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ABSTRACT

Jacobi Elliptic Solition Solutions for Lakshmanan-Porsezian Daniel (LPD) Model

A wave which propagates without changing its shape and speed is known as soliton. Whenever two of these waves collide they do not change their shape and speed. Solitons have many applications other fields like, fluid, biology and engineering. With the help of different non-linearities the soliton solutions can be calculated, some of the non-linearities are Kerr law, Power law, Parabolic law, Dual-power law, exponential law and logarithmic law.

This dissertation find out the exact Jacobi elliptic periodic and solitary wave solutions which includes triangular periodic solutions, soliton solutions, traveling wave solutions, bell and kink shape solitons using the extended F-expansion method with kerr law, parabolic law and anti cubic law for Lakshmanan Porsezian Daniel model.

Contents

Chapter 1

Introduction and Basic Concepts

Many mathematical models illustrate as a differential equations. The equations involving the derivatives are known as differential equations. These are further classified into linear/nonlinear (ordinary) differential equations and linear/ nonlinear partial differential equations. Linear and nonlinear partial differential equations play the pivotal roles in almost every field of sciences like quantum physics, plasma physics, optical fibers, astrophysics, cosmology, and fluid dynamics etc. Linear partial differential equations are equations for which the dependent variable and its derivatives appear in terms with degree at most one. Anything else is called nonlinear. The most general example of 1^{st} order linear PDE will be,

$$a(x,t)u_t + b(x,t)u_x + c(x,t)u = d(x,t) \tag{1.0.1}$$

Similarly the example of nonlinear PDE is,

$$u_{xx} + 3u_{xy} + u_{yy} + u_x^2 - u = e^{x-y} \tag{1.0.2}$$

It is extremely important to distinguished between the linear and nonlinear PDEs. The linear PDE problems can be solved using different techniques and different methods such as variables separation, homogenous, reducible to homogenous, Fourier se-

ries and Fourier transform, superposition, and Laplace transformation.
Some of the most important and famous PDEs including heat equation,

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} \tag{1.0.3}$$

where, $u(x,t)$ is the temperature of a metal bar, and κ is a constant.
Now the another example is wave equation, in which $u(x,t)$ is the displacement of a string, and t is the time,

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \tag{1.0.4}$$

One of the famous nonlinear partial differential equation (NLPDE) is Korteweg-de Vries equation (KdV) equation. The KdV equation is a (NLPDE) of two real variables, space x and time t.

$$\partial_t \phi + \partial_x^3 \phi - 6\phi \partial_x \phi = 0 \tag{1.0.5}$$

The most popular PDE in quantum physics is Schrödinger equation. The Schrödinger equation describes the state and evolution of quantum mechanical systems. Schrödinger equation is dependent on the physical situation . The most common example is time dependent Schrödinger equation (TDSE). This equation gives a brief detail of a system develop gradually with time.

$$i\hbar \frac{\partial}{\partial t} |\Psi(\mathbf{r},t)\rangle = \hat{H} |\Psi(\mathbf{r},t)\rangle \tag{1.0.6}$$

1.1 Solitons

A wave which travels long distances with little loss of energy or structure is called solitary wave. Solitons are solitary wave pulse solution of nonlinear Schrödinger equation. Soliton maintain its shape when it moves at constant speed and they conserve the energy and shape if two of the soliton waves interact with each other. For most dispersive equations these solitary waves go in various random directions and lose energy due to the radiation. The solitary waves keep their identities with same speed and shape. Soliton was first discovered by John Scott Russell in 1834, [34]. Russell observed the motion of the boat which was moving in the narrow canal and then suddenly the boat was stopped. That time no mass put in water, there he saw some tidal wave. He moved back to his laboratory and did some experiments; finally he came to know about some solitary waves which do not change their characteristics like shape, speed during the propagation, termed as soliton. Solitons are not limited in the water wave, the principle of the universal, solitons are appearing in the many nonlinear system.

1.2 Optical Solitons

The term soliton is used in the fiber optics [25]. Solitons play a vital role in optical Communication system. Optical solitons are light pulses that are localized and during their travel in some host medium they do not disperse or diffract. Optical solitons are of remarkable importance in telecommunication systems. There are so many types of solitons in optics. One of these forms is the **temporal solitons** in fibers that even in case of distances of several hundreds of kilometers hold their temporal width. When the group velocity dispersion is totally counteracted by nonlinear self-phase modulation effects, optical temporal solitons are formed, these are the non-spreading optical pulses. Whenever, there is group velocity dispersion temporal solitons are formed in the optical fibers. It creates impulses having non zero band width and the wire, wave move through refractive index that depends on frequency.

Another form of solitons is the **spatial solitons** that propagate several diffraction lengths but not broadening. If optical beam do not diffract when it propagates in a nonlinear medium, then we get spatial solitons. The diameter of the beam remains unchanged during propagation. Optical soliton plays a vital role in the field of fiber optics and telecommunication [16]. A lot of work has been done for last two decades. A special kind of equation known as the non-linear Schrödinger's equation (NLSE) was analyzed deeply and its integrability was discussed. With the help of NLSE the dynamics of optical solitons through an optical fiber were studied [6, 7, 10, 11, 25] and [32, 33].

1.3 Literature Review

In [3] Bakodah et al. explained bright and dark thirring optical solitons by using improved adomian decomposition method with some constraint conditions. Banaja et al. [4] investigate optical solitons using different nonlinearities in cascaded system. They used improved adomian decomposition scheme to find the optical solitons for NLSE. Biswas et al. [5] used Kerr law nonlinearity to find Thirring soliton. Biswas et al. gave some exciting results of the generalized form of the nonlinear Schrödinger equation. Biswas et al. [7] studied log law nonlinearity for gausson soliton. Biswas et al. [8] used parabolic law nonlinearity for bright and dark soliton. In [9], Biswas et al. apply parabolic and dual power law to originate the dark solitons for generalized nonlinear Schrödinger equation. With the help of Kerr law and power law nonlinearity, Biswas et al. [13] also achieved the cubic-quartic optical solitons solution of the non-linear Schrödinger's equation. Using first integral method, Eslami et al. [20], found optical solitons for the resonant NLSE with time-dependent coefficients. By using dual-power law nonlinearity, Bouzida et al. [14] observed chirped optical soliton in nano optical fibers. Ekici et al. [18] observed the optical solitons defined in the DWDM system. They found out the exact solitons through extended trial equation method. In [19], they found optical soliton with DWDM technology and four-wave mixing solitons. Fermi et al. [21], studied some nonlinear problems. In [23], Jawad et al. found the optical solitons with anti-cubic nonlinearity using three integration schemes. Mirzazadeh et al. settle up analytical study of solitons in the fiber waveguide using power law nonlinearity [30]. They also expose birefringent fibers in optical solitons using nonlinearity of Kerr law applying exp-function method [17]. Rizvi et al. [32] construct the optical 1-soliton solution in dual core fiber with the help of inter modal dispersion. They used some nonlinearities like Kerr and Power law with the help of ansatz approach.

Rizvi et al. [33] achieved the dark and singular 1- solitons in dual core fiber by the help of G'/G expansion scheme. They also listed the existence of soliton solution. Savescu et al. [36] found the optical solitons in magneto-optic wave guides with spatial-temporal dispersion. Savescu et al. [37] also discussed the optical solitons in nonlinear directional couplers with spatio-temporal dispersion. Optical solitons with spatio-temporal dispersion and coupled Hirota equation were found in [38] by Savescu et al. Taghizadeh et al. [39] found the soliton solutions in a-helix proteins. They have studied the nonlinearities in detail and gave a critical analysis for Davydov solitons. Topkara et al. [40] found optical solitons with non-kerr law nonlinearity with inter model dispersion, they used solitary wave ansatz method, was used to find the optical solitons for NLSE with time dependent coefficients. Triki et al. [41] found very exciting new envelope solitons solutions in nonlinear fiber optics which had been explained by using nonlinear model named Gerdjikov-Ivanov model with some constraint conditions. In [42], Younis et al. found optical solitons for ultrashort pulses in nano fibers. Birefringent fibers containing thirring solitons found by Zhou et al [43], that can be explained by vector coupled nonlinear Schrödinger equation with the help of kerr law nonlinearity. Zhou [44] studied nonlinear medium having higher order dispersion to find analytical soliton with different nonlinearities. Zhou also done a great work in magnet-electro-elastic circular rod as a consequences of analytical study of solitons [45]. Zhou at el. [46] presents Bright dark and singular solitons in optical fibers that leads to a new understanding of the complex structure of spatio-temporal dispersion and properties of such systems. They calculate the spatially dependent coefficients of the optical fibers. They [48] also established Biswas-Milovic model in nonlinear optics with analytical study of solitons. In [47] they start up with Biswas-Milovic equation and gave some exciting results in the field of optical solitons.

