Thermal Fluctuations of a Regular and Einstein-Aether Black Holes

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CIIT/FA14-BSM-001/LHR

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Declaration

I **Hafiza Zarish Arshad** registration number **CIIT/FA14-BSM-001/LHR** hereby declare that I have produced the work presented in this report, during the scheduled period of study.

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Certificate

It is certified that **<u>Hafiza Zarish Arshad (CIIT/FA14-BSM-001/LHR)</u>** has carried out all the work related to this report under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus, and the work fulfills the requirement for award of BS degree.

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Abstract

In this thesis, we study the effects of thermal fluctuations on regular black hole solutions with cosmological constant and Einstein-aether black hole with coupling constant respectively. We consider the logarithmic corrected entropy in order to analyzing the thermal fluctuations on regular black hole solutions and Einstein-aether black hole. We also obtain various thermodynamical quantities such as entropy, pressure, specific heats, Gibb's free energy and Helmholtz free energy. We also investigate the stability of regular black hole solution and Einstein-aether black hole in terms of γ, phase transition, grand canonical ensemble and canonical ensemble. We analyze that regular black hole are stable when we increase the value of cosmological constant and Einstein-aether black hole are stable when we increase the value of coupling constant respectively.

Chapter 1

Introduction

Black hole dynamics are one of the most valuable objects of modern physics. It is the most interesting prediction of relativity and was widely studied for many years. Black hole (BH) which has not any singularity exist in nature is called as “Regular Black hole”. In **1968**, Bardeen introduced the first regular black hole **RBH** which is named as Bardeen BH [1]. The existence of essential singularities is such a major demand of the field of relativity and after solving this issue, the RBHs are constructed. RBHs contain some problems like it had weak energy conditions, but in space-time we violate the strong energy conditions and depend on the weak energy conditions. We cannot construct BH symmetry without physical singularity. The laws of thermodynamics and BH dynamics were introduced by Bekenstein and Hawking. There are many phase transitions to BH, i.e. Reissner-Nordstrom BH, Kerr BH and Kerr-Newman BH [2].

According to recent observational data, we have to know that our universe is dominated by dark energy (DE), which has a state of an equation that is close to cosmological constant [3], while the dynamics of DE are more accurate, such as quintessence. On BH spacetime, this DE quintessence could be affected on given BHs. The modified BH is obtained by Schwarzschild BH in quintessence, and the thermodynamics had been investigated by Reissner-Nordstrom BH. In classical and Quantum gravity BH objects are much important [3, 4] and in physics a well-known properties of a BH are its thermodynamics. The most interesting phenomena in general relativity obtain the BH solution. Hawking proposed the entropy law which says that BH entropy and event horizon area A are proportional to each other [4]. The formula for BH entropy is

$$S = \frac{A}{4},$$

which shows that the maximal entropy contained by an object of the same volume. We notice that the entropy with BH has to be associated to prevent the negation of the second law of thermodynamics [5]. The violation is just because a BH hasn't a maximum entropy objects, so the entropy of the universe reduced when an object of that entropy crosses the horizon. The BH has maximum objects of entropy and they have more entropy then other objects with the same volume [6]. In **1970s** Hawking marked out that BH can radiate particles. The maximal entropy of BH is predict to need the improvement just because of the quantum fluctuations, which need to help for the development of holographic principle [7]. The relation between entropy and horizon area would be corrected by these quantum fluctuations, which is due to the hawking radiations because these radiations reducing the size of that BH. We have different approaches to evaluate these given corrections.

In quantum general relativity, we can obtain corrections to the BH thermodynamics by using the density of microstates for asymptotically flat BH [8]. By using Cardy's formula we can generate the logarithmic corrections for all those BH which have the explanation of microscopic degrees of freedom [9]. In the matter field analyzes, BHs are also generated from the entropy area formula, proposed by Bekenstein [10]. Then the correction of that entropy of dilation BHs is obtained, which led towards the logarithmic corrections. In BHs we see the thermal stability, which is one of the most important thermodynamical quantity. Due to their physical nature, the BH is dynamic and having thermodynamical frameworks.

In the present thesis, the organization of thesis is as follows:

i) Some basic definitions and mathematical formulae are given to understand the thesis work in nxt chapter.

ii) In Chapter **2**, we apply thermodynamics on regular BH solutions and calculate the corrected entropy, pressure, specific heats, Gibb's free energy and helmholtz free energy and check weather the BH is stable or not.

iii) Chapter **3** represents we discuss the thermodynamics of Einstein-aether BH. We construct the graphs by applying thermal fluctuations on it and discuss the parameters used in this BH.

iv) The last chapter contains the summary of the results.

Chapter 2

Preliminaries

This chapter has the basic concepts to understand the thesis work.

2.1 Fundamental Hypothesis And Postulates Of General Relativity

General relativity (GR) is based on the following set of fundamental principles, which guided its development. These are in the following:

- The general principle of relativity: The laws of physics must be the same for all observers (accelerated or not).

- Physical events are described in a four-dimensional spacetime manifold, the metric of which is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{2.1.1}$$

- Principle of covariance: The laws of physics must take the same form in all coordinate systems.

- Local Lorentz invariance: The laws of special relativity apply locally for all inertial observers.

- The principle that inertial motion is geodesic motion. The world lines of particles unaffected by physical forces are time-like or null geodesics of spacetime.

- Spacetime is curved. This permits gravitational effects, such as free fall to be described as a form of inertial motion.

- Spacetime curvature is created by stress-energy within the spacetime. This is described in general relativity by Einsteins field equations, which in the presence of matter are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\kappa\bigg((\rho + p)u_\mu u_\nu \pm p + g_{\mu\nu}\bigg), \qquad (2.1.2)$$

where $T_{\mu\nu}$ is the energymomentum tensor. In empty space, $T_{\mu\nu} = 0$ and the equations become

$$R_{\mu\nu} = 0. \qquad (2.1.3)$$

The equivalence principle, which served as a starting point for the development of general relativity, ended up being a consequence of the general principle of relativity and the principle that inertial motion is geodesic motion.

2.2 Schwarzschild Line-element

It is intended to find a line-element for an interval in the vicinity of a gravitating point particle, for instance, the sun. In the empty space, the spacetime would be flat and in spherical polar coordinates and time, the line element is given by

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2. \qquad (2.2.1)$$

In this equation, the velocity of light c is taken to be unity in order to use relativistic units. The isolated particle is assumed to be static and spherically symmetrical and hence, the gravitational field will depend on r alone and not on θ, ϕ. The line-element would be spatially spherically symmetric about the point and static. Thus, the line-element in the most general form is

$$ds^2 = -e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^\nu dt^2, \qquad (2.2.2)$$

where $\lambda = \lambda(r)$ and $\nu = \nu(r)$. However, the gravitational field due to the particle decreases as we move away from it and the line-element, Eq.(3.1.13) will become the Galilean line-element at $r = \infty$. Thus, $\lambda = 0 = \nu$ at $r = \infty$.

