q-2, 4n-q-3, 4n-q \\ for \qquad 2 \leq m \leq n, q \leq 4n-2m-1, \\ \\ 2m+q-2, 4n-q-3, 4n-q+1 \\ for \qquad 2 \leq m \leq n, q \leq 4n-2m, \\ \\ q, 4n-q-2, 4n-q \\ for \qquad q \leq 4n-2m-1, 1 \leq m \leq 2, \\ \\ q, 4n-q-2, 4n-q-1 \\ for \qquad m \leq 1, q \leq 4n-2, \\ \\ q, 4n-m-1, 4n-q+1 \\ for \qquad m \leq 1, q \leq 4n-4, \\ \end{cases}$$

$$\lambda(b_q^m)=\begin{cases} 2q,4n-2q-m,4n-2q \\ for \qquad m\leq 1, 1\leq q\leq 2n-2, \\ \\ 2m+2q-2,4n-2q+2,4n-2q \\ for \qquad 2\leq m\leq n-1, 1\leq q\leq 2n-m+1, \\ \\ 2m+2q-2,4n-2q-2,4n-2q+2 \\ for \qquad 1\leq m\leq n-1, q\leq 2n-m+2, \\ \end{cases}$$

$$\lambda(c_q)=\begin{cases} q-1,4n-2q,4n-2q \\ for \qquad q\leq 1 \\ \\ 2q-1,4n-2q-2,4n-2q+2 \\ for \qquad q\leq 2n, \\ 2q-1,4n-2q,4n-2q \\ for \qquad 2\leq q\leq 2n-1, \\ \end{cases}$$

Let v_s^m and v_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L)=r(v_f^m|L)$$

$$\Rightarrow (2m+s-2,4n-s-1,4n-s-3)=(2m+f-2,4n-f-1,4n-f-3)$$

$\Rightarrow s=f, \quad s=f, \quad s=f \quad$ which is contradiction.

Let v_s^m and v_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L)=r(v_f^m|L)$$

$\Rightarrow (s, 4n-m-s, 4n-m-s-1) = (f, 4n-m-f, 4n-m-f-1)$
$\Rightarrow s = f, \quad s = f, \quad s = f$ which is contradiction.

Let v_s^m and v_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(v_f^m|L)$$

$\Rightarrow (2m+s-2, 4n-s, 4n-s-3) = (2+f-2, 4n-f, 4n-f-3)$
$\Rightarrow s = f, \quad s = f, \quad s = f$ which is contradiction.

Let v_s^m and v_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(v_f^m|L)$$

$\Rightarrow (2m+s-2, 4n-s+1, 4n-s-3) = (2m+t-2, 4n-t+1, 4n-t-3)$
$\Rightarrow s = f, \quad s = f, \quad s = f$ which is contradiction.

Let v_s^m and v_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(v_f^m|L)$$

$\Rightarrow (s, 4n-s, 4n-s-2) = (f, 4n-f, 4n-f-2)$
$\Rightarrow s = f, \quad s = f, \quad s = f$ which is contradiction.

Let a_s^m and a_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^m|L) = r(a_f^m|L)$$

$\Rightarrow (2m+2s-2, 4n-2s, 4n-2s+2) = (2m+2f-2, 4n-2f, 4n-2f+2)$
$\Rightarrow s = t, \quad s = t, \quad s = t \quad$ which is contradiction.

Let a_s^m and a_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^m|L) = r(a_f^m|L)$$

$\Rightarrow (2s, 4n-2s, 4n-2s-2m) = (2f, 4n-2f, 4n-2f-2m)$
$\Rightarrow s = f, \quad s = f, \quad s = f \quad$ which is contradiction.

Let a_s^m and a_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^m|L) = r(a_f^m|L)$$

$\Rightarrow (2m+2s-2, 4n-2s+2, 4n-2s-2) = (2m+2t-2, 4n-2t+2, 4n-2t-2)$
$\Rightarrow s = f, \quad s = f, \quad s = f \quad$ which is contradiction.

Let v_s^m and a_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(a_f^m|L)$$

$\Rightarrow (2m+s-2, 4n-s-1, 4n-s-4) = (2i+2f-2, 4n-2f, 4n-2f-3)$
$\Rightarrow s = 2f, \quad s = 2f, \quad s = 2f \quad$ which is contradiction.

Let v_s^m and a_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(a_f^m|L)$$

$\Rightarrow (2m+2s-2, 4n-2s, 4n-2s-m) = (f, 4n-m-f, 4n-m-f-1)$
$\Rightarrow s = \frac{f-2m+2}{2}, \quad s = \frac{f+m}{2}, \quad s = \frac{f+m-2}{2}$ which is contradiction.

Let v_s^m and a_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(a_f^m|L)$$

$\Rightarrow (2m+s-2, 4n-s, 4n-s-3) = (2i+2f-2, 4n-2f, 4n-2f+2)$
$\Rightarrow s = 2f, \quad s = 2f, \quad s = 2f-5$ which is contradiction.

Let v_s^m and a_f^m are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(a_f^m|L)$$

$\Rightarrow (2m+s-2, 4n-s+1, 4n-s-3) = (2m+2f-2, 4n-2f+2, 4n-2f-2)$
$\Rightarrow s = 2f, \quad s = 2f-2, \quad s = 2f-1$ which is contradiction.

Let v_s^m and c_q are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^m|L) = r(c_q|L)$$

$\Rightarrow (2i+s-2, 4n-s-1, 4n-s-4) = (2f-1, 4n-2f, 4n-2f)$
$\Rightarrow s = 2f-2i+1, \quad s = 2f-1, \quad s = 2f-4$ which is contradiction.

So every vertex has unique representation w.r.t L, so L be a resolving set for $L[HC(n)], n \geq 2$.

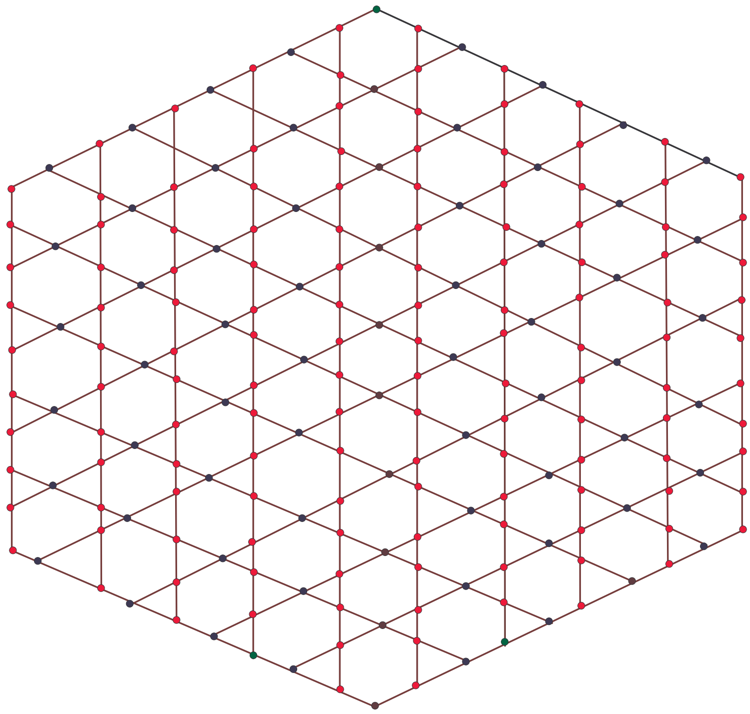

Figure 3.1: Line graph of honeycomb network

3.1.2 Poly Cyclic Aromatic Hydrocarbon

$PAH(n)$ is a family of hydrocarbon , therefore its structure contains cycles of length 6 (Benzene). The free valences of PAH H-saturated bonds may be considered as small pieces of graphene plate parts. Vice versa, an infinite PAH molecule can be interpreted as graphene. Benzene with 6 carbon (C) and hydrogen (H) 6 atoms is the successful use of polycyclic aromatic hydrocarbon molecules in modeling graphite surfaces PAH_1. PAH_2 is Coronene with 24 carbon atoms and 12 hydrogen atoms.

Theorem 3.1.4. For $G \cong L[PAH(n)]$; $n \geq 1$ then G has metric dimension greater than 2.