1.4 Dispersion

The dispersion is the change of the refractive index with the wavelength. Dispersion is the phenomenon that gives you the separation of colors in a prism. It also gives the chromatic aberration in the lenses.

The most common example of dispersion is rainbow, where dispersion causes the white light to be spatially separated into different parts of wavelengths (different colors). Although, dispersion can also have an effect in many other situations: for example, Group Velocity Dispersion (GVD) can cause pulses to propagate through the fiber, resulting in long-range signal attenuation; in addition, cancelation between GVD and non-linear effects leads to Soliton waves.

1.5 Group Velocity Dispersion

The group velocity dispersion is the phenomenon that the group velocity of light in a transparent medium depends on the optical frequency or wavelength. The term can also be used as a defined quantity, i.e. the derivative of the velocity of the inverse group with respect to the angular frequency.

$$\mathrm{GVD}(\omega_0) \equiv \frac{\partial}{\partial\omega}\left(\frac{1}{v_g(\omega)}\right)_{\omega=\omega_0}, \tag{1.5.1}$$

where ω and ω_0 are angular frequencies, and the group velocity $v_g(\omega)$ is defined as

$$v_g(\omega) \equiv \partial\omega/\partial k \tag{1.5.2}$$

The units of group velocity dispersion are $[time]^2$/[distance], often expressed in fs^2/mm. For optical fibers (e.g. in the context of optical fiber communications), the group velocity dispersion is usually defined as a derivative with respect to wavelength (rather than angular frequency). This can be obtained from the GVD parameter mentioned above.

$$\text{GVD}(\omega_0) \equiv \left(\frac{\partial^2 k}{\partial \omega^2}\right)_{\omega=\omega_0} \tag{1.5.3}$$

or in terms of the refractive index $n(\omega)$

$$\text{GVD}(\omega_0) \equiv \frac{2}{c}\left(\frac{\partial n}{\partial \omega}\right)_{\omega=\omega_0} + \frac{\omega_0}{c}\left(\frac{\partial^2 n}{\partial \omega^2}\right)_{\omega=\omega_0} \tag{1.5.4}$$

1.6 Non-Linear Schrödinger's Equation

In Non-linear Fiber Optics [10], optical solitons is the most emerging and significant field of study. During last 3 decades, the research work of this field have brought a revolutionary success. There is an equation of special kind and have great importance, known as the non-linear Schrödinger's equation (NLSE) was analyzed and deeply studied by mathematicians and physicists and its integrability was discussed. The mathematical model of the NLSE is given as:

$$ip_t + ap_{xx} + bF(|p|^2)p = 0.$$

where p is complex and dependents on the variables x and t, it serve as the wave profile. The variables x and t represent spatial and temporal variables respectively. The constant a is the coefficient of dispersion and b is the coefficient of non-linear term.

Some of the famous models of nonlinear Schrödinger's equation are:

1.7 Schrödinger- Hirota equation

The Schrödinger Hirota equation leads the propagation of optical solitons in a dispersive optical fiber. The mathematical form of Schrödinger-Hirota equation is,

$$iu_t + \frac{1}{2}u_{xx} + |u|^2 u = i\lambda u_{xxx} \tag{1.7.1}$$

Where u is the wave profile, λ is the dispersion coefficient of third order . The first term of the left hand side shows the evolution term, the 2^{nd} term is GVD), and the third term represents the Kerr law of nonlinearity that originate when it is dependent the intensity of light on the refractive index of the material [12].

1.8 Biswas- Milovic equation

Biswas Milovic equation is one famous model of this era. A lot of work has been done on it. The mathematical model of Biswas Milovic equation is given as,

$$i(r^m)_t + a(r^m)_{xx} + bF(|r|^2)r^m = 0, \tag{1.8.1}$$

The first term define the evolution of the soliton pulses, while the second term shows group velocity dispersion (GVD) , and the third term is the non Kerr Law media. The parameter m is the generalization of NLSE to BME. i.e, if $m = 1$, then the BME equation condenses to NLSE. We can say that the parameter m shows the error of the model faces with GVD and nonlinearity [7, 11].

1.9 Ginzburg-Landau equation

Ginzburg- Landau equations have been used to model a wide variety of physical systems. The Ginzburg- Landau equation was first derived as long-wave amplitude equation in binary mixtures near the onset of instability. The complex Ginzburg- Landau equation was first derived in the studies of Poiseuille flow and reaction diffusion systems.

$$\partial_T A = A + (1 + i\alpha)\nabla^2 A - (1 + i\beta)|A|^2 A \tag{1.9.1}$$

GLE is one of the most fascinating equations in physics and applied mathematics. This equation give the qualitative results, and often quantitative. This equation gives the interesting results about the evolution of amplitudes of unstable modes [15].

1.10 Sasa-Satsuma equation

The Sasa-Satsuma equation is an integrable perturbation of the nonlinear Schrödinger equation which models the effects of third-order dispersion, self- steepening and stimulated Raman scattering in the propagation of ultra- fast pulses in optical fiber transmission systems.

$$ip_t - p_{xx} - 2\sigma p|p|^2 + i\hat{\alpha}(|p|^2 p)_x + i\hat{\beta}(|p|^2)_x p + i\hat{\gamma} p_{xxx} = 0, \tag{1.10.1}$$

$\hat{\alpha}$ $\hat{\beta}$ $\hat{\gamma}$ are real parameters. $\sigma = \pm 1$ is the parameter that describe the difference between the focusing $(\sigma = 1)$ and the de-focusing $(\sigma = -1)$ cases of the nonlinear Schrödinger equation.

1.11 Types of Solitons

There are numerous types of solitons, some are discussed bellow:

One of the most popular type of soliton is **bright soliton**. The bright solitons are in the form of pulses and they are on a zero background. The refractive index depends on the dispersion area it decreases when frequency increase. The temporal bright solitons occur in this region. While in the normal dispersion region do the dark solitons appear. In optical fiber, nonlinear effects responsible for the formation of soliton. Group velocity dispersion has two different signs it is associated with the bright and dark solitons. In optical fiber, the group velocity dispersion is irregular and a wave has constant amplitude. Due to instability, the wave is unstable and breakdown into a sequence of pulses. These pulses are bright solitons. It has the property of maintaining its shape and speed, even after colliding with other such waves during propagation in local dispersion. These waves are also known as solitary waves and sometimes non-topological waves.

$$P(x,t) = A_1 \sec h^{\rho} \varsigma, \quad where \quad \varsigma = M(x - vt) \tag{1.11.1}$$

In above equations, M and A_1 are known as unconstrained parameters, where A_1 represents the amplitude (which is maximum at $x = 0$ and $t = 0$) M is said to be the inverse width of the soliton and v is basically the velocity of the solitary wave.

Another famous type of soliton is **dark soliton**. Dark solitons are the intensity dips that exist on the background of the continuous-wave. In case of self-focussing the spatial bright solitons exist while the dark solitons do occur in case of self de-focussing non-linearities. Bright solitons do not exist in normal group velocity dispersion in fibers. In this case, waves have constant amplitude. These waves have no variation, stable and localized pulses. These Waves has continuous background and pulses appear only as holes. The waves of these kind are known as dark solitons.

A Dark soliton is a wave packet, carries energy and propagate through the mediums such as solid, liquid, gas and plasma. Dark waves soliton travels at a much higher speed as compare to an ordinary waves, the energy of these waves disappear relatively rapidly with distance. Dark soliton generation is less straight forward than the bright soliton, because the dark solitons have both two requirement like local change in phase and amplitude change of the back ground wave. It was suggested that continuous dark soliton pulse train could be generated by electro-optic modulators. For a long time, there was no method to invent the dark soliton train to form a data type, because the dark soliton rotates his phase twice than the bright soliton phase. Also, on colliding with such waves it merges or partially cancel each other's effect. These waves are also named as shock waves and topological waves. The dark wave form is given by the function of distance x and time t only.

$$P(x,t) = \alpha \tanh^{\rho} \varsigma, \quad where \quad \varsigma = \beta(x - vt), for \rho > 0$$

Here, α and β are called unconstrained parameters and α represents the amplitude (which is maximum at $x = 0$ and $t = 0$) β is said to be the inverse width of the soliton. The dark solitons have fundamental as well as technological importance. Optical dark solitons are stable during the propagation. When temporal solitons are present in the optical fibers, nonlinear effects are weak and most of the case is valid for the cubic NLS equation. Solitons are always stable. Spatial solitons in waveguide or bulk media use the higher power. The nonlinear refractive index shows the non kerr change in real optical materials with the increase of the light power. Dark solitary waves can become unstable in the non kerr materials. A black dark soliton is always stable. A dark soliton has zero power at its center.