Rewriting the line-element, Eq.(2.2.2) as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{2.2.3}$$

with $x^0 = t,\ x^1 = r,\ x^2 = \theta,\ x^3 = \phi$. The metric tensor $g_{\mu\nu}$ is

$$g_{\mu\nu} = \begin{pmatrix} e^\nu & 0 & 0 & 0 \\ 0 & -e^\lambda & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{pmatrix} \tag{2.2.4}$$

The inverse metric tensor $g^{\mu\nu}$ is

$$g^{\mu\nu} = \begin{pmatrix} e^{-\nu} & 0 & 0 & 0 \\ 0 & -e^{-\lambda} & 0 & 0 \\ 0 & 0 & -r^{-2} & 0 \\ 0 & 0 & 0 & \frac{-1}{r^2 \sin^2\theta} \end{pmatrix} \tag{2.2.5}$$

Let us evaluate the Christoffel symbols $\Gamma^\alpha_{\mu\nu}$ and $\Gamma_{\alpha,\mu\nu}$ for the static and spherically symmetric line-element Eq.(2.2.3). Recalling that

$$\Gamma_{\alpha,\mu\nu} = \frac{1}{2(\partial_\nu g_{\alpha\mu} + \partial_\mu g_{\alpha\nu} - \partial_\alpha g_{\mu\nu})}, \tag{2.2.6}$$

$$\Gamma^\alpha_{\mu\nu} = g^{\alpha\lambda} \Gamma_{\lambda,\mu\nu}. \tag{2.2.7}$$

In view of the symmetry relation Eq.(2.2.6), only 40 are independent out of the 64 Christoffel symbols. Further, it is clear that since $g_{\mu\nu}$ is diagonal, the Christoffel symbols with all the three indices different, are zero. In addition, for the line-element (2.2.3), 31 Christoffel symbols are zero and the following nine are non-zero.

$$
\begin{aligned}
\Gamma^0_{10} &= \frac{1}{2}v', \quad \Gamma^2_{12} = \frac{1}{r}, \\
\Gamma^1_{00} &= \frac{1}{2}v'e^{v-\lambda}, \quad \Gamma^2_{33} = -r\sin\theta\cos\theta, \\
\Gamma^1_{11} &= \frac{1}{2}\lambda', \quad \Gamma^3_{13} = \frac{1}{r}, \\
\Gamma^1_{22} &= -re^{-\lambda}, \quad \Gamma^3_{32} = \cot\theta \\
\Gamma^1_{23} &= -r\sin^2\theta e^{-\lambda},
\end{aligned}
\tag{2.2.8}
$$

where $\nu' = \frac{dv}{dr}$ and $\lambda' = \frac{d\lambda}{dr}$. Einsteins law of gravitation for empty space is

$$R_{\mu\nu} = 0, \tag{2.2.9}$$

where $R_{\mu\nu}$ is the contracted curvature tensor. Making use of the Christoffel symbols set (2.2.8), we calculate the different components of $R_{\mu\nu}$ which are given by

$$R_{\mu\nu} = -\partial_\alpha\Gamma^\alpha_{\mu\nu} + \partial_\nu\Gamma^\alpha_{\mu\alpha} + \Gamma^\rho_{\mu\alpha}\Gamma^\alpha_{\nu\rho} - \Gamma^\alpha_{\mu\nu}\Gamma^\rho_{\alpha\rho}. \tag{2.2.10}$$

Thus,

$$
\begin{aligned}
R_{00} =& -\partial_\alpha \Gamma^\alpha_{00} + \Gamma^\rho_{0\alpha}\Gamma^\alpha_{0\rho} - \Gamma^\alpha_{00}\Gamma^\rho_{\alpha\rho}, \qquad (2.2.11)\\
=& -\partial_1 \Gamma^1_{00}\Gamma^0_{01} + \Gamma^0_{01}\Gamma^1_{00} - \Gamma^1_{00}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13})\\
=& -\partial_r(\frac{1}{2}v'e^{v-\lambda}) + 2\frac{1}{4}v'^2e^{v-\lambda} - \frac{1}{2}v'e^{v-\lambda}\left(\frac{1}{2}v' + \frac{1}{2}\lambda' + \frac{2}{r}\right)\\
=& -\frac{1}{2}[v\text{"} + v'(v'-\lambda')]e^{v-\lambda} + \frac{1}{2}v'^2e^{v-\lambda} - \frac{1}{4}v'e^{v-\lambda}(v'+\lambda') - \frac{v'}{r}e^{v-\lambda}.
\end{aligned}
$$

Analogously, we calculate R_{11}.

$$
\begin{aligned}
R_{11} =& -\partial_\alpha \Gamma^\alpha_{11} + \partial_1 \Gamma^\alpha_{1\alpha} + \Gamma^\rho_{1\alpha}\Gamma^\alpha_{1\rho} - \Gamma^\alpha_{11}\Gamma^\rho_{\alpha\rho} \qquad (2.2.12)\\
=& -\partial_1\Gamma^1_{11} + \partial_1\Gamma^0_{10} + \partial_1\Gamma^1_{11} + \partial_1\Gamma^2_{12} + \partial_1\Gamma^3_{13} + \Gamma^0_{1\alpha}\Gamma^\alpha_{10}\\
&+\Gamma^1_{1\alpha}\Gamma^\alpha_{11} + \Gamma^2_{1\alpha}\Gamma^\alpha_{12} + \Gamma^3_{1\alpha}\Gamma^\alpha_{13} - \Gamma^1_{11}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13})\\
=&\partial_1(\Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}) + \Gamma^0_{10}\Gamma^0_{10} + \Gamma^1_{11}\Gamma^1_{11} + \Gamma^2_{12}\Gamma^2_{12} + \Gamma^3_{13}\Gamma^3_{13}\\
&-\Gamma^1_{11}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13})\\
=&\partial_r\left(\frac{1}{2\nu'} + \frac{2}{r}\right) + \frac{\nu'^2}{4} + \frac{\lambda'^2}{4} + \frac{2}{r^2} - \frac{\lambda'}{2}\left(\frac{\nu'}{2} + \frac{\lambda'}{2} + \frac{2}{r}\right)\\
R_{11} =&\frac{1}{2}\nu'' + \frac{1}{4}\nu'^2 - \frac{1}{4}\nu'\lambda' - \frac{\lambda'}{r}.
\end{aligned}
$$

Similarly, we calculate R_{22}

$$
\begin{aligned}
R_{22} =& -\partial_\alpha \Gamma^\alpha_{22} + \partial_2 \Gamma^\alpha_{2\alpha} + \Gamma^\rho_{2\alpha}\Gamma^\alpha_{2\rho} - \Gamma^\alpha_{22}\Gamma^\rho_{\alpha\rho} \qquad (2.2.13)\\
=& -\partial_1\Gamma^1_{22} + \partial_2\Gamma^3_{23} + \Gamma^\rho_{2\alpha}\Gamma^1_{2\rho} + \Gamma^\rho_{22}\Gamma^2_{2\rho} + \Gamma^\rho_{23}\Gamma^3_{2\rho}\\
&-\Gamma^1_{22}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13})\\
=&\partial_r(-re^{-\lambda}) + \partial_2(cot\theta) + \Gamma^2_{21}\Gamma^1_{22} + \Gamma^1_{22}\Gamma^2_{21} + \Gamma^3_{23}\Gamma^3_{23}\\
&+re^{-\lambda}(\frac{1}{2\nu'} + \frac{1}{2\lambda'} + \frac{2}{r})\\
=&e^{-\lambda} - r\lambda'e^{-\lambda} - \csc^2\theta - 2e^{-\lambda} + \cot^2\theta\\
&+re^{-\lambda}(\frac{1}{2\nu'} + \frac{1}{2\lambda'} + \frac{2}{r})
\end{aligned}
$$

Lastly we calculate R_{33}.