Proof:

Here, we have to show that there doesn't exist any set A with two vertices i.e resolving set of G. Let on contrary G has metric dimension equal to 2

Let $A = \{m_{i+1}, z^i_{b+1}\}$ be a resolving set then

$$r(x^i_b|A) = r(y^i_{b+3}|A)$$

so A is not resolving set for the graph.
Let $A = \{y^{i+1}_{b+3}, y^{i+2}_{b+4}\}$ be a resolving set then

$$r(y^{i+2}_{b+2}|A) = r(y^{i+1}_{b+5}|A)$$

so A is not resolving set.

Let $A = \{s^i_{b+4}, y^{i+3}_{b+4}\}$ be a resolving set then

$$r(w^{i+3}_{b+1}|A) = r(y^{i+3}_{b+2}|A)$$

so A is not resolving set.

Let $A = \{m_{i+2}, y^{i+2}_b\}$ be a resolving set then

$$r(y^{i+1}_{b+2}|A) = r(y^{i+1}_{b+3}|A)$$

so A is not resolving set.
Let $A = \{x^{i+1}_{b+1}, y^{i+2}_{b+5}\}$ be a resolving set then

$$r(w^i_{b+3}|A) = r(w^i_{b+4}|A)$$

so A is not resolving set.

Let $A = \{t^{i}_{b+4}, x^{i+3}_{b+4}\}$ be a resolving set then

$$r(z^{i+3}_{b+1}|A) = r(y^{i+3}_{b+2}|A)$$

so A is not resolving set.

Let $A = \{w^{i}_{b+3}, y^{i+2}_{b+1}\}$ be a resolving set then

$$r(w^{i}_{b+2}|A) = r(y^{i+2}_{b+4}|A)$$

so A is not resolving set.

Let $A = \{z^{i}_{b+3}, x^{i+2}_{b+1}\}$ be a resolving set then

$$r(z^{i}_{b+2}|A) = r(x^{i+2}_{b+4}|A)$$

so A is not resolving set.

Let $A = \{x^{i+1}_{b+3}, x^{i+2}_{b+4}\}$ be a resolving set then

$$r(x^{i+2}_{b+2}|A) = r(x^{i+2}_{b+5}|A)$$

so A is not resolving set.

Let $A = \{z^{i}_{b+1}, w^{i}_{b+3}\}$ be a resolving set then

$$r(x^{i}_{b+5}|A) = r(y^{i}_{b+2}|A)$$

so A is not resolving set.

Let $A = \{z^{i+1}_{b+6}, z^{i+3}_{b+5}\}$ be a resolving set then

$$r(x^{i+2}_{b+10}|A) = r(x_{b+13}|A)$$

so A is not resolving set.

Let $A = \{m_{i+7}, w^{i}_{b+8}\}$ be a resolving set then

$$r(y^{i}_{b+14}|A) = r(y^{i}_{b+10}|A)$$

so A is not resolving set.

Let $A = \{m_{i+5}, x^{i+1}_{b+9}\}$ be a resolving set then

$$r(x^{i+1}_{b+8}|A) = r(y^{i+1}_{b+10}|A)$$

so A isn't be a resolving set.

Similarly,

here no set A with two vertices be a resolving set for $L[PHA(n)]$ network so it has metric dimension is greater than 2.

Theorem 3.1.5. For $G \cong L[PAH_n]$; $n \geq 1$ then G has metric dimension greater than 2.

Proof: The $L[PAH_n]$ with one vertex between every two vertices has vertex set,

$$V[L[PAH(n)]] = \{x^{i}_{b}: \ 1 \leq i \leq n, 1 \leq b \leq k, k = 4n-2i\}$$

$$\cup\{z^{i}_{b}: \ 1 \leq i \leq n-1, 1 \leq b \leq k, k = 2n-i\}$$

$\cup\{y^{i}_{b}: \ 1 \leq i \leq n, 1 \leq b \leq k, k = 4n-2i\} \cup \{w^{i}_{b}: \ 1 \leq i \leq n-1, 1 \leq b \leq k, k = 2n-i\}$

$$\cup\{c_i:\ 1 \le i \le 2n\}$$

$$\cup\{t_b^i:\ 1 \le i \le n\}$$

$$\cup\{s_b^i:\ 1 \le i \le n\}$$

Now let $A = \{a_{b+14}^i, w_{b+14}^i, c_b\}$ be the resolving set for the above graph.

$$\lambda(x_b^i) = \begin{cases} b, 4n-i-b, 4n-i-b-1 \\ for \qquad i \le 1, 1 \le b \le 4n-6 \\ \\ 2i+b-2, 4n-b-1, 4n-b-3 \\ for \qquad 2 \le i \le n, 1 \le b \le 4n-2i-2 \\ \\ 2i+b-2, 4n-b, 4n-b-3 \\ for \qquad 2 \le i \le n, b \le 4n-2i-1 \\ \\ 2i+b-2, 4n-b+1, 4n-b-3 \\ for \qquad 2 \le i \le n, b \le 4n-2i \\ \\ b, 4n-b, 4n-b-2 \\ for \qquad b \le 4n-2m-1, 1 \le m \le 2 \\ \\ b, 4n-b-1, 4n-b-2 \\ for \qquad i \le 1, b \le 4n-2 \\ \\ b, 4n-b+1, 4n-b-2 \\ for \qquad i \le 1, b \le 4n-4 \end{cases}$$

$$\lambda(z_b^i) = \begin{cases} 2b, 4n-2b, 4n-2b-i \\ for \qquad i \leq 1, 1 \leq b \leq 2n-2 \\ \\ 2i+2b-2, 4n-2b, 4n-2b+2 \\ for \qquad 2 \leq i \leq n-1, 1 \leq b \leq 2n-i+1 \\ \\ 2i+2b-2, 4n-2b+2, 4n-2b-2 \\ for \qquad 1 \leq i \leq n-1, b \leq 2n-i+2 \\ \end{cases}$$

$$\lambda(y_b^i) = \begin{cases} b, 4n-i-b-1, 4n-i-b \\ for \qquad i \leq 1, 1 \leq b \leq 4n-6 \\ \\ 2i+b-2, 4n-b-3, 4n-b-1 \\ for \qquad 2 \leq i \leq n, 1 \leq b \leq 4n-2i-2 \\ \\ 2i+b-2, 4n-b-3, 4n-b \\ for \qquad 2 \leq i \leq n, b \leq 4n-2i-1 \\ \\ 2i+b-2, 4n-b-3, 4n-b+1 \\ for \qquad 2 \leq i \leq n, b \leq 4n-2i \\ \\ b, 4n-b-2, 4n-b \\ for \qquad b \leq 4n-2m-1, 1 \leq m \leq 2 \\ \\ b, 4n-b-2, 4n-jb-1 \\ for \qquad i \leq 1, b \leq 4n-2 \\ \\ b, 4n-i-1, 4n-b+1 \\ for \qquad i \leq 1, b \leq 4n-4 \end{cases}$$

$$\lambda(w_b^i) = \begin{cases} 2b, 4n-2b-i, 4n-2b \\ for \quad i \leq 1, 1 \leq b \leq 2n-2 \\ \\ 2i+2b-2, 4n-2b+2, 4n-2b \\ for \quad 2 \leq i \leq n-1, 1 \leq b \leq 2n-i+1 \\ \\ 2i+2b-2, 4n-2b-2, 4n-2b+2 \\ for \quad 1 \leq i \leq n-1, b \leq 2n-i+2 \\ \end{cases}$$

$$\lambda(c_b) = \begin{cases} b-1, 4n-2b, 4n-2b \\ for \ \ i \leq 1 \\ \\ 2b-1, 4n-2b-2, 4n-2b+2 \\ for \ \ i \leq 2n \\ 2b-1, 4n-2b, 4n-2b \\ for \quad 2 \leq b \leq 2n-1 \\ \end{cases}$$

$$\lambda(t_b^i) = \begin{cases} 2i-1, 4n-b, 4n-2 \\ for \qquad i \leq 1, b \leq 1 \\ \\ 2i-1, 4n-b, 4n-3 \\ for \qquad i \leq 1, 1 \leq b \leq n \\ \\ 2i-2, 4n-2b, 4n-2b-2 \\ for \qquad 1 \leq i \leq n, b \leq n+i \\ \\ 4n-1, 4n-i-b+11, 4n-i-b+7 \\ for \;\; i \leq 2n+8, b \leq n+1 \\ \\ 4n-1, 4n-i-b+11, 4n-i-b+10 \\ for \qquad 2n+2 \leq i \leq 3n, 10 \leq b \leq 2n \\ \end{cases}$$

$$\lambda(s_b^i) = \begin{cases} 2i-1, 4n-2, 4n-b \\ for \quad i \leq 1, b \leq 1 \\ \\ 2i-1, 4n-3, 4n-b \\ for \quad i \leq 1, 1 \leq b \leq n \\ \\ 2i-2, 4n-2b-2, 4n-2b \\ for \quad 1 \leq i \leq n, b \leq n+i \\ \\ 4n-b, 4n-i-b+7, 4n-i-b+11 \\ for \ \ i \leq 2n+8, b \leq n+1 \\ \\ 4n-b, 4n-i-b+10, 4n-i-b+11 \\ for \quad 2n+2 \leq i \leq 3n, 10 \leq b \leq 2n \\ \end{cases}$$