1.12 Jacobi Elliptic Function

The Jacobi elliptic functions are of basic elliptic functions, and auxiliary theta functions. These are doubly periodic meromorphic functions on the complex plane. Jacobi elliptic functions are of great and historical importance. Jacobi elliptic function is analogous of the trigonometry functions because they are represented by the similar notation like sn for sin. Some of these functions have direct relevance to some applications like the equation of a pendulum. Jacobi elliptic functions were first introduced by Carl Gustav Jakob Jacobi in 1829. These functions are the standard forms of elliptic functions. These are functions of two variables denoted by cn(u ,k),dn(u ,k) and sn(u ,k). The first variable is given in terms of the amplitude which is written in terms of u and the second variable is the parameter which is the elliptic modulus [**?**].

1.13 Non-Linearities

A linear process can be defined as a process in which the response to the process and the stimulus given to the process are proportional. For example if a worker works twenty percent longer hours, he will expect to cover twenty percent more work. The linear models are very straight forward and simple mathematical models mostly they can be solved without the aid of computers. Before the era of computers the non-linear processes were also attempted by the aid of linear processes.

A process which is opposite to the linear process is known as nonlinear process. Thus a process which is not linear is called a non-linear process. There are many examples of a linear and non linear process. If a worker increases his working hours and works twice hours as compared to his daily routine work hours. There will remain doubts about his working efficiency and there will be more doubts that he will not cover double tasks as compared to his daily routine work tasks. This is an example of a non-linear process. Likewise, there is a common observation that income tax increases rapidly as compared to respective increase in one's income. To control the non-linear effects, simulation approach is used. With this approach the non-linear approach can be modeled effectively as easy as the linear process.

The non-linear effects arise in optical fiber while transmission because of an increase in the data rates, number of wavelengths, transmission lengths and in optical power levels. A new phase of obstacles has been opened because of the fiber non-linearity. The amount of data which is transmitted on an optical fiber has some limitations on being transmitted by these fiber non-linearities. The designers of the system must take some steps for minimizing the restricting (detrimental) effects of these non-linearities and this is possible only when they have knowledge of these limitations.

Another non-linearity is fiber non-linearity which arises because of two mechanisms. Refractive index present in the glass is the first factor. This factor is dependent on the optical energy going through the material. The second factor is the phenomena known as scattering. Among these two the refractive index is the most detrimental mechanism. The refractive index in an optical fiber has the general equation:

$$n = n_0 + n_2 . P / A_e f f \tag{1.13.1}$$

where n_0 is the refractive index of the fiber core at an optical power of low levels, n_2 is the coefficient of the non-linear refractive index $(2.35 \times 10^{-20} m^2/W forsilica)$, P is the power of the optical fiber in Watts, and A_{eff} represent area in square meters of the fiber core.

From the above equation this is to be noted that the non-linearities produced by the power of the refractive index can be eliminated by maximizing the effective area of the fiber A_{eff} and by minimizing the quantity (amount) of the power P. The distorting (detrimental) effects are eliminated by minimizing the power; whereas in the latest (modern) designs of the fibers the maximization of the effective area is the approach which is the most common.

The propagation of the solitons is governed by the famous non-linear Schrödinger's equation (NLSE) through the optical fibers. The NLSE is given by:

$$iq_t + aq_{xx} + bF(|q|^2)q = 0 \tag{1.13.2}$$

where F is an algebraic function of real values, q is the variable which depends upon

x and t, q_t is the partial derivative of q with respect to t, and q_{xx} is the second partial derivative of q with respect to x. Here in $(2.7.2)$ the first term represents the time evolution term, the second term represents the group velocity dispersion and the third term is the reason (cause) of non-linearity. Such equations are sometimes termed as the non-linear evolution equations.

There are many distinct forms of non-linearities of the function F. In this thesis, the main emphasis will be on the distinct forms of the function $F(s)$ during the propagation of the solitons through an optical fiber. The non-linearities are given as follows:

1.13.1 Kerr Law $F(s) = s$

This is the simplest form of non-linearity. This non-linearity is also known as the cubic non-linearity. Through the Inverse Scattering Transform technique the NLSE can be integrated in case of Kerr Law. Most of the optical fibers obey the kerr law of non-Linearity.

1.13.2 Power Law $F(s) = s^p$

In power law, it is necessary take p in the the interval $0 < p < 2$ to avoid the wave collapse. Similarly, to avoid self-focussing singularity it is compulsory that $p \neq 2$. This is the non-linearity that occurs in non-linear plasmas. This non-linearity appears in optics and semiconductors.

1.13.3 Parabolic Law $F(s) = s + \nu s^2$

In case of non-linear interaction between Langmuir waves and electrons, there exists the parabolic law non-linearity. This non-linearity is also known as the cubic-quintic non-linearity.

1.13.4 Dual- Power Law $F(s) = s^p + \xi s^{2p}$

This model describes the saturation of the non-linear refractive index. Using this form of non-linearity many optical non-linearities are formed. When$1 < p < 2$, the solitons of this model become unstable. If we take $p \geq 2$, then these solitons collapse with each other.

1.13.5 Saturation Law $F(s) = \frac{\lambda s}{1+\lambda s}$

For $\lambda > 0$ this law is used to describe the variation of the dielectrics constant present in the gas vapors, the laser beam is propagated [27]. In case of semiconductor-doped fibers in-spite of the Kerr law, the saturation non-linearity is used for modeling the soliton propagation.

1.13.6 Exponential Law $F(s) = \frac{1}{2\lambda}(1 - e^{-2\lambda s})$

Exponential law non-linearity arises in case of unmagnetized, homogenous plasmas and laser-produced plasmas. The coupling equation depicts the slowly varying complex amplitude. If the coupling equation is combined with the plasma motion of low frequency, the saturable law of non-linearity is obtained.

1.13.7 Log Law $F(s) = a\ln(b^2 s)$

This is an important non-linearity which occurs in many different fields of physics. In case of this model there is an advantage which is the absence of the radiation from the periodic soliton; this because of the discrete spectrum of the linearized problem.

1.13.8 Higher Order Polynomial Law $F(s) = s + \nu s^2 + \gamma s^3$

This law of non-linearity is an extended form of the parabolic law, which we explained earlier.

1.13.9 Triple-Power Law $F(s) = s^p + \nu s^{2p} + \gamma s^{3p}$

This non-linearity is an extension of the dual-power law and the generalized form of the higher order polynomial law of non-linearity.

In the following section, we will explain the F-expansion Method.

1.14 The extended F- Expansion Method

This method is very useful tool used in optical soliton to find the exact solutions of the nonlinear evolution equation. By F-expansion method various Jacobi-elliptic solutions are obtained. This method is very short, straightforward and hence it is a powerful method.

We consider a nonlinear partial deferential equation with x and t as independent variables of u:

$$N(u, u_t, u_x, u_{xx}, \ldots) = 0 \tag{1.14.1}$$

Where N is a polynomial in its arguments. Here $u(x,t) = u(\rho)$, where ρ is the wave variable and given as $\rho = x - ct$, c represents the velocity of the wave. Using these PDE is converted to the ordinary differential equation:

$$N(u, -cu^{'}, u^{'}, -cu^{''}, u^{''}, \ldots) = 0 \tag{1.14.2}$$

the solutions of u can be expressed as a finite series in the form

$$u(\rho) = a_o + \sum_{i=1}^{n}(a_i F^i + \frac{b_i}{F^i}) \tag{1.14.3}$$

where a_i ,($i = 0, 1, 2, \ldots, n$), n is the positive integer which can be evaluated by the homogeneous balance method, and $F(\rho)$ satisfies

$$(F'(\rho))^2 = PF^4 + QF^2 + R \tag{1.14.4}$$

Where P, Q, and R are the constants Substituting the above two relations into the ordinary differential equation we get a set of equations upon solving the equations simultaneously we have different solutions like exact, jacobi-elliptic and trigonometric-function solution.

Chapter 2

Jacobi Elliptic Soliton Solutions for Lakshmanan Porsezian Daniel model

2.1 Mathematical Model

The mathematical model of LPD is given as,

$$\begin{aligned} & iq_t + aq_{xx} + bq_{xt} + cF(|q|^2)q \\ = & \sigma q_{xxxx} + \alpha(q_x)^2 q^* + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q_{xx}^* + \delta|q|^4 q \end{aligned} \tag{2.1.1}$$

In the following model $q(x,t)$ shows the complex valued wave function, while x is the space and t is the time. The terms on the left hand side of the LPD model represents the temporal evolution of the nonlinear wave, a represents the GVD, b is the STD, and the functional F is a real-valued algebraic function.