$$\begin{aligned}
R_{33} =& -\partial_\alpha \Gamma^\alpha_{33} + \partial_3 \Gamma^\alpha_{3\alpha} + \Gamma^\rho_{3\alpha}\Gamma^\alpha_{3\rho} - \Gamma^\alpha_{33}\Gamma^\rho_{3\rho} \\
=& -\partial_1 \Gamma^1_{33} + \partial_2 \Gamma^2_{33} + \Gamma^\rho_{31}\Gamma^1_{3\rho} + \Gamma^\rho_{32}\Gamma^2_{3\rho} + \Gamma^\rho_{33}\Gamma^3_{3\rho} \\
& -\Gamma^1_{33}\big(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}\big) - \Gamma^2_{33}\big(\Gamma^3_{23}\big) \\
=& -\partial_r(-r\sin^2\theta e^{-\lambda}) - \partial_2(-\sin\theta\cos\theta) + \Gamma^3_{13} + \Gamma^1_{33} + \Gamma^3_{23} + \Gamma^2_{33} \\
& +\Gamma^1_{33}\Gamma^3_{31} + \Gamma^2_{33}\Gamma^3_{32} - \Gamma^1_{33}(\frac{\nu'}{2} + \frac{\lambda'}{2} + \frac{2}{r}) - \Gamma^2_{33}(\cot\theta) \\
=& \sin^2\theta e^{-\lambda} - r\lambda'\sin^2\theta e^{-\lambda} + \cos 2\theta + 2(-\sin^2\theta e^{-\lambda} - \cot\theta\sin\theta\cos\theta) \\
& +r\sin^2\vartheta e^{-\lambda}(\frac{\nu'}{2} + \frac{\lambda'}{2} + \frac{2}{r}) + \sin\theta\cos\theta\cot\theta \\
=& \sin^2\theta e^{-\lambda}(1 - r\lambda') + \cos 2\theta + \frac{1}{2r}\sin^2\theta e^{-\lambda}(\nu' + \lambda') \\
& +2\sin^2\theta e^{-\lambda} - 2\sin^2\theta e^{-\lambda} - 2\cos^2\theta + \cos^{2\theta} \\
=& \sin^2\theta e^{-\lambda}\big[1 + \frac{1}{2r\nu'} - \frac{1}{2r\lambda'}\big] + \cos 2\theta - \cos^2\theta \\
=& \sin^2\theta e^{-\lambda}\big[1 + \frac{1}{2r\nu'} - \frac{1}{2r\lambda'}\big] - \sin^2\theta
\end{aligned} \tag{2.2.14}$$

Also

$$R_{33} = \sin^2\theta R_{22} \tag{2.2.15}$$

We summarized the final form of components as

$$\begin{aligned}
R_{00} =& -\frac{1}{2e^{\nu-\lambda}}(\nu'' + \frac{1}{2\nu'^2} - \frac{1}{2\nu'\lambda'} + \frac{2\nu'}{r}), \\
R_{11} =& \frac{1}{2\nu''} + \frac{1}{4\nu'^2} + \frac{1}{4\nu'\lambda'} - \frac{\lambda'}{r}, \\
R_{22} =& e^{-\lambda}(1 + \frac{1}{2r\nu'} - \frac{1}{2r\lambda'}) - 1, \\
R_{33} =& \sin^2\theta R^{22}.
\end{aligned} \tag{2.2.16}$$

Now the field Eq.(2.2.9) becomes using Eq.(2.2.16) as

$$e^{-\lambda}(\nu'' + (\frac{1}{2})\nu'^2 - (\frac{1}{2})\nu'\lambda' + \frac{2\nu'}{r}) = 0, \tag{2.2.17}$$

$$\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{\lambda'}{r} = 0, \tag{2.2.18}$$

$$e^{-\lambda}(1 + \frac{1}{2r\nu'} - \frac{1}{2r\lambda'}) - 1 = 0, \tag{2.2.19}$$

$$\big[e^{-\lambda}(1 + \frac{1}{2r(\nu' - \lambda'})) - 1\big]\sin^2\theta = 0. \tag{2.2.20}$$

Obviously, Eq.(2.2.19) and Eq.(2.2.20) are identical. The three independent Eqs.(2.2.17), (2.2.18) and (2.2.19) enable us to determine the unknown functions ν and λ. Dividing Eq.(2.2.17) by $e^{-\lambda}$, we obtain

$$\nu'' + \frac{1}{2\nu'^2} - \frac{1}{2\nu'\lambda'} + \frac{2\lambda'}{r} = 0. \tag{2.2.21}$$

Rewriting Eq.(2.2.18), we get

$$\nu'' + \frac{\nu'}{2} - \frac{\nu'\lambda'}{2} - \frac{2\lambda'}{r} = 0. \tag{2.2.22}$$

Subtracting Eq.(2.2.22) from Eq.(2.2.21), we get

$$\frac{2}{r(\nu' + \lambda')} = 0. \tag{2.2.23}$$

Integrating Eq.(2.2.23), we obtain

$$\nu + \lambda = C, \tag{2.2.24}$$

where C is a constant of integration. The gravitational field (disturbance from flat space-time) due to the particle, diminishes as we record to the infinite distance. It implies that the line-element, Eq.(2.2.2), must assume the form of Galilean line-element at $r = \infty$. Thus, we must take $\nu = 0 = \lambda$ at $r = \infty$. Thus, $C = 0$ in Eq.(3.1.17). Therefore,

$$\nu + \lambda = 0 \tag{2.2.25}$$

or $\nu = -\lambda$ In view of Eq.(2.2.25), the Eq.(2.2.19) becomes

$$e^{-\lambda}(1 - r\lambda') - 1 = 0, \tag{2.2.26}$$

$$\frac{d}{dr}(re^{-\lambda}) = 1, \tag{2.2.27}$$

$$re^{-\lambda} = r + C_1, \tag{2.2.28}$$

$$e^{-\lambda} = 1 + \frac{C_1}{r}. \tag{2.2.29}$$

putting $C_1 = -2m$, we get

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2m}{r}. \tag{2.2.30}$$

The constant of integration is put equal to $-2m$, in order to facilitate the physical interpretation of m as the mass of gravitating particle. The line-element. Eq.(2.2.2), due to static, isolated gravitating mass point is, in relativistic units ($c = G = 1$)

$$ds^2 = \Big(1 - \frac{2m}{r}\Big)dt^2 - \Big(1 - \frac{2m}{r}\Big)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi). \tag{2.2.31}$$

In c.g.s. units, if the mass is M, the line-element is

$$ds^2 = \Big(1 - \frac{2MG}{rc^2}\Big)dt^2 - \Big(1 - \frac{2MG}{rc^2}\Big)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi). \tag{2.2.32}$$

As it was obtained first by Schwarzschild in the closed form, it is called Schwarzschilds exterior solution. As is obvious, in the limit $r \to \infty$, it goes over to the metric of spacetime of special relativity. This solution is of fundamental importance. We included the cosmological constant Λ, then for empty space, the solution corresponding to the field equations

$$R_{\mu\nu} = \Lambda g_{\mu\nu}, \tag{2.2.33}$$

can be obtained by taking $T_{\mu\nu} = 0$ and the resulting metric is

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)dt^2 - \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2.2.34}$$

Comparing the two metrics, Eq.(2.2.31) and Eq.(2.2.34), we see that the effect of term Λ on the field surrounding the attracting particle, increases with r, i.e., the size of the region considered. But as we will see in the next section, the motion of planets is described with great accuracy by Eq.(2.2.32), we infer that the cosmological constant Λ, is so small that it does not produce tangible effects within regions of the size comparable to the solar system.

2.2.1 Schwarzschild Singularity

Schwarzschild solution, Eq.(2.2.31) has the following singularities:

- $r = 0$. But this type of singularity is usual and also occurs in the Newtonian theory of gravitation.
- $r = 2m$. The value of r is called the **Schwarzschilds radius**. At $r = 2m$, g_{00} becomes zero and g_{11} becomes infinite. For points $0 \leq r \leq 2m$, $ds^2 < 0$, i.e., the interval is purely space-like.