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (e, 4n-i-u, 4n-i-e-1) = (v, 4n-i-v, 4n-i-v-1)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (2i-2+e, 4n-1-e, 4n-e-3) = (2i+v-2, 4n-v-1, 4n-v-3)$

$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (2i-2+e, 4n-e, 4n-e-3) = (2i-2+v, 4n-v, 4n-v-3)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n))]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (2i-2+e, 4n+1-e, 4n-e-3) = (2i-2+v, 4n+1-v, 4n-v-3)$
$\Rightarrow v = e, \quad v = e, \quad v = e \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (e, 4n-e, 4n-2-e) = (v, 4n-v, 4n-2-v)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (e, 4n-1-e, 4n-2-e) = (v, 4n-1-v, 4n-2-v)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (e, 4n+1-e, 4n-i-1) = (v, 4n-j+1, 4n-j-1)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (e, 4n-i-e, 4n-i-e-1) = (2i+v-2, 4n-v-1, 4n-v-3)$
$\Rightarrow e = 2i+j-2, \quad e = v-i+1, \quad e = v-i+4 \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (2i-2+e, 4n-1-e, 4n-e-3) = (2i-2+v, 4n-v, 4n-v-3)$
$\Rightarrow e = v, \quad e = v+1, \quad e = v \quad$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (2i-2+e, 4n+1-e, 4n-e-3) = (v, 4n-v, 4n-v-2)$
$\Rightarrow e = v-2i+2, \quad e = v+1, \quad e = v-1$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (e, 4n-e-1, 4n-e-2) = (v, 4n-v+1, 4n-v-1)$
$\Rightarrow e = v, \quad e = v-2, \quad e = v-1$ which is contradiction.

Let x_e^i and z_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(z_v^i|A)$$

$\Rightarrow (e, 4n-1-e, 4n-e-2) = (2i, 4n-2v, 4n-2v-i)$
$\Rightarrow e = 2i, \quad e = 2v-1, \quad e = 2v+i-2$ which is contradiction.

Let x_e^i and x_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(x_v^i|A)$$

$\Rightarrow (e, 4n-1-e, 4n-2-e) = (v, 4n-v+1, 4n-v-1)$
$\Rightarrow e = v, \quad e = v-2, \quad e = v-1$ which is contradiction.

Let z_e^i and z_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(z_e^i|A) = r(z_v^i|A)$$

$\Rightarrow (2i, 4n-2e, 4n-2e-i) = (2i, 4n-2v, 4n-2v-i)$
$\Rightarrow 2i = 2i, \quad e = v, \quad e = v$ which is contradiction.

Let z_e^i and z_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(z_e^i|A) = r(z_v^i|A)$$

$\Rightarrow (2i+2e-2, 4n-2e, 4n-2e+2) = (2i+2v-2, 4n-2v, 4n-2v+2)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let z_e^i and z_v^i are two distinct vertices from $V([L(PAH(n)]$ then

$$r(z_e^i|A) = r(z_v^i|A)$$

$\Rightarrow (2i+2e-2, 4n-2e+2, 4n-2e-2) = (2i+2v-2, 4n-2v+2, 4n-2v-2)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let z_e^i and z_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(z_e^i|A) = r(z_v^i|A)$$

$\Rightarrow (2i+2e-2, 4n+2-2e, 4n-2-2e) = (2i+2v-2, 4n-2v, 4n-2v+2)$
$\Rightarrow e = v, \quad e = v+1, \quad e = v-2$ which is contradiction.

Let x_e^i and z_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(x_e^i|A) = r(z_v^i|A)$$

$\Rightarrow (2i+2e-2, 4n-1-e, 4n-e-3) = (2i+2v-2, 4n-2v, 4n-2v+2)$
$\Rightarrow e = \frac{2-2v}{2}, \quad e = 2v-1, \quad e = 2v-5$ which is contradiction.

Let y_e^i and y_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(y_e^i|A) = r(y_v^i|A)$$

$\Rightarrow (e, 4n-i-1-e, 4n-i-e) = (v, 4n-i-v-1, 4n-i-v)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let y_e^i and y_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(y_e^i|A) = r(y_v^i|A)$$

$\Rightarrow (2i+e-2, 4n-e-3, 4n-e-1) = (2i+v-2, 4n-v-3, 4n-v-1)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let y_e^i and y_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(y_e^i|A) = r(y_v^i|A)$$

$\Rightarrow (2i+e-2, 4n-e-3, 4n-e) = (2i+v-2, 4n-v-3, 4n-v)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let y_e^i and y_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(y_e^i|A) = r(y_v^i|A)$$

$\Rightarrow (2i+e-2, 4n-e-3, 4n-e+1) = (2i+v-2, 4n-v-3, 4n-v+1)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let y_e^i and y_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(y_e^i|A) = r(y_v^i|A)$$

$\Rightarrow (e, 4n-e-2, 4n-e) = (v, 4n-v-2, 4n-v)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let y_e^i and y_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(y_e^i|A) = r(y_v^i|A)$$

$\Rightarrow (e, 4n-2-e, 4n-1-e) = (v, 4n-v-2, 4n-v-1)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let y_e^i and y_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(y_e^i|A) = r(y_v^i|A)$$

$\Rightarrow (e, 4n-1-e, 4n-e+1) = (v, 4n-j-1, 4n-j+1)$

$\Rightarrow e = t, \quad e = t, \quad e = t \quad$ which is contradiction.

Let w_e^i and w_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(w_e^i|A) = r(w_v^i|A)$$

$\Rightarrow (2i+2e-2, 4n-2e, 4n-2e+2) = (2i+2v-2, 4n-2v, 4n-2v+2)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let w_e^i and w_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(w_e^i|A) = r(w_v^i|A)$$

$\Rightarrow (2i+2e-2, 4n-2-2e, 4n-2e+2) = (2i+2v-2, 4n-2v-2, 4n-2v+2)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let w_e^i and w_v^i are two distinct vertices from $V[L(PAH(n))]$ then

$$r(w_e^i|A) = r(w_v^i|A)$$

$\Rightarrow (2i+2e-2, 4n-2-2e, 4n-2e+2) = (2i+2v-2, 4n-2v+2, 4n-2v)$
$\Rightarrow e = v, \quad e = v-2, \quad e = v-1 \quad$ which is contradiction.

Let c_e and c_v are two distinct vertices from $V[L(PAH(n))]$ then

$$r(c_e^i|A) = r(c_v^i|A)$$

$\Rightarrow (2e-1, 4n-2e, 4n-2e) = (2v-1, 4n-2v, 4n-2v)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let t_e^i and t_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(t_e^i|A) = r(t_v^i|A)$$

$\Rightarrow (4n-e, 4n-i-e+11, 4n-i-e+10) = (4n-v, 4n-i-v+11, 4n-i-v+10)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let t_e^i and t_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(t_e^i|A) = r(t_v^i|A)$$

$\Rightarrow (4n-e, 4n-i+11-e, 4n+7-e-i) = (4n-v, 4n-i-v+11, 4n+7-v-i)$
$\Rightarrow e = v, \quad e = v, \quad e = v$ which is contradiction.