On the other side σ is the fourth order dispersion, and δ is the two-photon absorption. The starting hypothesis is the traveling wave argument given by,

$$q(x,t) = u(\rho)e^{\imath\phi} \tag{2.1.2}$$

$\rho(x,t)$ is the wave profile given as,

$$\rho = x - vt \tag{2.1.3}$$

while $\phi(x,t)$ is the phase component given as,

$$\phi = -\kappa x + \omega t + \theta \tag{2.1.4}$$

By putting Eq. (2.1.2) and its derivatives along with Eq. (2.1.3) and Eq. (2.1.4) into Eq. (2.1.1), we get real and imaginary parts given below,

$$\sigma(\kappa^4 u-6\kappa^2 u''+u'''')+\alpha(uu'^2-\kappa^2 u^3)+\beta(uu'^2-\kappa^2 u^3)+\gamma(-\kappa^2 u^3+u^2 u'')+$$
$$\lambda(-\kappa^2 u^3+u^2 u'')+\delta(u^5)+uw-a(-\kappa^2 u+u'')-b(kwu-u''v)-cF(u^3)=0 \tag{2.1.5}$$

and

$$vu'(1-b\kappa)+u'(4\sigma\kappa^3+2a\kappa-bw)+2\kappa u^2 u'(-\alpha-\gamma+\lambda)-4\sigma\kappa u'''=0 \tag{2.1.6}$$

The linearly independent functions is zero in Eq. (2.1.6), so it gives the constraint conditions,

$$\sigma=0 \tag{2.1.7}$$

$$\lambda=\alpha+\gamma \tag{2.1.8}$$

and consequently the soliton speed,

$$v=\frac{b\omega-2a\kappa}{1-b\kappa} \tag{2.1.9}$$

whenever $1\neq bk$

Consequently Eq. (2.1.5) reduces to

$$u^2 u''(\gamma+\lambda)+uu'^2(\alpha+\beta)-cF(u^3)+\delta(u^5)+\kappa^2 u^3(-\alpha-\beta-\gamma-\lambda)+$$
$$u(\omega+a\kappa^2-b\kappa\omega)+bu''v-au''=0 \tag{2.1.10}$$

2.2 Kerr Law

The Kerr law defined on u is given as,

$$F(u) = u \tag{2.2.1}$$

We will apply the extended $F-$expansion method to the LPD model along with Kerr law nonlinearity,

$$\begin{gathered} iq_t + aq_{xx} + bq_{xt} + c|q|^2 q = \\ \sigma q_{xxxx} + \alpha (q_x)^2 q^* + \beta |q_x|^2 q + \gamma |q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta |q|^4 q \end{gathered} \tag{2.2.2}$$

In this scenario the Eq. (2.1.10) becomes

$$\begin{split} u^2 u''(\gamma + \lambda) + uu'^2(\alpha + \beta) - c(u^3) + \delta(u^5) + \kappa^2 u^3(-\alpha - \beta - \gamma - \lambda) + \\ u(\omega + a\kappa^2 - b\kappa\omega) + bu''\nu - au'' = 0 \end{split} \tag{2.2.3}$$

By extended $F-$expansion method our starting hypothesis is,

$$u = a_o + \sum_{i=1}^{n} [a_i F^i + \frac{b_i}{F^i}] \tag{2.2.4}$$

and,

$$F'^2 = PF^4 + QF^2 + R \tag{2.2.5}$$

In Eq. (2.2.3) balancing the u'' with u^3, and find n = 1, so we assume the solution of the Eq. (2.2.4) as,

$$u = a_o + a_1 F + \frac{b_1}{F} \tag{2.2.6}$$

So the equation (2.2.3) takes the form,

$$-au'' + b\nu u'' + u(\omega + ak^2 - b\kappa w) + k^2u^3(-2\lambda - \beta) + \delta u^5 - cu^3 + uu'^2(\alpha + \beta) + u^2u''(\gamma + \lambda) = 0 \qquad (2.2.7)$$

After solving the above equation we get the set of algebraic equations given below,

$$a_1^3P\beta + 2a_1^3P\lambda + \delta a_1^5 + 2a_1^3P\gamma + a_1^3P\alpha = 0 \qquad (2.2.8)$$

$$4a_oa_1^2P\lambda + a_oa_1^2P\alpha + a_oa_1^2P\beta + 5\delta a_oa_1^4 + 4a_oa_1^2P\gamma = 0 \qquad (2.2.9)$$

$$2a_o^2a_1P\lambda + 4a_1^2b_1P\lambda + a_1^3Q\alpha + a_1^3Q\beta + a_1^3Q\lambda - \kappa^2a_1^3\beta + a_1^3Q\gamma - ca_1^3 - a_1^2b_1P\alpha - a_1^2b_1P\beta - 2\kappa^2a_1^3\lambda + 5\delta a_1^4b_1 + 10\delta a_o^2a_1^3 - 2aa_1P + 4a_1^2b_1P\gamma + 2a_o^2a_1P\gamma + 2b\nu a_1P = 0 \qquad (2.2.10)$$

$$2a_oa_1^2Q\lambda + 4a_oa_1b_1P\lambda - 3\kappa^2a_oa_1^2\beta - 6\kappa^2a_oa_1^2\lambda + a_oa_1^2Q\beta - 2a_oa_1b_1P\alpha - 2a_oa_1b_1P\beta + 10\delta a_o^3a_1^2 - 3ca_oa_1^2 + 4a_oa_1b_1P\gamma + 2a_oa_1^2Q\gamma + a_oa_1^2Q\alpha + 20\delta a_oa_1^3b_1 = 0 \qquad (2.2.11)$$

$$-a_1b_1^2P\alpha + a_1\omega + a_1^3R\beta + 30\delta a_o^2a_1^2b_1 + 5\delta a_o^4a_1 - a_1^2b_1Q\alpha - a_1^2b_1Q\beta + 2a_1b_1^2P\gamma - 3ca_1^2b_1 + a_1a\kappa^2 + 3a_1^2b_1Q\gamma + a_o^2a_1Q\lambda - 3ca_o^2a_1 + 10\delta a_1^3b_1^2 + a_1^3R\alpha - aa_1Q + b\nu a_1Q - 6\kappa^2a_o^2a_1\lambda + a_o^2a_1Q\lambda + 3a_1^2b_1Q\lambda - a_1b_1^2P\beta - a_1b\kappa\omega + 2a_1b_1^2P\lambda - 3\kappa^2a_o^2a_1\beta - 6\kappa^2a_1^2b_1\lambda - 3\kappa^2a_1^2b_1\beta = 0 \qquad (2.2.12)$$

$$4a_oa_1b_1Q\lambda + a_ob_1^2P\alpha + a_ob_1^2P\beta + a_oa_1^2R\alpha + a_oa_1^2R\beta - \kappa^2a_o^3\beta - 2\kappa^2a_o^3\lambda + a_o\omega + \delta a_o^5 -$$
$$ca_o^3 - a_ob\kappa\omega - 2a_oa_1b_1Q\alpha - 2a_oa_1b_1Q\beta + a_oa\kappa^2 + 4a_oa_1b_1Q\gamma - 12\kappa^2a_oa_1b_1\lambda -$$
$$6\kappa^2a_oa_1b_1\beta + 30\delta a_oa_1^2b_1^2 + 20\delta a_o^3a_1b_1 - 6ca_oa_1b_1 = 0 \quad (2.2.13)$$

$$10\delta a_1^2b_1^3 - 3ca_1b_1^2 - ab_1Q + b_1^3P\alpha + a_o^2b_1Q\lambda - a_1b_1^2Q\beta - a_1b_1^2Q\alpha + b_1a\kappa^2 +$$
$$5\delta a_o^4b_1 - a_1^2b_1R\beta + b_1\omega + a_o^2b_1Q\gamma - 3\kappa^2a_1b_1^2\beta - 3ca_o^2b_1 + b_1^3P\beta - 6\kappa^2a_1b_1^2\lambda -$$
$$3\kappa^2a_o^2b_1\beta - b_1b\kappa\omega - a_1^2b_1R\alpha + 2a_1^2b_1R\lambda - 6\kappa^2a_o^2b_1\lambda + 3a_1b_1^2Q\lambda + 3a_1b_1^2Q\gamma +$$
$$30\delta a_o^2a_1b_1^2 + 2a_1^2b_1R\gamma + b\nu b_1Q = 0 \quad (2.2.14)$$

$$2a_ob_1^2Q\lambda + 20\delta a_oa_1b_1^3 - 3ca_ob_1^2 + 2a_ob_1^2Q\gamma + 4a_oa_1b_1R\gamma - 6\kappa^2a_ob_1^2\lambda -$$
$$3\kappa^2a_ob_1^2\beta + a_ob_1^2Q\alpha + 10\delta a_o^3b_1^2 - 2a_oa_1b_1R\alpha - 2a_oa_1b_1R\beta + a_ob_1^2Q\beta + 4a_oa_1b_1R\lambda = 0$$
$$(2.2.15)$$

$$-cb_1^3 - \kappa^2b_1^3\beta - 2ab_1R + b_1^3Q\alpha + 10\delta a_o^2b_1^3 + 5\delta a_1b_1^4 + 2b\nu b_1R +$$
$$b_1^3Q\gamma + 4a_1b_1^2R\gamma + b_1^3Q\beta + 2a_o^2b_1R\gamma + 2a_o^2b_1R\lambda + b_1^3Q\lambda - a_1b_1^2R\alpha -$$
$$a_1b_1^2R\beta + 4a_1b_1^2R\lambda - 2\kappa^2b_1^3\lambda = 0 \quad (2.2.16)$$

$$4a_ob_1^2R\lambda + 5\delta a_ob_1^4 + 4a_ob_1^2R\gamma + a_ob_1^2R\alpha + a_ob_1^2R\beta = 0 \quad (2.2.17)$$

$$2b_1^3R\lambda + \delta b_1^5 + b_1^3R\beta + b_1^3R\alpha + 2b_1^3R\gamma = 0 \quad (2.2.18)$$