Let us estimate the singular region for the sun. If it has the radius r, then

$$r = 2m = \frac{2mG^2}{c} = 2.956 \qquad (2.2.35)$$

The physical radius of sun is $6.960105km$, thus, the singular region is extreme deeply embedded in it. For a region inside, the Schwarzschild solution of Einsteins field equations, does not hold as $T_{\mu\nu}$ will not be zero. This is the situation long before the singular region starts. The Schwarzschilds radius is always much smaller than the physical radius of the body, say earth, electron.

However, it may be noted that Einstein and Rosen have shown that the singularity in the Schwarzschild solution, Eq.(2.2.31), can be removed by the transformation

$$r - 2m = u^2 \tag{2.2.36}$$

The line-element, Eq.(2.2.31), then becomes

$$ds^2 = -4(u^2 + 2m)du^2 - (u^2 + 2m)^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{u^2}{u^2 + 2m} \tag{2.2.37}$$

The singularity is no more there in the line-element, but the singularity $r = 2m$ has been shifted from the line-element, Eq.(2.2.31) to the transformation, Eq.(2.2.36). The Jacobian of this transformation is zero at $u = 0$. But in general relativity, only non-singular transformations are allowed so that the Jacobian has to be non-zero and finite within its domain. Furthermore, the line-element, Eq.(2.2.37), is free of singularity only if m is positive, implying thereby that only positive values of mass are permissible. However, this is only a tentative argument.

2.3 Black Hole

A BH is a place in space where gravity pulls so much that even particles and electromagnetic radiations such as light can not get out [11]. BH are perhaps the most perfectly thermal objects in the universe, and yet their thermal properties are not fully understood. They are described very accurately by a small number of macroscopic parameters (e.g., mass, angular momentum, and charge), but the microscopic degrees of freedom that lead to their thermal behavior have not yet been adequately identified [12].

2.3.1 Event Horizon

A BH is therefore bounded by a well-defined surface or edge known as the **event horizon**, within which nothing can be seen and nothing can escape [13]. It is a boundary around the singularity that encloses the black hole.

2.3.2 Regular Black hole

A class of BHs such as Schwarzschild, RN, Kerr and Kerr-Newman BHs provide a true singularity. Since at the point of true singularity, spacetime losses its existence and all laws of physics turn out to be invalid, so it is impossible to understand the inside of a BH. A RBH is a solution with special matter cores that would substitute the true singularities of the Schwarzschild, RN, Kerr and Kerr-Newman BHs. RBH are very important class of globally regular, static, spherically symmetric BHs that are obtained when coupling of general relativity and nonlinear dynamics are considered. Bardeen [15] gave the idea of a central matter core, by proposing a solution of Einsteins equation in which there is a BH with horizons but without a singularity is known as **RBH**.

2.3.3 Entropy

Bekenstein proposed that a BH should have an entropy and that it should be proportional to its horizon area $S = \frac{A}{4}$ [14].

Chapter 3

Thermodynamics Of Regular Black Hole

In this chapter, we apply the thermodynamics on regular BH solutions and calculate the corrected entropy, pressure, specific heats, Gibb's free energy and helmholtz free energy and check weather the BH is stable or not.

3.1 Regular Black Hole Solutions

The generalize form of BH solutions is given by [10],

$$ds^2 = -A(r)dt^2 + A^{-1}(r)dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2,$$

where $A(r) = (1 - 2\frac{M(r)}{r})$. A class of RBH solution with cosmological constant Λ in non-linear electrodynamics is developed by Mo Wen-Juan et al. [15]. We reduce this metric to well known BH by the choice of $M(r)$.

$$M(r) = \left(\frac{mr^3}{(r^2+q^2)^{\frac{3}{2}}} - \frac{q^2 r^3}{2(r^2+q^2)^2} + \frac{r^3}{6}\right), \tag{3.1.1}$$

by inserting the Eq.(3.1.1) in A, then the equation becomes

$$A = 1 - \frac{2mr^2}{(r^2+q^2)^{\frac{3}{2}}} + \frac{r^2 q^2}{(r^2+q^2)^2} - \frac{\Lambda r^2}{3}, \tag{3.1.2}$$

where m, Λ, q represent mass, cosmological constant and electric charge respectively. For $\Lambda = 0$, the solution reduces to the case discussed by Ayon-Beato and Garcia [40] in which they have shown that the field strength and all curvature invariants are regular everywhere radius is an important feature of the metric function, which depend upon the different variety of parameters. We obtain non-minimal RBH by considering the real roots and letting $A = 0$, then we obtain the equation.

$$1 - \frac{2mr^2}{(r^2+q^2)^{\frac{3}{2}}} + \frac{r^2 q^2}{(r^2+q^2)^2} - \frac{\Lambda r^2}{3} = 0. \tag{3.1.3}$$

If the $\Lambda > 0$, then we have mainly three horizons which depending upon the values of parameters in RBHs, and the horizons are the name as, the horizon, event horizon cosmological horizon. If $\Lambda \leq 0$, then the two types of horizons obtained that is an event horizon. It depends also upon different values of parameters. The cosmological horizon is not obtained in that case [7]. Thermal quantities are studied on the outer horizon and we call it as r_+. We obtained mass of the BH after obtaining the roots is given as

$$m(r) = \frac{(r^2+q^2)^{\frac{3}{2}}}{2r^2} + \frac{q^2}{2(r^2+q^2)^{\frac{1}{2}}} - \frac{\Lambda(r^2+q^2)^{\frac{3}{2}}}{6}. \tag{3.1.4}$$

This equation implies that $r \neq 0$. The area of BH horizon is related with entropy, which is given as

$$S_0 = \pi r^2, V = \frac{4}{3}\pi r^3. \tag{3.1.5}$$

As we know the formula for the temperature of BH

$$T = \frac{A'(r)}{4\pi}, \tag{3.1.6}$$

by using Eq.(3.1.2), we take derivative and obtain the equation,

$$T = \frac{1}{4\pi}\Big(\frac{2q^2(q-r)r(q+r)}{(q^2+r^2)^3} + \frac{2mr(-2q^2+r^2)}{(q^2+r^2)^{\frac{5}{2}}} - \frac{2r\Lambda}{3}\Big), \tag{3.1.7}$$

where do we use mass m, which is given above, we analyze the thermodynamics of RBH of mass m, horizon radius r, parameter q and cosmological constant Λ. For different parameters, we use following explicit values. The following three conditions obtained for real positive temperature.

- $q = 0.5, \Lambda = 0.01$
- $q = 0.5, \Lambda = 0$
- $q = 0.5, \Lambda = -0.01$

3.1.1 Logarithmic Corrected Thermodynamics

The corrected form of entropy is given as

$$S = S_0 - \frac{b}{2}\ln[S_0 T^2], \tag{3.1.8}$$

We introduce the parameter b in this RBH and the limit of $b \to 0$, the original result can be obtained and $b \to 1$ yields the usual correction. By inserting Eq.(3.1.5) and Eq.(3.1.7) in Eq.(3.1.8) we obtain the equation.

$$S = \pi r^2 - \frac{b}{2}\ln\left[\frac{\left(\frac{1}{4\pi}\left(\frac{2q^2(q-r)r(q+r)}{(q^2+r^2)^3} + \frac{2mr(-2q^2+r^2)}{(q^2+r^2)^{\frac{5}{2}}} - \frac{2r\Lambda}{3}\right)^2\right)}{16\pi}\right]. \tag{3.1.9}$$

We calculate the pressure for the presence of logarithmic corrections, which cause the reduction of the BHs.

$$P = T\left(\frac{\partial S}{\partial V}\right), \tag{3.1.10}$$

by substituting Eq.(3.1.7), derivative of Eq.(3.1.9) in Eq.(3.1.10), we obtain

$$\begin{aligned} P = & \Bigg[-2\pi r^2(q^2+r^2)(q^2+r^2)\Bigg(3m\big(2q^4+q^2r^2-r^4+\sqrt{q^2+r^2}(-3q^2 \\ & (q-r)(q+r)+(q^2+r^2)^3\Lambda)\Bigg)+b\Bigg(3m(q^2+r^2)(4q^4-10q^2r^2 \\ + & r^4)+2\sqrt{q^2+r^2}(-3q^2(q^4-4q^2r^2+r^4)\Lambda)\Bigg)\Bigg] \times (24\pi^2r^2 \\ & (q^2+r^2)^{\frac{9}{2}})^{-1}. \end{aligned} \tag{3.1.11}$$

From **Fig.(3.1)**, we see that the behavior of pressure for three cases of Λ. When we compare $b = 0$ and $b = 1$, we observe that P decreases due to logarithmic corrections. But when we increase the cosmological constant then P also increases, and at that position where $\Lambda = 0$, the value of P is high. Then we conclude that P will decrease due to logarithmic correction and the value of the Cosmological constant.