Let t_e^i and s_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(t_e^i|A) = r(s_v^i|A)$$

$\Rightarrow (4n-e, 4n-i+11-e, 4n-i-e+10) = (4n-v, 4n-i-v+10, 4n-i-v+11)$
$\Rightarrow e = v, \quad e = v+1, \quad e = v-1$ which is contradiction.

Let s_e^i and s_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(s_e^i|A) = r(s_v^i|A)$$

$\Rightarrow (4n-e, 4n-i-e+10, 4n-i-e+11) = (4n-v, 4n-i-v+10, 4n-i-v+11)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

Let t_e^i and t_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(t_e^i|A) = r(t_v^i|A)$$

$\Rightarrow (2i-2, 4n-2e, 4n-2e-2) = (2i-2, 4n-2u, 4n-2v-2)$
$\Rightarrow 2i-2 = 2i-2, \quad e = v, \quad e = v \quad$ which is contradiction.

Let s_e^i and s_v^i are two distinct vertices from $V[L(PAH(n)]$ then

$$r(s_e^i|A) = r(s_v^i|A)$$

$\Rightarrow (4n-e, 4n+7-e-i, 4n-i+11-e) = (4n-v, 4n+7-v-i, 4n-i-v+11)$
$\Rightarrow e = v, \quad e = v, \quad e = v \quad$ which is contradiction.

So every vertex has unique representation w.r.t A, this be a resolving set for $L[PAH_n]$, $n \geq 2$.

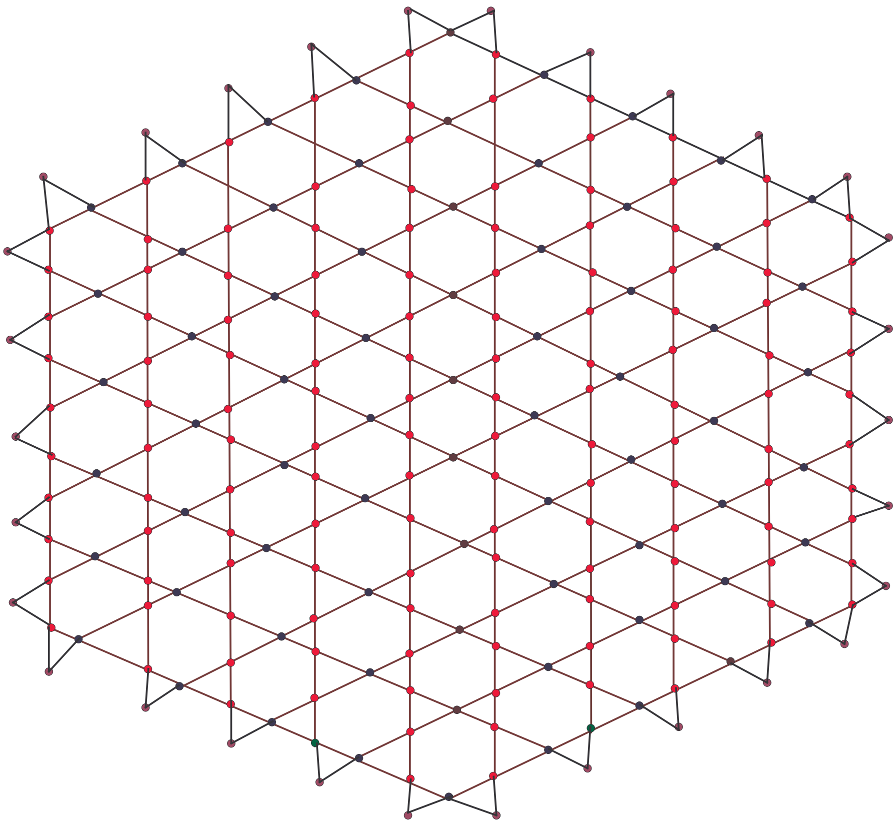

Figure 3.2: Line graph of polycyclic aromatic hydrocarbon

3.1.3 Aztec Diamond Network

The area derived from the staircase forms a height of n when it is known as the aztec diamond of n by a straight edges. We can define it as a combination of unit(1) squares on the plane with edges lying on the square grid lines and justifying its center (a, b) $\frac{1}{2}[|2x-1|+|2y-1|]$. Az(n) is a common symbol of this network.

Theorem 3.1.6. For $G \cong AZ(p)$; $p \geq 1$ then $dim(G) > 2$.

Proof: Suppose on contrary $L[AZ(n)]$ has E as its resolving set with cardinality is 2. Let $E = \{a_1, v_c^{d+11}\}$ be a resolving set.

$$r(a_{d+2}|E) = r(v_c^{d+9}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{v^{d}_{c+3}, v^{d+11}_{c+3}\}$ be a resolving set.

$$r(v^{d+6}_{c+3}|E) = r(v^{d+6}_{c+2}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{b_{d+10}, v^{d+5}_{c+6}\}$ be a resolving set.

$$r(v^{d+3}_{c+8}|E) = r(v^{d+4}_{c+8}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{a_1, a_{c+8}\}$ be a resolving set.

$$r(v^{d+9}_{c+5}|E) = r(v^{d+10}_{c+5}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{v^{d+7}_{c+9}, v^{d+1}_{c+8}\}$ be a resolving set.

$$r(v^{d+5}_{c+7}|E) = r(v^{d+5}_{c+8}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{g_{d+3}, v^1_{c+7}\}$ be a resolving set.

$$r(v^1_{c+5}|E) = r(v^{d+1}_{c+5}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{q_{c+5}, m_{d+3}\}$ be a resolving set.

$$r(v^{d+7}_{c+4}|E) = r(v^{d+8}_{c+4}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{q_{d+3}, a_{d+10}\}$ be a resolving set.

$$r(v^{d+10}_{c+8}|E) = r(v^{d+11}_{c+8}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{m_1, a_{d+2}\}$ be a resolving set.

$$r(m_{d+2}|E) = r(v^{d+9}_{c}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{v^{d+2}_{c+5}, v^{d+9}_{c+6}\}$ be a resolving set.

$$r(v^{d+4}_{c+5}|E) = r(v^{d+3}_{c+6}|E)$$

$\Rightarrow$ which is contradiction.

Let $E = \{q_{d+9}, v_{c+9}^{d+4}\}$ be a resolving set.

$$r(v_{c+10}^{d+5}|E) = r(v_{c+11}^{d+5}|E)$$

$\Rightarrow$ which is contradiction.

Similarly,

there is no resolving set with two basis elements. Hence $dim(G) > 2$ for $p \geq 2$.

Theorem 3.1.7. For $G \cong L[AZ(p)]$; $p \geq 1$ then G has metric dimension 3.

Proof: The $L[AZ(n)]$ has vertex set

$$V(L(AZ(p))) = \{k_c^d : 2p \geq d \geq 1, 2p \geq c \geq 1\} \cup \{g_d : 2p \leq d \leq 1\}$$

$$\cup \{a_d : 2p \geq d \leq 1, 2p \geq c \geq 1\} \cup \{m_d : 2p \geq d \leq 1\}$$

$$\cup \{q_d : 2p \geq d \leq 1\}$$

Now let $E = \{g_1, g_1, v_c^{2l}\}$ be the resolving set for the above graph.

$$\lambda(k_c^d) = \begin{cases} 2p-d, c-1, 2p-d \\ for \;\; 1 \le d \le \frac{n}{2}+1, j \le 2p-d+1 \\ \\ c-1, d, 2p-c+1 \\ for \;\; l+1 \le d \le 2p, 2p-d+2 \le j \le 2d-(p+1) \\ \\ 2l-d, d, 2l-j+1 \\ for \;\; 1 \le d \le l, k-4 \le c \le k-3 \\ \\ 2l-d, c, 2l-c+2 \\ for \;\; 1 \le d \le l, 2 \le c \le p+1 \\ \\ c-1, c-1, 2p-d \\ for \;\; 2 \le d \le 2p-1, 2p+2-d \le c \le 2p \\ \\ c-1, c, 2p-c+2 \\ for \;\; p+2 \le c \le 2p \\ \end{cases}$$

$$\lambda(a_d) = \begin{cases} d, 2p-d+1, 2p+1 \\ for \;\; 1 \le d \le 2 \\ \\ d-1, 2p-d+1, 2p+1 \\ for \;\; 3 \le d \le 2p \\ \end{cases}$$

$$\lambda(g_d) = \begin{cases} d, 2p+1, 2p-d+1 \\ for\ \ 1 \leq d \leq 2 \\ \\ d-1, 2p+1, 2p-d+1 \\ for\ \ 3 \leq d \leq 2p \\ \end{cases}$$

$$\lambda(m_d) = \begin{cases} 2p, d-1, 2p+1 \\ for\ \ d = 1 \\ \\ 2p, d-1, 2p \\ for\ \ 2 \leq d \leq 2p \\ \end{cases}$$

$$\lambda(q_d) = \begin{cases} 2p, 2p+1, d-1 \\ for\ \ d = 1 \\ \\ 2p, 2p, d-1 \\ for\ \ 2 \leq d \leq 2p \\ \end{cases}$$