Now solving the above equations, we get the following results,

$$a_o = 0 \quad (2.2.19)$$

$$a_1 = \sqrt{\frac{2P\gamma + P\alpha + 2P\lambda + P\beta}{-\delta}}; \delta < 0 \tag{2.2.20}$$

$$b_1 = \sqrt{\frac{R\beta + 2R\gamma + R\alpha + 2R\lambda}{-\delta}}; \delta < 0 \tag{2.2.21}$$

Hence Eq. (2.2.6) becomes

$$u = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{-\delta}}[\sqrt{P}F + \frac{\sqrt{R}}{F}]; \delta < 0 \tag{2.2.22}$$

Thus Eq. (2.1.2) becomes

$$q(x,t) = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{-\delta}}[\sqrt{P}F + \frac{\sqrt{R}}{F}]e^{i\phi} \tag{2.2.23}$$

This equation has many solution which are determined as under,

2.3 Cases

For sine function

1)$P = m^2; R = 1; F = sn\xi$

$$q_1 = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{-\delta}}[msn\xi + \frac{1}{sn\xi}]e^{i\phi} \tag{2.3.1}$$

For cosine function,

2)$P = -m^2; R = 1 - m^2; F = cn\xi$

$$q_2 = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{-\delta}}[\sqrt{-m^2}(cn\xi) + \frac{\sqrt{1 - m^2}}{cn\xi}]e^{i\phi} \tag{2.3.2}$$

For different elliptic functions,

3)$P=-1; R=m^2-1; F=dn\xi$

$$q_3=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{-\delta}}[\sqrt{-1}(dn\xi)+\frac{\sqrt{m^2-1}}{dn\xi}]e^{i\phi} \tag{2.3.3}$$

4)$P=1-m^2; R=-m^2; F=nc\xi$

$$q_4=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{-\delta}}[\sqrt{1-m^2}(nc\xi)+\frac{\sqrt{-m^2}}{nc\xi}]e^{i\phi} \tag{2.3.4}$$

5)$P=m^2-1; R=-1; F=nd\xi$

$$q_5=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{-\delta}}[\sqrt{m^2-1}(nd\xi)+\frac{\sqrt{-1}}{nd\xi}]e^{i\phi} \tag{2.3.5}$$

6)$P=-m^2(1-m^2); R=2m^2-1; F=sd\xi$

$$q_6=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{-\delta}}[\sqrt{-m^2(1-m^2)}(sd\xi)+\frac{\sqrt{2m^2-1}}{sd\xi}]e^{i\phi} \tag{2.3.6}$$

7)$P=1; R=1-m^2; F=cs\xi$

$$q_7=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{-\delta}}[\sqrt{1}(cs\xi)+\frac{\sqrt{1-m^2}}{cs\xi}]e^{i\phi} \tag{2.3.7}$$

8)$P=1; R=-m^2(1-m^2); F=ds\xi$

$$q_8=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{-\delta}}[\sqrt{1}(ds\xi)+\frac{\sqrt{-m^2(1-m^2)}}{ds\xi}]e^{i\phi} \tag{2.3.8}$$

We can compute many triangular and periodic solutions if we take modulus $m\to 0$, if the modulus $m\to 1$ bell and kink type soliton solutions can be found, because if we take $m\to 0$, Jacobian elliptic functions degenerate to

$sn\xi\to sin\xi$, $cn\xi\to cos\xi$, $dn\xi\to 1$

if $m\to 1$, The jacobian -elliptic functions degenerate to the hyperbolic functions

$sn\xi\to tanh\xi$, $dn\xi\to sech\xi$, $cn\xi\to sech\xi$,

here sech and tanh are hyperbolic secant and hyperbolic tangent function, respectively.

2.4 Parabolic Law

For parabolic law nonlinearity the functional F is given as,

$$F(u) = u + \eta u^2 \tag{2.4.1}$$

Now again we will apply the extended F-expansion method to the LPD model along with parabolic law nonlinearity,

$$\begin{aligned} & iq_t + aq_{xx} + bq_{xt} + c(|q|^2 + \eta|q|^4)q \\ & \quad = \sigma q_{xxxx} + \alpha(q_x)^2 q^* + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta|q|^4 q \end{aligned} \tag{2.4.2}$$

After solving the above equation, and using the parabolic law in Eq. (2.1.10), the Eq. (2.1.10) reduces to,

$$\begin{aligned} & u^2 u''(\gamma+\lambda) + uu'^2(\alpha+\beta) - c(u^3 + \eta u^5) + \delta(u^5) + \kappa^2 u^3(-2\lambda-\beta) + \\ & \quad u(\omega + a\kappa^2 - b\kappa\omega) + bu''\nu - au'' = 0 \end{aligned} \tag{2.4.3}$$

By homogenous balancing method, we get n = 1, so we assume the solution of the Eq. (2.2.4) as,

$$u = a_o + a_1 F + \frac{b_1}{F} \tag{2.4.4}$$

After solving the Eq. (2.4.3) we get the set of algebraic equations,

$$a_1^3 P\beta + 2a_1^3 P\lambda + \delta a_1^5 + 2a_1^3 P\gamma + a_1^3 P\alpha - c\eta a_1^5 = 0 \tag{2.4.5}$$

$$4a_oa_1^2P\lambda + a_oa_1^2P\alpha + a_oa_1^2P\beta + 5\delta a_oa_1^4 + 4a_oa_1^2P\gamma - 5cetaa_oa_1^4 = 0 \qquad (2.4.6)$$

$$\begin{aligned}&2a_o^2a_1P\lambda + 4a_1^2b_1P\lambda + a_1^3Q\alpha + a_1^3Q\beta + a_1^3Q\lambda - \kappa^2a_1^3\beta + a_1^3Q\gamma - ca_1^3 - a_1^2b_1P\alpha - \\ &a_1^2b_1P\beta - 2\kappa^2a_1^3\lambda + 5\delta a_1^4b_1 + 10\delta a_o^2a_1^3 - 2aa_1P + 4a_1^2b_1P\gamma + 2a_o^2a_1P\gamma + 2b\nu a_1P - \\ &5c\eta a_1^4b_1 - 10c\eta a_o^2a_1^3 = 0 \qquad (2.4.7)\end{aligned}$$

$$\begin{aligned}&2a_oa_1^2Q\lambda + 4a_oa_1b_1P\lambda - 3\kappa^2a_oa_1^2\beta - 6\kappa^2a_oa_1^2\lambda + a_oa_1^2Q\beta - 2a_oa_1b_1P\alpha - 2a_oa_1b_1P\beta + \\ &10\delta a_o^3a_1^2 - 3ca_oa_1^2 + 4a_oa_1b_1P\gamma + 2a_oa_1^2Q\gamma + a_oa_1^2Q\alpha + 20\delta a_oa_1^3b_1 - 20c\eta a_oa_1^3b_1 - 10c\eta a_o^3a_1^2 = 0\end{aligned} \qquad (2.4.8)$$