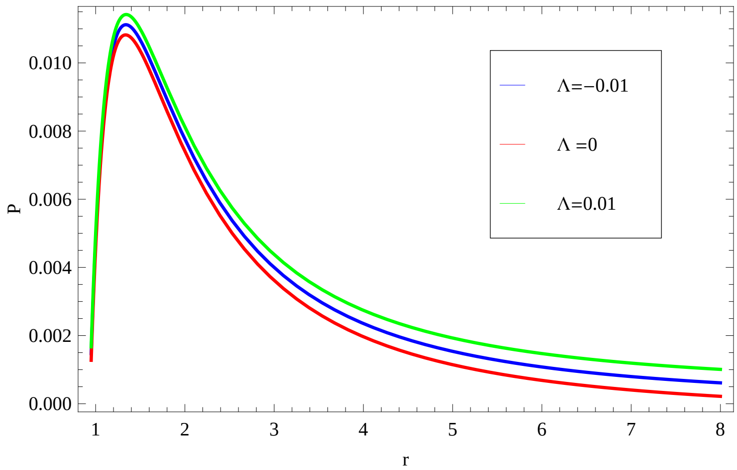

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Figure 3.1: Plot of pressure versus horizon radius.

3.1.2 Stability and Phase Transition of RBH

In BH thermodynamics, heat capacity is an important measurable physical quantity, which is used for the identification of the amount of heat required for the change in temperature. We estimate the nature of heat capacity and analyze that either it is stable or unstable. There are two types of heat capacities C_V and C_P. We obtain these heat capacities by keeping volume and pressure constant as

$$C_V = T\Big(\frac{\partial S}{\partial T}\Big)_V \tag{3.1.12}$$

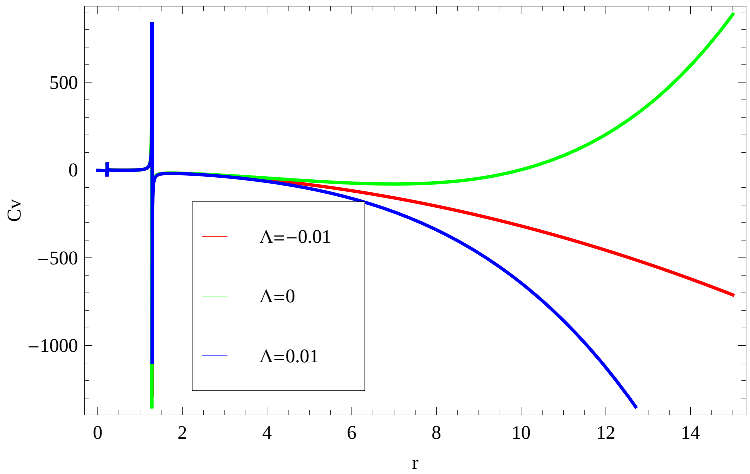

Figure 3.2: Plot of specific heat versus horizon radius.

by using Eq.(3.1.7) and Eq.(3.1.9), we obtain the equation,

$$\begin{aligned} C_V &= \Bigg(2\pi r^2(q^2+r^2)\Bigg(3m\Bigg(2q^4+q^2r^2-r^4+\sqrt{q^2+r^2}(-3q^2(q-r) \\ &\quad (q+r)+(q^2+r^2)^3\Lambda)-b\Bigg(3m(q^2+r^2)(4q^4-10q^2r^2+r^4)+ \\ &\quad 2\sqrt{q^2+r^2}(-3q^2(q^4-4q^2r^2+r^4)+(q^2r^2)^4\Lambda)\Bigg)\Bigg)\Bigg)\Bigg)\times\Bigg(3m(q^2+r^2) \\ &\quad (2q^4-11q^2r^2+2r^4)+\sqrt{q^2+r^2}\Bigg(-3q^2(q^4-8q^2r^2+3r^4)+ \\ &\quad (q^2+r^2)^4\Lambda\Bigg)\Bigg)^{-1}. \end{aligned} \tag{3.1.13}$$

There is another way to find the stability of BH thermodynamics locally. In Eq(3.1.13) we investigate that either specific heat is stable or unstable. The RBH is locally stable when $\Lambda = 0$ at $C_V > 40$. The **Fig.(3.2)** represents the plot of specific heat versus horizon radius to discuss the phase transition. We can find phase transition point at $C_V = 0$ and BH is stable at $\Lambda = -0.01$, and BH is locally unstable for $C_V < 0$ at $\Lambda = 0.01$. We discuss the range of BH horizon local thermodynamical stability for each case. We analyze that when $\Lambda = 0$ the horizon radius for local stability is 2.25. When $\Lambda = 0.01$ the horizon radius for local stability is 1.85, and for negative cosmological constant the radius horizon $r > 4.9$. Hence we can conclude that for negative cosmological constant the range of horizon radius for local stability of BH is increased as compared to positive and zero cosmological constant, respectively. We observe that the phase transition for the positive cosmological constant is nearly equal to RBH as compare to zero and negative cosmological constant. Hence we can say that if we increase the value of cosmological constant, then the phase transition moving towards the RBH and vice versa.

Now we have to calculate the specific heat with constant pressure by using Eq.(3.1.4) and Eq.(3.1.7) and substituting into Eq.(3.1.14),

$$C_P = \Big(\frac{\partial M}{\partial T}\Big)_P. \tag{3.1.14}$$

$$C_P = \frac{(2\pi(q^2+r^2)^{\frac{5}{2}}(2q^6+3q^4r_+^2+q^2r_+^4-r_+^6+r_+^4(q^2+r_+^2)^2\Lambda))}{(r_+(-2q^8-11q^6r_+^2-12q^4r_+^4-8q^2r_+^6+r^8+r_+^4(q^2+r_+^2)^2(3q^2+r_+^2)\Lambda} \tag{3.1.15}$$

When we obtained the result of heat capacities, then we comprise the ratio between C_P and C_V called which can be written as

$$\gamma = \frac{C_P}{C_V} \tag{3.1.16}$$

$$\begin{aligned}
\gamma &= \Bigg(2\pi(q^2+r_+^2)^{\frac{5}{2}}(2q^6+3q^4r_+^2+q^2r_+^4-r_+^6+r_+^4(q^2+r_+^2)^2\Lambda)(2q^8-7q^6r^2 \\
&- 13q^4r^4-11q^2r^6+2r^8+5q^2r^4(q^2+r^2)^2\Lambda)\Bigg)\Bigg(r(-2q^8-11q^6r^2-12q^4r^4 \\
&- 8q^2r^6+r^8+r^4(q^2+r^2)^2(3q^2+r^2)\Lambda)\Bigg(\pi r^2(2q^8+5q^6r^2+4q^4r^4-r^8+r^4 \\
&\quad (q^2+r^2)^3\Lambda)+b(-4q^8+2q^6r^2+9q^4r^4+11q^2r^6-r^8-r^4(q^2+r^2)^2 \\
&\quad (6q^2+r^2)\Lambda)\Bigg)\Bigg)^{-1}.
\end{aligned}$$