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (2p-d, w-1, 2p-d) = (2p-d, z-1, 2p-d)$
$\Rightarrow 2p-d = 2p-d, \quad w = z, \quad 2p-d = 2p-d$
which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (w-1, d, 2p-w+1) = (z-1, d, 2p-z+1)$
$\Rightarrow w = z, \quad d = d, \quad w = z$
which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (2p-d, d, 2p-w+1) = (2p-d, d, 2p-z+1)$
$\Rightarrow 2p-d = 2p-d, \quad d = d, \quad w = z$
which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (2p-d, w, 2p-w+2) = (2p-d, z, 2p-z+2)$
$\Rightarrow 2p-d = 2p-d, \quad w = z, \quad w = z$
which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (w-1, w-1, 2p-d) = (z-1, z-1, 2p-d)$
$\Rightarrow w = z, \quad w = z, \quad 2p-d = 2p-d$
which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (w-1, w-1, 2p-d) = (z-1, z-1, 2p-d)$
$\Rightarrow w = z, \quad w = z, \quad 2p-d = 2p-d$
which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (w-1, w, 2p-w+2) = (z-1, z, 2p-z+2)$
$\Rightarrow w = z, \quad w = z, \quad w = z$
which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (w-1, w-1, 2p-d) = (z-1, z, 2p-z+2)$
$\Rightarrow w = z, \quad w = z-1, \quad z = d+2$

which is contradiction.

Let k_w^d and k_z^d are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(k_z^d|E)$$

$\Rightarrow (w-1, w, 2p-w+2) = (2p-d, z, 2p-z+2)$
$\Rightarrow w = z, \quad w = 2n-d+1, \quad w = z$
which is contradiction.

Let k_w^d and c_z are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(c_z|E)$$

$\Rightarrow (w-1, w-1, 2p-d) = (z-1, 2p-z+1, 2p+1)$
$\Rightarrow w = z, \quad w = 2l-z+2, \quad d = 1$
which is contradiction.

Let c_w and c_z are two distinct vertices from $L[V(AZ(p))]$ then

$$r(c_w|E) = r(c_z|E)$$

$\Rightarrow (w-1, 2p-w+1, 2p+1) = (z-1, 2p-z+1, 2p+1)$
$\Rightarrow w = z, \quad w = z, \quad 1 = 1$
which is contradiction.

Let c_w and c_z are two distinct vertices from $L[V(AZ(p))]$ then

$$r(c_w|E) = r(c_z|E)$$

$\Rightarrow (w-1, 2p-w+1, 2p+1) = (z, 2p-z+1, 2p+1)$
$\Rightarrow w = z+1, \quad w = z, \quad 1 = 1$
which is contradiction.

Let k_w^d and m_z are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^d|E) = r(m_z|E)$$

$\Rightarrow (w-1, w, 2p-w+2) = (2p, z-1, 2p)$
$\Rightarrow w = 2p+1, \quad w = z-1, \quad w = 2$
which is contradiction.

Let k_w^d and q_z are two distinct vertices from $L[V(AZ(p))]$ then

$$r(k_w^i|E) = r(q_z|E)$$

$\Rightarrow (w-1, w, 2p-w+2) = (2p, 2p, z-1)$
$\Rightarrow w = 2p+1, \quad w = 2p, \quad w = 2p-z+3$
which is contradiction.

Every vertex has unique representation w.r.t E, so E is a resolving set for $L[AZ(p)]$; $p \geq 2$.

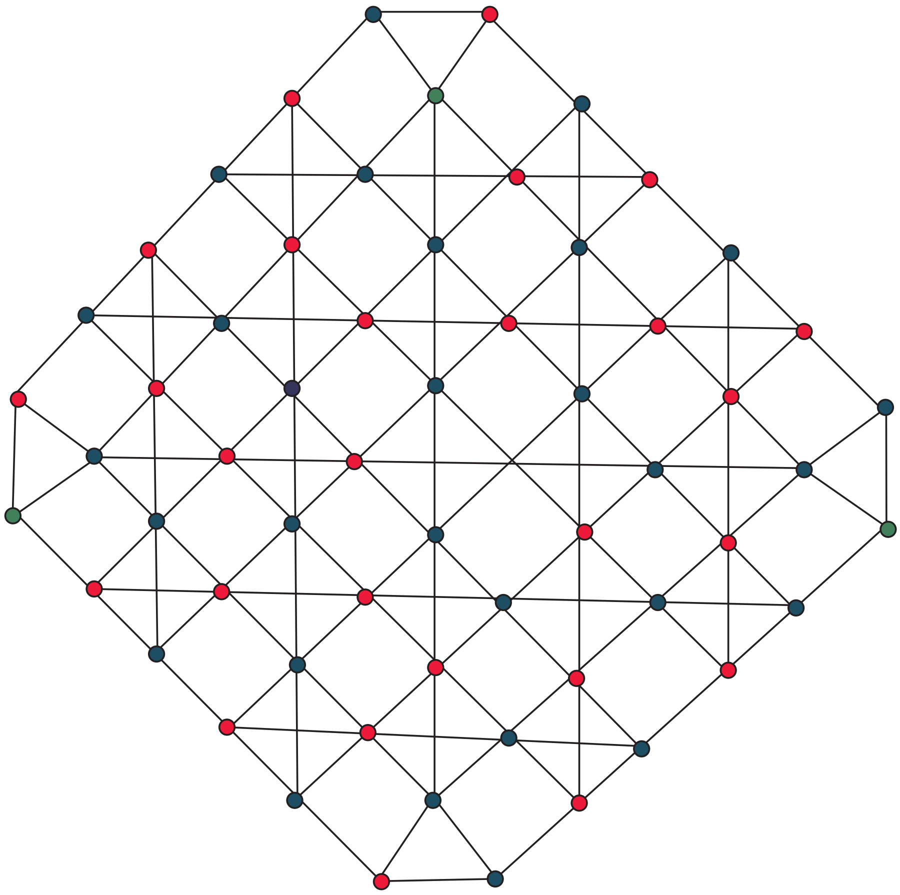

Figure 3.3: Line graph of aztec diamond

3.1.4 Triangular Benzenoid

Theorem 3.1.8. For $G \cong L[T_n]$; $n = 1$ then G has metric dimension 2.

Proof: $L[T(1)]$ is a cycle with 6 vertices as it is not path so its metric dimension is not 1 [4] and as it is C_6 so have metric dimension 2.

Theorem 3.1.9. For $G \cong L[T_n]$; $n \geq 2$ then G has metric dimension greater than 2.

Proof:

Here, we have to show that, there doesn't exist any set P with two vertices that is a resolving set of G. Let on contrary G has metric dimension equal to 2

Let $P = \{q_{j+9}, d_{j+8}\}$ be a resolving set then

$$r(q_{j+8}|P) = r(e^{i+1}_{j+15}|P)$$

therefore P be not a resolving set for the graph.

Let $P = \{e^{i+2}_{j+5}, e^{i+3}_{j+6}\}$ be a resolving set then

$$r(e^{i+2}_{j+7}|P) = r(e^{i+3}_{j+4}|P)$$

therefore P be not a resolving set for the graph.

Let $P = \{d^{i+4}_{j+3}, e^{i+5}_{j+7}\}$ be a resolving set then

$$r(e^{i+4}_{j+6}|P) = r(e^{i+4}_{j+7}|P)$$

therefore P be not a resolving set for the graph.

Let $P = \{d^{i+2}_{j+6}, d^{i+3}_{j+5}\}$ be a resolving set then

$$r(e^{i+4}_{j+9}|P) = r(e^{i+4}_{j+10}|P)$$

therefore P be not a resolving set for the graph.