$$\begin{aligned}&-a_1b_1^2P\alpha + a_1\omega + a_1^3R\beta + 30\delta a_o^2a_1^2b_1 + 5\delta a_o^4a_1 - a_1^2b_1Q\alpha - a_1^2b_1Q\beta + 2a_1b_1^2P\gamma - \\ &3ca_1^2b_1 + a_1a\kappa^2 + 3a_1^2b_1Q\gamma + a_o^2a_1Q\lambda - 3ca_o^2a_1 + 10\delta a_1^3b_1^2 + a_1^3R\alpha - aa_1Q + \\ &b\nu a_1Q - 6\kappa^2a_o^2a_1\lambda + a_o^2a_1Q\lambda + 3a_1^2b_1Q\lambda - a_1b_1^2P\beta - a_1b\kappa\omega + 2a_1b_1^2P\lambda - \\ &3\kappa^2a_o^2a_1\beta - 6\kappa^2a_1^2b_1\lambda - 3\kappa^2a_1^2b_1\beta - 30c\eta a_o^2a_1^2b_1 - 10c\eta a_1^3b_1^2 - 5c\eta a_o^4a_1 = 0\end{aligned} \qquad (2.4.9)$$

$$\begin{aligned}&4a_oa_1b_1Q\lambda + a_ob_1^2P\alpha + a_ob_1^2P\beta + a_oa_1^2R\alpha + a_oa_1^2R\beta - \kappa^2a_o^3\beta - 2\kappa^2a_o^3\lambda + a_o\omega + \delta a_o^5 - \\ &ca_o^3 - a_ob\kappa\omega - 2a_oa_1b_1Q\alpha - 2a_oa_1b_1Q\beta + a_oa\kappa^2 + 4a_oa_1b_1Q\gamma - 12\kappa^2a_oa_1b_1\lambda - 6\kappa^2a_oa_1b_1\beta + \\ &30\delta a_oa_1^2b_1^2 + 20\delta a_o^3a_1b_1 - 6ca_oa_1b_1 - 20c\eta a_o^3a_1b_1 - 30c\eta a_oa_1^2b_1^2 - c\eta a_o^5 = 0\end{aligned} \qquad (2.4.10)$$

$$10\delta a_1^2 b_1^3 - 3ca_1b_1^2 - ab_1Q + b_1^3P\alpha + a_o^2b_1Q\lambda - a_1b_1^2Q\beta - a_1b_1^2Q\alpha + b_1a\kappa^2 +$$
$$5\delta a_o^4 b_1 - a_1^2b_1R\beta + b_1\omega + a_o^2b_1Q\gamma - 3\kappa^2a_1b_1^2\beta - 3ca_o^2b_1 + b_1^3P\beta - 6\kappa^2a_1b_1^2\lambda -$$
$$3\kappa^2a_o^2b_1\beta - b_1b\kappa\omega - a_1^2b_1R\alpha + 2a_1^2b_1R\lambda - 6\kappa^2a_o^2b_1\lambda + 3a_1b_1^2Q\lambda + 3a_1b_1^2Q\gamma +$$
$$30\delta a_o^2a_1b_1^2 + 2a_1^2b_1R\gamma + b\nu b_1Q - 10c\eta a_1^2b_1^3 - 5c\eta a_o^4b_1 - 30c\eta a_o^2a_1b_1^2 = 0 \quad (2.4.11)$$

$$2a_ob_1^2Q\lambda + 20\delta a_oa_1b_1^3 - 3ca_ob_1^2 + 2a_ob_1^2Q\gamma + 4a_oa_1b_1R\gamma - 6\kappa^2a_ob_1^2\lambda - 3\kappa^2a_ob_1^2\beta + a_ob_1^2Q\alpha +$$
$$10\delta a_o^3b_1^2 - 2a_oa_1b_1R\alpha - 2a_oa_1b_1R\beta + a_ob_1^2Q\beta + 4a_oa_1b_1R\lambda - 20c\eta a_oa_1b_1^3 - 10c\eta a_o^3b_1^2 = 0$$
$$(2.4.12)$$

$$-cb_1^3 - \kappa^2b_1^3\beta - 2ab_1R + b_1^3Q\alpha + 10\delta a_o^2b_1^3 + 5\delta a_1b_1^4 + 2b\nu b_1R +$$
$$b_1^3Q\gamma + 4a_1b_1^2R\gamma + b_1^3Q\beta + 2a_o^2b_1R\gamma + 2a_o^2b_1R\lambda + b_1^3Q\lambda - a_1b_1^2R\alpha -$$
$$a_1b_1^2R\beta + 4a_1b_1^2R\lambda - 2\kappa^2b_1^3\lambda - 5c\eta a_1b_1^4 - 10c\eta a_o^2b_1^3 = 0 \quad (2.4.13)$$

$$4a_ob_1^2R\lambda + 5\delta a_ob_1^4 + 4a_ob_1^2R\gamma + a_ob_1^2R\alpha + a_ob_1^2R\beta - 5c\eta a_ob_1^4 = 0 \quad (2.4.14)$$

$$2b_1^3R\lambda + \delta b_1^5 + b_1^3R\beta + b_1^3R\alpha + 2b_1^3R\gamma - c\eta b_1^5 = 0 \quad (2.4.15)$$

After solving the above equations, we get the solutions,

$$a_o = 0 \quad (2.4.16)$$

$$a_1 = \sqrt{\frac{2P\gamma + P\alpha + 2P\lambda + P\beta}{c\eta - \delta}}; c\eta \geq \delta \quad (2.4.17)$$

$$b_1 = \sqrt{\frac{R\beta + 2R\gamma + R\alpha + 2R\lambda}{c\eta - \delta}}; c\eta \geq \delta \tag{2.4.18}$$

putting a_0 , a_1, and b_1 in Eq. (2.4.4), we get,

$$u = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{c\eta - \delta}}[\sqrt{P}F + \frac{\sqrt{R}}{F}]; c\eta \geq \delta \tag{2.4.19}$$

Thus Eq. (2.1.2) becomes,

$$q(x,t) = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{c\eta - \delta}}[\sqrt{P}F + \frac{\sqrt{R}}{F}]e^{i\phi} \tag{2.4.20}$$

This equation has many solution which are determined as under,

2.5 Cases

For sine function

1)$P = m^2; R = 1; F = sn\xi$

$$q_1 = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{c\eta - \delta}}[msn\xi + \frac{1}{sn\xi}]e^{i\phi} \tag{2.5.1}$$

For cosine function

2)$P = -m^2; R = 1 - m^2; F = cn\xi$

$$q_2 = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{c\eta - \delta}}[\sqrt{-m^2}(cn\xi) + \frac{\sqrt{1 - m^2}}{cn\xi}]e^{i\phi} \tag{2.5.2}$$

For different elliptic functions,

3)$P = -1; R = m^2 - 1; F = dn\xi$

$$q_3 = \sqrt{\frac{\alpha + \beta + 2\gamma + 2\lambda}{c\eta - \delta}}[\sqrt{-1}(dn\xi) + \frac{\sqrt{m^2 - 1}}{dn\xi}]e^{i\phi} \tag{2.5.3}$$

$4)P=1-m^2;R=-m^2;F=nc\xi$

$$q_4=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{c\eta-\delta}}[\sqrt{1-m^2}(nc\xi)+\frac{\sqrt{-m^2}}{nc\xi}]e^{i\phi} \tag{2.5.4}$$

$5)P=m^2-1;R=-1;F=nd\xi$

$$q_5=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{c\eta-\delta}}[\sqrt{m^2-1}(nd\xi)+\frac{\sqrt{-1}}{nd\xi}]e^{i\phi} \tag{2.5.5}$$

$6)P=-m^2(1-m^2);R=2m^2-1;F=sd\xi$

$$q_6=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{c\eta-\delta}}[\sqrt{-m^2(1-m^2)}(sd\xi)+\frac{\sqrt{2m^2-1}}{sd\xi}]e^{i\phi} \tag{2.5.6}$$

$7)P=1;R=1-m^2;F=cs\xi$

$$q_7=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{c\eta-\delta}}[\sqrt{1}(cs\xi)+\frac{\sqrt{1-m^2}}{cs\xi}]e^{i\phi} \tag{2.5.7}$$

$8)P=1;R=-m^2(1-m^2);F=ds\xi$

$$q_8=\sqrt{\frac{\alpha+\beta+2\gamma+2\lambda}{c\eta-\delta}}[\sqrt{1}(ds\xi)+\frac{\sqrt{-m^2(1-m^2)}}{ds\xi}]e^{i\phi} \tag{2.5.8}$$

We can compute many triangular and periodic solutions if we take modulus $m\to 0$, if the modulus $m\to 1$ bell and kink type soliton solutions can be found, because if we take $m\to 0$, Jacobian elliptic functions degenerate to

$sn\xi\to sin\xi$, $cn\xi\to cos\xi$, $dn\xi\to 1$

if $m\to 1$, The jacobian -elliptic functions degenerate to the hyperbolic functions

$sn\xi\to tanh\xi$, $dn\xi\to sech\xi$, $cn\xi\to sech\xi$,

here sech and tanh are hyperbolic secant and hyperbolic tangent function, respectively.