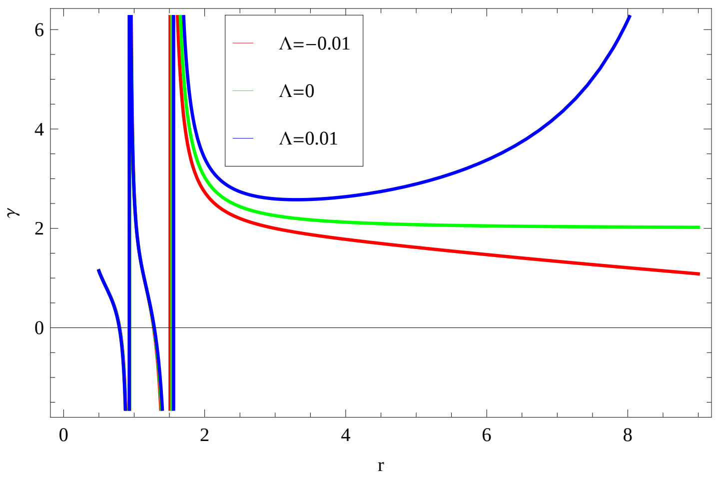

Figure 3.3: Plot of gamma versus horizon radius.

We have three special cases, from **Fig.(3.3)** we analyze that the value of Γ increases due to logarithmic correction. We obtained maximum value for negative cosmological constant and $\gamma \rightarrow 2.2$ for a large horizon. Further, we observe that $\gamma \rightarrow 2.35$, when $\Lambda = 0$ for a large horizon. It is interesting to note that the value of γ decreases and become 0 at $r = 9.95$. The value of γ become negative for a large horizon, then the value of γ shows that a stable behavior for negative and 0 values of cosmological constant but represents the unstable behavior of positive cosmological constant. We conclude that value of cosmological constant decreases then the value of γ is increasing, but it experiences an unstable behavior for higher values of the cosmological constant. Hence, we conclude that the value of γ increases due to logarithmic correction.

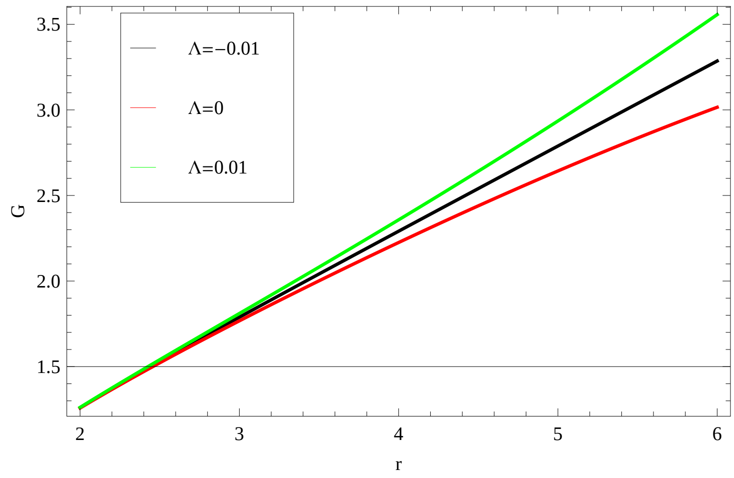

Figure 3.4: Plot of Gibb's free energy versus horizon radius.

3.1.3 Grand Canonical Ensemble

One of the important thermodynamic quantities which can be used to investigate the small and large BH phase transition called as Gibb's Free energies, which is defined as

$$G = M - TS, \tag{3.1.17}$$

$$\begin{aligned} G &= \frac{1}{12}\Big(\frac{6q^2}{\sqrt{q^2+r^2}} + \frac{6(q^2+r^2)^{\frac{3}{2}}}{r^6} - 2((q^2+r^2)^{\frac{3}{2}})\Lambda \\ &+ 3(-2q^8 - 11q^6r^2 - 12q^4r^4 - 8q^2r^6 + r^8 + r^4(q^2+r^2)^2 \times (3q^2+r^2) \\ &\quad \Lambda) \times (2\pi r^2) + b\ln[16\pi] - b\ln[(2q^6 + 3q^4r^2 + q^2r^4 - r^6 + r^4 \\ &\quad (q^2+r^2)^2\Lambda)^2((q^2+r^2)^6)^{-2}]) \times (2\pi r^2(q^2+r^2))^{-4}. \end{aligned} \tag{3.1.18}$$

From **Fig.(3.4)**, we see that the behavior of Gibb's Free energy effected due to the correction of the variation of horizon radius. So it is clear that logarithmic correction enhanced Gibb's free energy. We analyze that Gibb's Free energy is minimum at $r_+ \simeq 0.8$ due to logarithmic correction. Logarithmic correction reduces the Gibbs free energy.

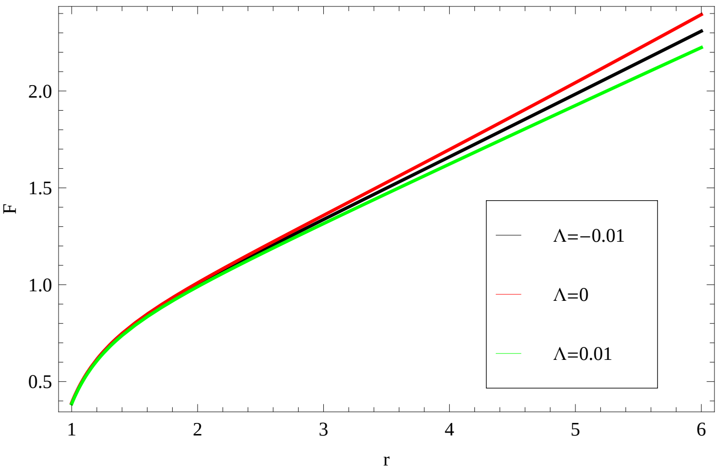

Figure 3.5: Plot of Force versus horizon radius.

3.1.4 Canonical Ensemble

Gibb's free energy can be used to obtain Helmholtz free energy (F) by the following relation.

$$F = G - (P)(V), \tag{3.1.19}$$

by using Eq.(3.1.18), Eq.(3.1.5) and Eq.(3.1.11) in Eq.(3.1.19), we have

$$\begin{aligned}
F &= \frac{1}{12}\Big(\frac{6q^2}{\sqrt{q^2+r^2}} + \frac{6(q^2+r^2)^{\frac{3}{2}}}{r^6} - 2((q^2+r^2)^{\frac{3}{2}})\Lambda + +3(-2q^8 - 11q^6r^2 \\
&- 12q^4r^4 - 8q^2r^6 + r^8 + r^4(q^2+r^2)^2(3q^2+r^2)\Lambda)(2(\pi)r^2) + bLog[16\pi] \\
&- bLog[\frac{(2q^6+3q^4r^2+q^2r^4-r^6+r^4(q^2+r^2)^2\Lambda)^2}{(q^2+r^2)^6}])(2\pi r^2(q^2+r^2))^{-4} \\
&+ (2\pi r^2(2q^8+5q^6r^2+4q^4r^4-r^8+r^4(q^2+r^2)^3\Lambda + b(-4q^8+2q^6r^2 \\
&+ 9q^4r^4+11q^2r^6-r^8-r^4(q^2+r^2)^2(6q^2+r^2)\Lambda))(\pi r(q^2+r^2))^{-4}
\end{aligned} \tag{3.1.20}$$

From **Fig.(3.5)** wee see that in the presence of logarithmic correction the F decreases. For a negative cosmological constant the F increases till ($r = 7$) to be compared with 0 and positive cosmological constant. For a large horizon, F decrease at $r = 18$ then it becomes negative. Hence, we conclude that BH is more thermodynamically unstable if a value of increases in the large horizon and it become unstable when the cosmological constant is negative.