Let $P = \{e^{i+5}_{j+6}, e^{i+7}_{j+3}\}$ be a resolving set then

$$r(e^{i+5}_{j+5}|P) = r(e^{i+5}_{j+7}|P)$$

therefore P be not a resolving set for the graph.

Let $P = \{e^i_{j+10}, q_{j+5}\}$ be a resolving set then

$$r(e^i_{j+9}|P) = r(e^i_{j+10}|P)$$

so P be not a resolving set for the graph.

here,no set P with 2 vertices is a resolving set for $L[T_n]$ network so its metric dimension is greater than 2.

Theorem 3.1.10. For $G \cong L[T_n]$; $n \geq 2$ then G has metric dimension greater than 2.

Proof: The $L[T_n]$ with one vertex between every two vertices has vertex set,

$$V[L[T(n)] = \{e^i_w : 1 \leq i \leq n, 1 \leq w \leq k, k = 2n - 2i + 2\}$$

$$\cup\{f^i_w : i \leq 1, w \leq 2n + 1 - m, 1 \leq m \leq 2\}$$

$$\cup\{d^i_w :\ 1 \leq i \leq n-1, 1 \leq w \leq k, k = n - i + 1\} \cup \{q^i_w : w \leq 2n\}$$

Now let $P = \{c_1, u^i_{w+19}, a^i_{w+9}\}$ be the resolving set for the above graph.

$$\lambda(e_w^i) = \begin{cases} w, 2n-w, 2n+1-w \\ for \qquad i \leq 1, 1 \leq w \leq 2n-2, \\ \\ 2i+w-2, 2n-1-w, 2n-w+1 \\ for \qquad 2 \leq i \leq n-1, 1 \leq w \leq 2n-2i, \\ \\ w, i, 2n-w+2 \\ for \qquad i \leq 1, w \leq 2n+1-m, 1 \leq m \leq 2 \\ \\ 2i-2+w, 2n-1-w, 2n+2-w \\ for \qquad 2 \leq i \leq n-1 \\ w \leq 2n-2i+1, j = odd, w \leq 2n-2i+2, j = even \end{cases}$$

$$\lambda(f_w^i) = \begin{cases} w, 2n+1-w, 2n-w \\ for \qquad i \leq n, 1 \leq w \leq 2n-2, \\ \\ w, 2n-w+2, i \\ for \qquad i = 1, w \leq 2n-m+1, 1 \leq m \leq 2 \end{cases}$$

$$\lambda(d_w^i) = \begin{cases} 2w, 2n-2w, 2n+2-2w \\ for \quad i \leq 1, 1 \leq w \leq n-i \\ \\ 2i+2w-2, 2n-2w, 2n+2-2w \\ for \quad 2 \leq i \leq n-1, w = n-i \\ \\ 2i+2w-2, 2n-2w, 2n-2w+3 \\ for \quad n-1 \geq i \geq 2, w = n-i+1, \\ \end{cases}$$

$$\lambda(q_w^i) = \begin{cases} 2w-1, 2n-2w+2, 2n-2w+2 \\ for \quad ,2 \leq w \leq n-1, \\ \\ 2w-1, 2n-2w+4, 2n-2w+3 \\ for \ \ w = n+1, \\ \\ 2w-1, 2n-2w+4, 2n-2w+3 \\ for \ \ w = 1, \\ \end{cases}$$

Let e_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (w, 2n-w, 2n-w+1) = (c, 2n-c, 2n-c+1)$

$\Rightarrow w = c, \quad w = c, \quad w = c$ which is contradiction.

Let e_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (2i-2+w, 2n-1-w, 2n-w+1) = (2i+c-2, 2n-c-1, 2n-c+1)$
$\Rightarrow w = c, \quad w = c, \quad w = c$ which is contradiction.

Let e_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (2i+w-2, 2n-w-1, 2n-w+2) = (2i+c-2, 2n-c-1, 2n-c+2)$
$\Rightarrow w = c, \quad w = c, \quad w = c$ which is contradiction.

Let e_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (w, i, 2n-w+2) = (c, i, 2n-c+2)$
$\Rightarrow w = c, \quad i = i, \quad w = c$ which is contradiction.

Let e_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (2i+w-2, 2n-w-1, 2n-w+1) = (2i+c-2, 2n-c-1, 2n-c+2)$
$\Rightarrow w = c, \quad w = c, \quad w = c+1$ which is contradiction.

Let e_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (w, 2n-w, 2n-w+1) = (c, i, 2n-c+2)$
$\Rightarrow w = c, \quad w = 2n-i, \quad w = c-1$ which is contradiction.

Let f_w^i and f_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (w, 2n-w+1, 2n-w) = (c, 2n-c+1, 2n-c)$
$\Rightarrow w = c, \quad w = c, \quad w = c-1$ which is contradiction.

Let f_w^i and f_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(e_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (w, 2n-w+2, i) = (c, 2n-c+2, i)$
$\Rightarrow w = c, \quad w = c, \quad i = i$ which is contradiction.

Let d_b^i and d_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(d_b^i|P) = r(d_c^i|P)$$

$\Rightarrow (2w, 2n-2w, 2n-2w+2) = (2c, 2n-2c, 2n-2c+2)$
$\Rightarrow b = c, \quad b = c, \quad b = c$ which is contradiction.

Let d_w^i and d_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(d_w^i|P) = r(d_c^i|P)$$

$\Rightarrow (2w, 2n-2w, 2n-2w+2) = (2c, 2n-2c, 2n-2c+2)$
$\Rightarrow w = c, \quad w = c, \quad w = c$ which is contradiction.

Let d_w^i and d_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(d_w^i|P) = r(d_c^i|P)$$

$\Rightarrow (2i+2w-2, 2n-2w, 2n-2w+3) = (2i+2c-2, 2n-2c, 2n-2c+3)$
$\Rightarrow w = c, \quad w = c, \quad w = c$ which is contradiction.

Let q_w^i and q_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(q_w^i|P) = r(q_c^i|P)$$

$\Rightarrow (2w-1, 2n-2w+2, 2n-2w+2) = (2j-1, 2n-2c+2, 2n-2c+2)$
$\Rightarrow w = c, \quad w = c, \quad w = c$ which is contradiction.

Let q_w^i and q_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(q_w^i|P) = r(q_c^i|P)$$

$\Rightarrow (2w-1, 2n-2w+4, 2n-2w+3) = (2c-1, 2n-2c+4, 2n-2c+3)$

$\Rightarrow w = c, \quad w = c, \quad w = c \quad$ which is contradiction.

Let q_w^i and d_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(q_w^i|P) = r(d_c^i|P)$$

$\Rightarrow (2w-1, 2n-2w+2, 2n-2w+2) = (2c, 2n-2c, 2n-2c+2)$
$\Rightarrow w = \frac{2c+1}{2}, \quad w = c+1, \quad w = c \quad$ which is contradiction.

Let q_w^i and d_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(q_w^i|P) = r(d_c^i|P)$$

$\Rightarrow (2w-1, 2n-2w+4, 2n-2w+3) = (2c, 2n-2c, 2n-2c+2)$
$\Rightarrow w = \frac{2i+2c-1}{2}, \quad w = c-2, \quad w = \frac{2c+1}{2} \quad$ which is contradiction.

Let d_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(d_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (2w, 2n-2w, 2n-2w+2) = (c, 2n-c, 2n-c+1)$
$\Rightarrow w = \frac{c}{2}, \quad w = c, \quad w = 2c-1 \quad$ which is contradiction.

Let d_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(d_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (2i+2w-2, 2n-2w, 2n+2-2w) = (2i+c-2, 2n-1-c, 2n+1-c)$
$\Rightarrow w = \frac{c}{2}, \quad w = \frac{c+1}{2}, \quad w = \frac{c+1}{2}$ which is contradiction.

Let d_w^i and e_c^i are two distinct vertices from $V[L(T(n)]$ then

$$r(d_w^i|P) = r(e_c^i|P)$$

$\Rightarrow (2i+2w-2, 2n-2w, 2n+3-2w) = (2i+c-2, 2n-1-c, 2n+2-c)$
$\Rightarrow w = \frac{c}{2}, \quad w = \frac{c+1}{2}, \quad w = \frac{c+1}{2}$ which is contradiction.