2.6 Anti Cubic law

The anti cubic nonlinearity is defined as,

$$F(u) = \mu_1 u^{-2} + \mu_2 u + \mu_3 u^2 \tag{2.6.1}$$

We use extended $F-$expansion method to solve LPD model along with anti cubic nonlinearity,

$$\begin{aligned} iq_t + aq_{xx} + bq_{xt} + c|q|^2 q = \\ \sigma q_{xxxx} + \alpha (q_x)^2 q^* + \beta |q_x|^2 q + \gamma |q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta |q|^4 q \end{aligned} \tag{2.6.2}$$

In this scenario the Eq. (2.1.10) becomes

$$\begin{aligned} u^2 u''(\gamma+\lambda) + uu'^2(\alpha+\beta) - c(\mu_1 u^{-3} + \mu_2 2u^3 + \mu_3 u^5) + \delta(u^5) + \kappa^2 u^3(-\alpha-\beta-\gamma-\lambda) + \\ u(\omega + a\kappa^2 - b\kappa\omega) + bu''v - au'' = 0 \end{aligned} \tag{2.6.3}$$

Since

$$u = a_o + \sum_{i=1}^{n} [a_i F^i + \frac{b_i}{F^i}] \tag{2.6.4}$$

Again we get n = 1 by homogenous balancing method on Eq. (2.6.3), so we assume the solution of the Eq. (2.6.4) as,

$$u = a_o + a_1 F + \frac{b_1}{F} \tag{2.6.5}$$

After solving the equation (2.6.3) we get the set of algebraic equations which yield the solutions ,

2.7 Case 1

$$a_o = 0, b_1 = 0 \tag{2.7.1}$$

$$a_1 = \sqrt{\frac{2P\gamma + P\alpha + 2P\lambda + P\beta}{c\mu_3 - \delta}}; c\mu_3 > \delta \tag{2.7.2}$$

Hence Eq. (2.6.5) takes the form,

$$u = \sqrt{\frac{P(2\gamma + 2\lambda + \alpha + \beta)}{c\mu_3 - \delta}}; c\mu_3 > \delta, \tag{2.7.3}$$

Thus Eq. (2.1.2) becomes,

$$q(x,t) = \sqrt{\frac{P(2\gamma + 2\lambda + \alpha + \beta)}{c\mu_3 - \delta}} e^{\imath\phi} \tag{2.7.4}$$

2.8 Case 2

$$a_1 = 0, b_1 = b_1 \tag{2.8.1}$$

and,

$$a_0 = \sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} \tag{2.8.2}$$

Hence Eq. (2.6.5) takes the form,

$$u = \sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{F} \tag{2.8.3}$$

Thus Eq. (2.1.2) becomes,

$$q(x,t) = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{F})e^{i\phi} \tag{2.8.4}$$

Eq. (2.7.4)and Eq. (2.8.4) have many solutions which are determined as under,

2.9 Case 1

For sine function

1)$P = m^2; F = sn\xi$

$$q_1 = m[\sqrt{\frac{(2\gamma + 2\lambda + \alpha + \beta)}{c\mu_3 - \delta}}]sn\xi e^{i\phi} \tag{2.9.1}$$

For cosine function,

2)$P = -m^2; F = cn\xi$

$$q_2 = m[\sqrt{\frac{(2\gamma + 2\lambda + \alpha + \beta)}{\delta - c\mu_3}}]cn\xi e^{i\phi} \tag{2.9.2}$$

for different elliptic functions,

3)$P = -1; F = dn\xi$

$$q_3 = [\sqrt{\frac{(2\gamma + 2\lambda + \alpha + \beta)}{\delta - c\mu_3}}]cn\xi e^{i\phi} \tag{2.9.3}$$

4)$P=1-m^2; F=nc\xi$

$$q_4=[\sqrt{\frac{(1-m^2)(2\gamma+2\lambda+\alpha+\beta)}{c\mu_3-\delta}}]nc\xi e^{i\phi} \tag{2.9.4}$$

5)$P=m^2-1; F=nd\xi$

$$q_5=[\sqrt{\frac{(m^2-1)(2\gamma+2\lambda+\alpha+\beta)}{c\mu_3-\delta}}]nd\xi e^{i\phi} \tag{2.9.5}$$

6)$P=-m^2(1-m^2); F=sd\xi$

$$q_6=[\sqrt{\frac{-m^2(1-m^2)(2\gamma+2\lambda+\alpha+\beta)}{c\mu_3-\delta}}]nd\xi e^{i\phi} \tag{2.9.6}$$

7)$P=1; F=cs\xi$

$$q_7=\sqrt{\frac{(2\gamma+2\lambda+\alpha+\beta)}{c\mu_3-\delta}}cs\xi e^{i\phi} \tag{2.9.7}$$

8)$P=1; F=ds\xi$

$$q_8=\sqrt{\frac{(2\gamma+2\lambda+\alpha+\beta)}{c\mu_3-\delta}}ds\xi e^{i\phi} \tag{2.9.8}$$

2.10 Case 2

For sine function

1) $F=sn\xi$

$$q_1=(\sqrt{\frac{b\nu-a}{3\alpha+3\beta+10\gamma+10\lambda}}+\frac{b_1}{sn\xi})e^{i\phi} \tag{2.10.1}$$

For cosine function,

2)$F = cn\xi$

$$q_2 = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{cn\xi})e^{i\phi} \tag{2.10.2}$$

for different elliptic functions,

3)$F = dn\xi$

$$q_3 = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{dn\xi})e^{i\phi} \tag{2.10.3}$$

4)$F = nc\xi$

$$q_4 = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{nc\xi})e^{i\phi} \tag{2.10.4}$$

5)$F = nd\xi$

$$q_5 = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{nd\xi})e^{i\phi} \tag{2.10.5}$$

6)$F = sd\xi$

$$q_6 = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{sd\xi})e^{i\phi} \tag{2.10.6}$$

7)$F = cs\xi$

$$q_7 = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{cs\xi})e^{i\phi} \tag{2.10.7}$$

8)$F = ds\xi$

$$q_8 = (\sqrt{\frac{b\nu - a}{3\alpha + 3\beta + 10\gamma + 10\lambda}} + \frac{b_1}{ds\xi})e^{i\phi} \tag{2.10.8}$$

We can compute many triangular and periodic solutions if we take modulus $m \to 0$, if the modulus $m \to 1$ bell and kink type soliton solutions can be found, because if we take $m \to 0$, Jacobian elliptic functions degenerate to

$sn\xi \to sin\xi$, $cn\xi \to cos\xi$, $dn\xi \to 1$

if $m \to 1$, The jacobian -elliptic functions degenerate to the hyperbolic functions

$sn\xi \to tanh\xi$, $dn\xi \to sech\xi$, $cn\xi \to sech\xi$,

here sech and tanh are hyperbolic secant and hyperbolic tangent function, respectively.

Chapter 3

Optical Solitons in Dual-Core Fibers with Inter-Modal Dispersion

3.1 The Model

The mathematical illustration for decoupled NLSE of optical solitons in dual core fibers with inter modal dispersion is given as,

$$i(\frac{\partial\psi_1}{\partial t}+a_1\frac{\partial\psi_2}{\partial x})+b_1\frac{\partial^2\psi_1}{\partial x_2}+c_1\frac{\partial^2\psi_1}{\partial x\partial t}+d_1F(|\psi_1|^2)\psi_1+k_1\psi_2=0 \tag{3.1.1}$$

$$i(\frac{\partial\psi_2}{\partial t}+a_2\frac{\partial\psi_1}{\partial x})+b_2\frac{\partial^2\psi_2}{\partial x_2}+c_2\frac{\partial^2\psi_2}{\partial x\partial t}+d_2F(|\psi_2|^2)\psi_2+k_2\psi_1=0 \tag{3.1.2}$$

In the above coupled model ψ_1 and ψ_2 represents the field envelopes, x is the propagation co-ordinate, a_j are group velocities , and b_j are group velocity dispersion. While c_j and k_j represents spatio-temporal dispersion and linear coupling coefficients respectively, for $j=1,2$. The functional F is real-valued algebraic function.

$$\psi_j(x,t)=u_j(\rho)e^{i\phi}, where\, j=1,2 \tag{3.1.3}$$

where $\rho(x,t)$ represents the shape of the wave profile,

$$\rho=x-vt \tag{3.1.4}$$

and the phase component $\phi(x,t)$ is defined as

$$\phi=-\kappa x+\omega t+\theta \tag{3.1.5}$$

with κ representing the soliton frequency, and ω the wave number, while θ is the phase constant.

Now we will find all the derivatives and terms involving Eq. (3.1.1) and Eq. (3.1.2).