Chapter 4

Thermodynamics of Einstein-Aether Black hole

In this chapter, we discuss the thermodynamics of Einstein-aether BH. We construct the graphs by applying thermal fluctuations on it and discuss the parameters used in this BH.

4.1 Einstein-Aether Black Hole

The metric for Einstein-aether BH spacetime is spherically symmetric and static, which can be written as

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta + \sin^2\theta d\phi^2).$$

The Einstein aether BH have exact solutions.From the given metric, we take $c_{14} \neq 0$ and $c_{123} = 0$. We impose following constraints on given metric. $0 \leq c_{14} < 2$, $2 + c_{13} + 3c_2 > 0$, $0 \leq c_{13} < 1$ where $c_{14} \equiv c_1 + c_4$as

$$A(r) = 1 - \frac{2M}{r} - J(\frac{M}{r})^2, \tag{4.1.1}$$

where $J = \frac{c_1 - \frac{c_2}{2}}{1-c_1}$ and killing horizon at this point located as

$$r_{KH} = M(1 + \sqrt{J+1}),$$

where $c_i (i = 1, 2, 3, ...)$ are the coupling constant of the Einstein Aether theory [15]. We reduce the given metric into the Schwarzschild BH by Inserting $c_{13} = \frac{c_{14}}{2}$. BH have an universal horizon behind their killing horizon(r_{KH}), for the co-efficient of aether BH $c_{13} = 0$ or $c_{13} = \frac{c_{14}}{2}$. There are two types of horizons defined in this portion. Universal horizon has the ability to trap particles with arbitrary high velocity and killing horizon is invisible to these particles. We find the location of killing horizon by putting $f(r) = 0$

$$1 - \frac{2M}{r} - J(\frac{M}{r})^2 = 0.$$

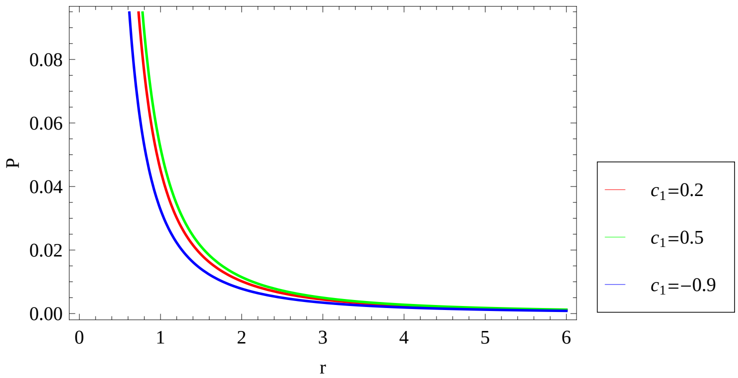

Figure 4.1: Plot of pressure versus horizon radius.

By solving this equations we obtained two solutions of mass M but we take positive mass and use it in further equations.

$$M = \frac{-2r + 2c_{13}r - \sqrt{2}\sqrt{2r^2 - 2c_{13}r^2 - c_{14}r^2 + c_{13}c_{14}r^2}}{2c_{13} - c_{14}}. \tag{4.1.2}$$

We have to find temperature for Einstein Aether BH by substituting derivative of Eq.(4.1.1) in Eq.(3.1.6) , we obtain

$$T = \frac{M[(-2c_{13} + c_{14})M + 2r(-1 + c_{13})]}{4\pi r^3(-1 + c_{13})}, \tag{4.1.3}$$

where mass M is given in Eq.(4.1.2). As earlier, we discussed the logarithmic corrections, then we can obtain the following corrected entropy by using Eq.(3.1.5) and Eq.(4.1.3) in Eq.(3.1.8), we have

$$S = S_0 - \frac{b}{2}\ln[\frac{M^2[(-2c1 + c_2)M + 2r(-1 + c_1)]^2}{16\pi r^4(-1 + c_1)^2}]. \tag{4.1.4}$$

We can calculate the pressure from Eq.(3.1.10) by inserting derivative of volume in Eq.(3.1.5) and derivative of entropy Eq.(4.1.4).

$$P = \frac{M[b((-2c_{13} + c_{14})M + (-1 + c_{13})r) + \pi r^2((-2c_{13} + c_{14})M + 2(-1 + c_{13})r)]}{8\pi^2 r^6(-1 + c_{13})} \tag{4.1.5}$$

From **Fig.(4.1)** we can observe the behavior of pressure for various values of parameters. If we compare $b = 0$ and $b = 1$, we see that due to logarithmic

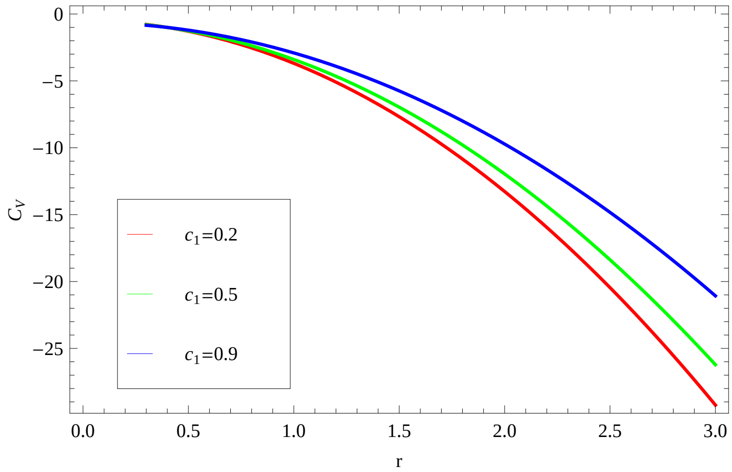

Figure 4.2: Plot of specific heat versus horizon radius.

corrections the value of pressure decreases. We observe that pressure is maximum when the coupling constant $c_{13} > 3.95$. It means that when we increase the coupling constant then the pressure also increases but the range of the horizon for positive pressure decreases.

We studied earlier the stability in above section. Now we have to calculate the specific heat by constant volume and constant pressure by using derivatives of Eq.(4.1.2), Eq.(4.1.3) and Eq.(4.1.4) in Eq.(3.1.12) and Eq.(3.1.14) respectively.

$$\begin{aligned} C_V &= (2(b((-2c_1+c_2)M+(-1+c_1)r)+\pi r^2((-2c_1+c_2)M+ \\ &\quad 2(-1+c_1)r)))(6c_1M-3c_2M+4r-4c_1r)^{-1}. \qquad (4.1.6) \\ C_P &= \frac{2\pi r\sqrt{2}\sqrt{(-1+c_1)(-2+c_2)r^2}}{-2+c_2}. \qquad (4.1.7) \end{aligned}$$

Fig.(4.2) shows that behavior of specific heat at constant volume after the implementation of logarithmic corrections on it. We see clearly that when we increase the value of coupling constant then specific heat decreases locally.