So every vertex has been unique representation w.r.t to P, so P be a resolving set for $L[T_n]$, $n \geq 2$.

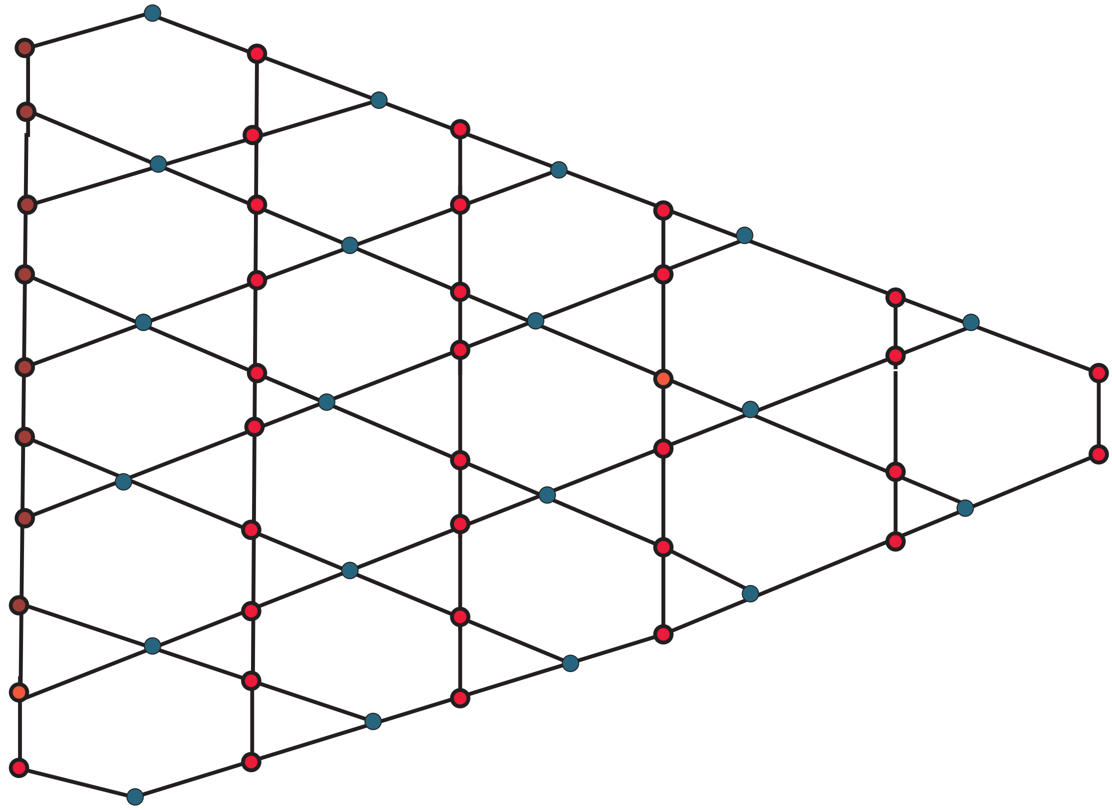

Figure 3.4: Line graph of triangular benezoid

3.1.5 Honeycomb cup network

Theorem 3.1.11. For $G \cong L[HCC_n]$; $n \geq 1$ then G has metric dimension greater than 2.

Proof:

Here, we have to show that, there doesn't exist any set Q with two vertices that is a resolving set of G. Let on contrary G has metric dimension equal to 2

Let $Q\{c_{y+1}, a^x_{y+1}\}$ be a resolving set then

$$r(v^x_y|Q) = r(v^x_{y+3}|Q)$$

so Q is not resolving set for the graph.

Let $Q=\{v_{y+3}^{x+1},v_{y+4}^{x+2}\}$ be a resolving set then

$$r(v_{y+2}^{x+2}|Q)=r(v_{y+5}^{x+1}|Q)$$

so Q is not resolving set.

Let $Q=\{u_{y+3}^{x+1},u_{y+4}^{x+2}\}$ be a resolving set then

$$r(u_{y+2}^{x+2}|Q)=r(u_{y+5}^{x+1}|Q)$$

so Q is not resolving set.

Let $Q=\{c_{y+2},u_{y}^{x+2}\}$ be a resolving set then

$$r(u_{y+2}^{x+1}|Q)=r(u_{y+3}^{x+1}|Q)$$

so Q is not resolving set.

Let $Q=\{v_{y+1}^{x+1},u_{y+5}^{x+2}\}$ be a resolving set then

$$r(b_{y+3}^{x}|Q)=r(b_{y+4}^{x}|Q)$$

so Q is not resolving set.

Let $Q=\{b_{y+3}^{x},u_{y+1}^{x+2}\}$ be a resolving set then

$$r(b_{y+2}^{x}|Q)=r(u_{y+4}^{x+2}|Q)$$

so Q is not resolving set.

Let $Q=\{a_{y+3}^{x}, v_{y+1}^{x+2}\}$ be a resolving set then

$$r(a_{y+2}^{x}|Q)=r(v_{y+4}^{x+2}|Q)$$

so Q is not resolving set.

Let $Q=\{v_{y+3}^{x+1}, v_{y+4}^{x+2}\}$ be a resolving set then

$$r(v_{y+2}^{x+2}|Q)=r(v_{y+5}^{x+2}|Q)$$

so Q is not resolving set.

Let $Q=\{a_{y+1}^{x}, b_{y+3}^{x}\}$ be a resolving set then

$$r(v_{y+5}^{x}|Q)=r(u_{y+2}^{x}|Q)$$

so Q is not resolving set.

Let $Q=\{a_{y+6}^{x+1}, a_{y+5}^{x+3}\}$ be a resolving set then

$$r(v_{y+10}^{x+2}|Q)=r(v_{y+13}|Q)$$

so Q is not resolving set.

Let $Q=\{c_{y+7}, b_{y+8}^{x}\}$ be a resolving set then

$$r(u_{y+14}^{x}|Q)=r(u_{y+10}^{x}|Q)$$

so Q is not resolving set.

Let $Q = \{c_{y+5}, v^{x+1}_{y+9}\}$ be a resolving set then

$$r(v^{x+1}_{y+8}|Q) = r(v^{x+1}_{y+10}|Q)$$

so Q is not resolving set.

Similarly
there is no set Q with two vertices is a resolving set for $L[HCC(n)]$ network so its metric dimension is greater than 2.

Theorem 3.1.12. For $G \cong L[HCC(n)]$; $n \geq 1$ then G has metric dimension 3.

Proof:
The $L[HCC(n)]$ with one vertex between every two vertices has vertex set,

$$V(L(HCC(n))) = \{v^x_y : 1 \leq x \leq n+1, 1 \leq j \leq k, k = 4n-2x+2\}$$

$$\cup\{a^x_y : 1 \leq x \leq n, 1 \leq y \leq k, j = 2n-x+1\}$$

$$\cup\{u^x_y : 1 \leq x \leq n+1, 1 \leq y \leq k, k = 4n-2x+2\} \cup \{b^x_y : 1 \leq x \leq n-1, 1 \leq y \leq k, j = 2n-x+1\}$$

$$\cup\{c_y : 1 \leq x \leq 2n+1\}$$

Now let $Q = \{c_1, a^i_{j+15}, b^i_{j+15}\}$ be the resolving set for the above graph.

$$\lambda(v_y^x) = \begin{cases} y, 4n-y+1, 4n-y \\ for \quad x \leq 1, 1 \leq y \leq 4n-2 \\ \\ 2x+y-2, 4n-y+1, 4n-y-1 \\ for \quad 2 \leq x \leq n+1, 1 \leq y \leq 4n-2x \\ \\ 2x-2+y, 4n+2-y, 4n-y-1 \\ for \ \ 2 \leq x \leq n+1, y \leq 4n-2x+1 \\ \\ 2i+y-2, 4n-y+3, 4n-y-1 \\ for \ \ 2 \leq x \leq n+1, y \leq 4n-2x+2 \\ \\ y, 4n-y+2, 4n-y \\ for \ \ x \leq 1, y \leq 4n-x \\ \\ y, 4n-y+3, 4n-y+1 \\ for \ \ x \leq 1, y \leq 4n-2x \end{cases}$$

$$\lambda(a_y^x) = \begin{cases} 2y, 4n-2y+2, 4n-2y+1 \\ for \quad x \leq n, 1 \leq y \leq 2n-1 \\ \\ 2x+2y-2, 4n-2y+2, 4n-2y \\ for \quad 2 \leq x \leq n, 1 \leq y \leq 2n-x \\ \\ 2x+2y-2, 4n-2y+4, 4n-2y \\ for \ \ 1 \leq x \leq n, 1 \leq y \leq 2n-x+1 \end{cases}$$

$$\lambda(u_y^x) = \begin{cases} y, 4n-y, 4n-y+1 \\ for \quad x \leq 1, 1 \leq y \leq 4n-2 \\ \\ 2x+y-2, 4n-y-1, 4n-y+1 \\ for \;\; 2 \leq x \leq n+1, 1 \leq y \leq 4n-2x \\ \\ 2x-2+y, 4n-y-1, 4n-y+2 \\ for \;\; 2 \leq x \leq n+1, j \leq 4n-2x+1 \\ \\ 2i-2+y, 4n-1-y, 4n-y+3 \\ for \;\; 2 \leq x \leq n+1, y \leq 4n-2x+2 \\ \\ y, 4n-y, 4n-y+2 \\ for \;\; x \leq 1, y \leq 4n-x \\ \\ y, 4n-y+1, 4n-y+3 \\ for \;\; x \leq 1, y \leq 4n-2x \\ \end{cases}$$

$$\lambda(b_y^x) = \begin{cases} 2y, 4n-2y+1, 4n-2y+2 \\ for \quad x \leq n, 1 \leq y \leq 2n-1 \\ \\ 2x+2y-2, 4n-2y, 4n-2y+2 \\ for \;\; 2 \leq x \leq n, 1 \leq y \leq 2n-x \\ \\ 2x+2y-2, 4n-2y, 4n-2y+4 \\ for \;\; 2 \leq x \leq n, 1 \leq y \leq 2n-x+1 \\ \end{cases}$$

$$\lambda(c_y)=\begin{cases} y-1,4n-2y+2,4n-2y+2 \\ for\ \ y\leq 1 \\ \\ 2y-1,4n-2y+2,4n-2y+2 \\ for\ \ 2\leq y\leq 2n \\ \\ 2y-1,4n-2y+4,4n-2y+4 \\ for\ \ y\leq 2n+1 \\ \end{cases}$$

Let v_s^x and v_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(v_s^x|Q)=r(v_t^x|Q)$$

$\Rightarrow (2x+s-2,4n-s+1,4n-s-1)=(2x+2t-2,4n+1-t,4n-t-1)$
$\Rightarrow s=t,\quad s=t,\quad s=t\quad$ which is contradiction.

Let v_s^x and v_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(v_s^x|Q)=r(v_t^x|Q)$$

$\Rightarrow (2x+s-2,4n-s+2,4n-s-1)=(2x+2t-2,4n-t+2,4n-t-1)$
$\Rightarrow s=t,\quad s=t,\quad s=t\quad$ which is contradiction.

Let v_s^x and v_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(v_s^x|Q)=r(v_t^x|Q)$$

$\Rightarrow (2x+s-2, 4n-s+3, 4n-s-1) = (2x+2t-2, 4n-t+3, 4n-t-1)$
$\Rightarrow s=t, \quad s=t, \quad s=t$ which is contradiction.

Let v_s^x and v_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(v_s^x|Q) = r(v_t^x|Q)$$

$\Rightarrow (s, 4n-s+2, 4n-s) = (t, 4n-t+2, 4n-t)$
$\Rightarrow s=t, \quad s=t, \quad s=t$ which is contradiction.

Let v_s^x and v_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(v_s^x|Q) = r(v_t^x|Q)$$

$\Rightarrow (s, 4n-s+3, 4n-s+1) = (t, 4n-t+3, 4n-t+1)$
$\Rightarrow s=t, \quad s=t, \quad s=t$ which is contradiction.

Let v_s^x and v_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(v_s^x|Q) = r(v_t^x|Q)$$

$\Rightarrow (s, 4n-s+1, 4n-s) = (t, 4n-t+1, 4n-t)$
$\Rightarrow s=t, \quad s=t, \quad s=t$ which is contradiction.

Let v_s^x and u_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(v_s^x|Q) = r(u_t^x|Q)$$

$\Rightarrow (s, 4n-s+1, 4n-s) = (t, 4n-t, 4n-t+1)$
$\Rightarrow s = t, \quad s = t-1, \quad s = t+1$ which is contradiction.

Let a_s^x and a_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(a_s^x|Q) = r(a_t^x|Q)$$

$\Rightarrow (2x+s-2, 4n-2s+2, 4n-2s) = (2x+2t-2, 4n-2t+2, 4n-2t)$
$\Rightarrow s = t, \quad s = t, \quad s = t$ which is contradiction.

Let a_s^x and a_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(a_s^x|Q) = r(a_t^x|Q)$$

$\Rightarrow (2x+s-2, 4n-2s+2, 4n-2s) = (2x+2t-2, 4n-2t+4, 4n-2t)$
$\Rightarrow s = t-2, \quad s = t, \quad s = t$ which is contradiction.

Let c_s^x and c_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(c_s^x|Q) = r(c_t^x|Q)$$

$\Rightarrow (2s-1, 4n-2s+2, 4n+2-2s) = (2t-1, 4n-2t+2, 4n-2t+2)$
$\Rightarrow s = t-2, \quad s = t, \quad s = t$ which is contradiction.

Let u_s^x and u_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(u_s^x|Q) = r(u_t^x|Q)$$

$\Rightarrow (s, 4n-s, 4n-s+1) = (t, 4n-t, 4n-t+1)$
$\Rightarrow s = t, \quad s = t, \quad s = t$ which is contradiction.

Let u_s^x and u_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(u_s^x|Q) = r(u_t^x|Q)$$

$\Rightarrow (2x+s-2, 4n-s-1, 4n-s+2) = (2x+t-2, 4n-t-1, 4n-t+2)$
$\Rightarrow s = t, \quad s = t, \quad s = t$ which is contradiction.

Let u_s^x and u_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(u_s^x|Q) = r(u_t^x|Q)$$

$\Rightarrow (2x+s-2, 4n-s-1, 4n-s+3) = (2x+t-2, 4n-t-1, 4n-t+3)$
$\Rightarrow s = t, \quad s = t, \quad s = t$ which is contradiction.

Let b_s^x and b_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(b_s^x|Q) = r(b_t^x|Q)$$

$\Rightarrow (2x+s-2, 4n-2s, 4n-2s+2) = (2x+2t-2, 4n-2t, 4n-2t+2)$
$\Rightarrow s = t-2, \quad s = t, \quad s = t$ which is contradiction.

Let b_s^x and b_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(b_s^x|Q) = r(b_t^x|Q)$$

$\Rightarrow (2x+s-2, 4n-2s, 4n-2s+4) = (2x+2t-2, 4n-2t, 4n+4-2t)$
$\Rightarrow s = t-2, \quad s = t, \quad s = t$ which is contradiction.

Let c_s and b_t^x are two distinct vertices from $V(L(HCC(n))$ then

$$r(c_s|Q) = r(b_t^x|Q)$$

$\Rightarrow (2s-1, 4n-2s+4, 4n-2s+4) = (2x+2t-2, 4n-2t, 4n-2t+4)$
$\Rightarrow s = \frac{2x+2t-1}{2}, \quad s = t+2, \quad s = t$ which is contradiction.

So every vertex has been unique representation w.r.t to A, so A be a resolving set for $L[HCC_n]$, $n \geq 2$.

Chapter 4

Conclusions

4.1 Conclusions

In the foregoing section we investigated the Honeycomb network $HC(n)$, Aztec diamond network, Poly cyclic Aromatic Hydrocarbon, triangular benzenoid and honeycomb cup network then determined the metric dimension of line graph of honeycomb,aztec diamond ,Poly cyclic Aromatic Hydrocarbon, triangular benzenoid and honeycomb cup network and we have proved that $dim[L(HC(n)] = 3$

$dim[L(AZ(n)] = 3$

$dim[L(PAH(n)] = 3$

$dim[L(T(n)] = 3$

$dim[L(HCC(n)] = 3$

Chapter 5
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