By using above Eq.(4.1.6) and Eq.(4.1.7) we calculate the ratio between

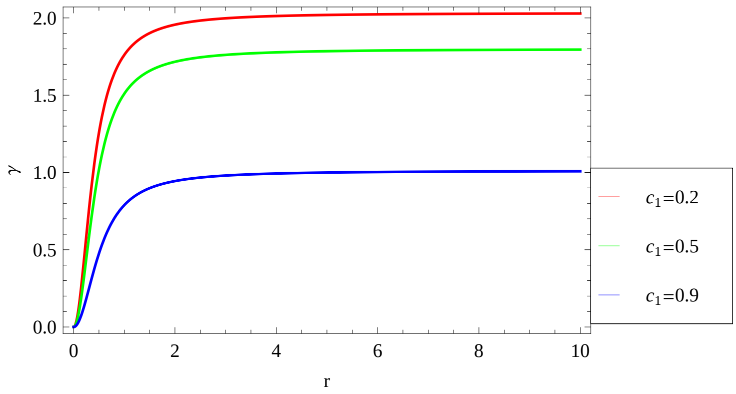

Figure 4.3: Plot of gamma versus horizon radius.

them, and we have

$$\begin{aligned}\gamma &= (2(-1+c_{13})\pi r^2(3(-2+c_{14})r+\sqrt{2}\sqrt{(-1+c_{13})(-2+c_{14})r^2}))\\ &\quad ((((-2+c_{14})(b(-1+c_{13})r+\sqrt{2}b\sqrt{(-1+c_{13})(-2+c_{14})r^2}+\\ &\quad \sqrt{2}\pi r^2\sqrt{(-1+c_{13})(-2+c_{14})r^2}))^{-\frac{1}{2}}.\end{aligned}$$

Fig.(4.3) shows that due to logarithmic corrections the γ increases. We acquire the higher value of γ for coupling constant $c = 0.2$ and $\gamma \to 1.85$ for large horizon. Further we notice $\gamma \to 1.6$ when $c = 0.5$, and fie $c = 0.9$ the $\gamma \to 0.75$. We can say that γ shows stable behavior when for all three cases of coupling constant. Hence, we conclude that if the value of coupling constant decreases then the value corresponding to γ increases and exhibits the more stable behavior.

Grand canonical ensemble is also known as Gibb's free energy and canonical ensemble as Helmholtz free energy as we discussed above sections. By using Eq.(4.1.2), Eq.(4.1.3) and Eq.(4.1.4) in Eq.(3.1.17).

$$G = \frac{1}{4(2c_1-c_2)\pi r^2}(2\pi r^2((-6+4c_1+c_2)r+3\sqrt{2}\sqrt{-1+c_1)(-2+c_2)r^2} \tag{4.1.8}$$

From **Fig.(4.4)** we observe that the behavior of Gibb's free energy for

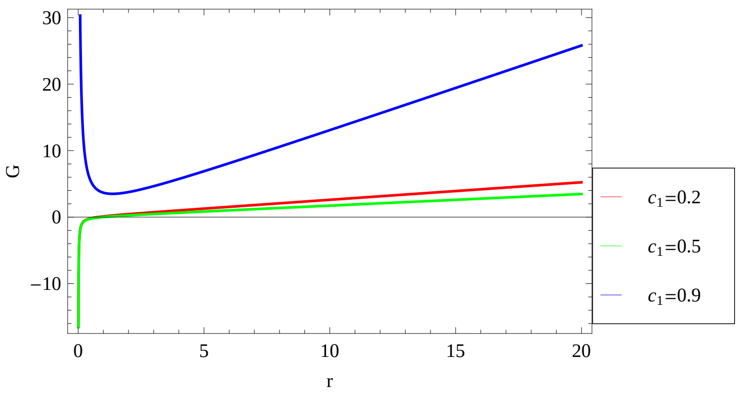

Figure 4.4: Plot of Gibb's free energy versus horizon radius.

special three cases of coupling constant. Due to logarithmic corrections, we see that the Gibb's free energy is minimum at $r \simeq 0.45$, which means that Gibb's free energy is decreased by logarithmic corrections. From the graph, we see that the coupling constant play a vital role to reduce the value of Gibb's free energy. We conclude that when we increase the value of coupling constant, then free energy becomes more globally stable.[7]

For Helmholtz free energy we are using Eq.(4.1.8), Eq.(4.1.5) and Eq.(3.1.5) in Eq.(3.1.19),

$$\begin{aligned} F &= \frac{1}{12\pi r^3}\Big(\frac{-2M}{-1+c_{13}}(b((-2c_{13}+c_{14})M+(-1+c_{13})r)+\pi r^2((-2c_{13} \\ &\quad +c_{14})M+2(-1+c_{13})r)\Big)+\frac{3r}{2c_{13}-c_{14}}(2\pi r^2((-6+4c_{13}+c_{14})r \\ &\quad +3\sqrt{2}\sqrt{(-1+c_{13})(-2+c_{14})r^2})+b((-2+c_{14})r+\sqrt{2}\Big((-1+c_{13}) \\ &\quad (-2+c_{14})r^2\Big)^{\frac{1}{2}}\Big(\ln[4\pi]Log[((-2+c_{14})r+\sqrt{2}((-1+c_{13}) \\ &\quad (-2+c_{14})r^2)^{\frac{1}{2}})((-2c_{13}+c_{14})^2r^2)^{-1}]\Big). \end{aligned} \tag{4.1.9}$$

Fig.(4.5) represent the behavior of Helmholtz free energy for specific values of parameters. We observe that due to logarithmic corrections, the value of free energy reduces for every case. We see that Helmholtz free energy is decreased when we apply logarithmic corrections on it and when the value

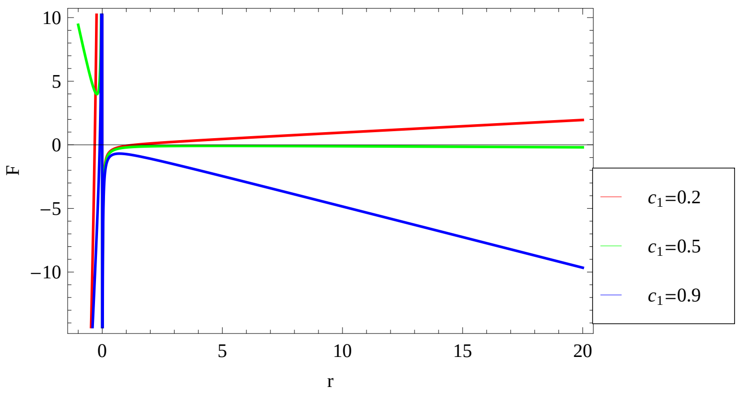

Figure 4.5: Plot of Helmholtz free energy versus horizon radius.

of radius horizon higher. Hence, we conclude that Einstein-aether BH is more thermodynamically stable if the value of coupling constant increases in the large horizon and it becomes thermodynamically unstable for lower values of coupling constant.

Chapter 5

Summary

In this thesis, we have investigated the thermodynamical structure of BH solutions and Einstein-aether black holes with new techniques in the presence of logarithmic corrections. It was shown that the behavior of the temperature and stability conditions for these functions of describing phase transition of these black holes. We discussed the effect of thermal fluctuation on BH solutions and Einstein-aether BH. After the implementation of logarithmic corrections, we have noted the behavior of pressure and specific heats. When we decrease the value of cosmological constant and coupling constant, we observed that the value of pressure also decreases due to logarithmic corrections. We also calculated the ratio of specific heat at constant pressure and specific heat at constant volume γ. We observed that the value of γ increases due to logarithmic correction.

For BH solutions, we have observed that the value of γ increases for the negative value of the cosmological constant. The value of γ decreases when we take the positive cosmological constant, and the value of γ is stable when we take zero cosmological constant. For Einstein Aether BH, we showed that same thing find using thermal fluctuations applying on this BH. In this BH we observe that when we increase the coupling constant ten the value of Pressure, specific heat, Gibb's free Energy, and Helmholtz free energy changed. It is concluded that the both BH solutions and Einstein-aether BH becomes stable as well as if we increase the value of cosmological constant and coupling constant respectively and vice versa.

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