First we will find the derivative of ψ with respect of t,

$$\frac{\partial \psi_1}{\partial t} = (i\omega u_1 - \nu u_1')e^{i\phi} \tag{3.1.6}$$

then the derivative of ψ with respect of x,

$$\frac{\partial \psi_1}{\partial x} = (-i\kappa u_1 + u_1')e^{i\phi} \tag{3.1.7}$$

Then,

$$\frac{\partial^2 \psi_1}{\partial x^2} = (-\kappa^2 u_1 - 2\imath\kappa u_1' + u_1'')e^{\imath\phi} \tag{3.1.8}$$

$$\frac{\partial \psi_1}{\partial x \partial t} = (\kappa\omega u_1 + \imath\omega u_1' + \imath\kappa u_1'\nu - u_1''\nu)e^{\imath\phi} \tag{3.1.9}$$

After putting all the derivatives into Eq. (3.1.1) and Eq. (3.1.2), we get

$$\begin{aligned} &-\omega u_1 - iu_1\nu + a_1\kappa u_2 + ia_1u_2' - b_1\kappa^2 u_1 - 2ib_1\kappa u_1' + b_1u_1'' + \\ &c_1\kappa\omega u_1 + ic_1\omega u_1' + ic_1\kappa u_1'\nu - c_1u_1''\nu + d_1F(u_1{}^2)u_1 + \kappa_1 u_2 = 0 \end{aligned} \tag{3.1.10}$$

and

$$\begin{aligned} &-\omega u_2 - iu_2\nu + a_2\kappa u_1 + ia_2u_1' - b_2\kappa^2 u_2 - 2ib_2\kappa u_2' + b_2u_2'' + \\ &c_2\kappa\omega u_2 + ic_2\omega u_2' + ic_2\kappa u_2'\nu - c_2u_2''\nu + d_2F(u_2{}^2)u_2 + \kappa_2 u_1 = 0 \end{aligned} \tag{3.1.11}$$

By decomposing into real and imaginary parts, The real part equations for the two components are,

$$-\omega u_1 + a_1\kappa u_2 - b_1\kappa^2 u_1 + b_1u_1'' + c_1\kappa\omega u_1 - c_1u_1''\nu + d_1F(u_1{}^2)u_1 + \kappa_1 u_2 = 0 \tag{3.1.12}$$

and

$$- \omega u_2 + a_2 \kappa u_1 - b_2 \kappa^2 u_2 + b_2 u_2'' + c_2 \kappa \omega u_2 - c_2 u_2'' v + d_2 F(u_2{}^2) u_2 + \kappa_2 u_1 = 0 \tag{3.1.13}$$

The imaginary part equations for the two components lead to the velocity of the solitons,

$$v = \frac{a_1 + \omega c_1 - 2\kappa b_1}{1 - c_1 \kappa} \tag{3.1.14}$$

and

$$v = \frac{a_2 + \omega c_2 - 2\kappa b_2}{1 - c_2 \kappa} \tag{3.1.15}$$

whenever $1 \neq c_1 \kappa$ and $1 \neq c_2 \kappa$

We will apply extended $F-$expansion method along with Kerr law nonlinearity in Eq. (3.1.12) and Eq. (3.1.13) respectively,

3.2 Kerr Law

For Kerr law nonlinearity the functional F is given by,

$$F(u) = u \tag{3.2.1}$$

which transform the Eq. (3.1.12) and Eq. (3.1.13) into,

$$-\omega u_1 + a_1 \kappa u_2 - b_1 \kappa^2 u_1 + b_1 u_1'' + c_1 \kappa \omega u_1 - c_1 u_1'' v + d_1 u_1{}^3 + \kappa_1 u_2 = 0 \tag{3.2.2}$$

and

$$-\omega u_2 + a_2\kappa u_1 - b_2\kappa^2 u_2 + b_2 u_2'' + c_2\kappa\omega u_2 - c_2 u_2''\nu + d_2 {u_2}^3 + \kappa_2 u_1 = 0 \quad (3.2.3)$$

By extended $F-$expansion method the starting hypothesis is,

$$u = a_o + \sum_{i=1}^{n} [a_i F^i + \frac{b_i}{F^i}] \quad (3.2.4)$$

and,

$$F'^2 = PF^4 + QF^2 + R \quad (3.2.5)$$

In Eq. (3.2.2) and (3.2.3) balancing the u'' with u^3, we get n = 1, so we assume as,

$$u_1 = \alpha_o + \alpha_1 F + \frac{\gamma_1}{F} \quad (3.2.6)$$

and

$$u_2 = \beta_0 + \beta_1 F + \frac{\gamma_2}{F} \quad (3.2.7)$$

After solving the Eq. (3.2.2) we get the set of algebraic equations,

$$-d_1\alpha_1^3 - 2b_1\alpha_1 P + 2c_1\nu\alpha_1 P = 0 \quad (3.2.8)$$

$$-3d_1\alpha_o\alpha_1^2 = 0 \quad (3.2.9)$$

$$-b_1\alpha_1 Q + b_1 k^2\alpha_1 - 3d_1\alpha_o^2\alpha_1 - 3d_1\alpha_1^2\gamma_1 - \kappa a_1\beta_1 - c_1 k\omega\alpha_1 + c_1\nu\alpha_1 Q + \omega\alpha_1 - (-d_1\alpha_1^3 - 2b_1\alpha_1 P + 2c_1\nu\alpha_1 P)\beta_1 = 0 \quad (3.2.10)$$

$$\omega\alpha_o + b_1k^2\alpha_o - \kappa a_1\beta_0 - d_1\alpha_o^3 - c_1\kappa\omega\alpha_o - 6d_1\alpha_o\alpha_1\gamma_1$$
$$-(-d_1\alpha_1^3 - 2b_1\alpha_1P + 2c_1\nu\alpha_1P)\beta_0 = 0 \tag{3.2.11}$$

$$-b_1\alpha_1Q - c_1\kappa\omega\gamma_1 + \omega\gamma_1 - \kappa a_1\gamma_2 + b_1\kappa^2\gamma_1 - 3d_1\alpha_o^2\gamma_1 -$$
$$3d_1\alpha_1\gamma_1^2 + c_1\nu\alpha_1Q - (-d_1\alpha_1^3 - 2b_1\alpha_1P + 2c_1\nu\alpha_1P)\gamma_2 = 0 \tag{3.2.12}$$

$$-3d_1\alpha_o\gamma_1^2 = 0 \tag{3.2.13}$$

$$d_1\gamma_1^3 - 2b_1\alpha_1R + 2c_1\nu\alpha_1R = 0 \tag{3.2.14}$$

Now solving the above equations simultaneously we get,

$$\alpha_o = 0 \tag{3.2.15}$$

$$\alpha_1 = \sqrt{\frac{2P(b_1 - c_1\nu)}{d_1}} \tag{3.2.16}$$

$$\gamma_1 = \sqrt[3]{\frac{2\alpha_1R(b_1 - c_1\nu)}{d_1}} \tag{3.2.17}$$

Thus Eq. (3.2.6) takes the form,

$$u_1 = \sqrt{\frac{2P(b_1 - c_1\nu)}{d_1}}F + \sqrt[3]{\frac{2\alpha_1R(b_1 - c_1\nu)}{d_1}}\frac{1}{F} \tag{3.2.18}$$

Following the same procedure we find u_2 which is,

$$u_2 = \sqrt{\frac{2P(b_2 - c_2\nu)}{d_2}}F + \sqrt[3]{\frac{2\beta_1 R(b_2 - c_2\nu)}{d_2}}\frac{1}{F} \tag{3.2.19}$$

By using Eq. (3.2.18) and Eq. (3.2.19) into Eq. (3.1.3), we get

$$\psi_1 = (\sqrt{\frac{2P(b_1 - c_1\nu)}{d_1}}F + \sqrt[3]{\frac{2\alpha_1 R(b_1 - c_1\nu)}{d_1}}\frac{1}{F})e^{i\phi} \tag{3.2.20}$$

and

$$\psi_2 = (\sqrt{\frac{2P(b_2 - c_2\nu)}{d_2}}F + \sqrt[3]{\frac{2\beta_1 R(b_2 - c_2\nu)}{d_2}}\frac{1}{F})e^{i\phi} \tag{3.2.21}$$

Eq. (3.2.20) and Eq. (3.2.21) have many solutions which are determined as under,

3.3 Cases

For sine function,

1)$P = m^2; R = 1; F = sn\xi$

$$\psi_1 = m(\sqrt{\frac{2(b_1 - c_1\nu)}{d_1}}(sn\xi) + \sqrt[3]{\frac{2\alpha_1(b_1 - c_1\nu)}{d_1}}\frac{1}{sn\xi})e^{i\phi} \tag{3.3.1}$$

and

$$\psi_2 = m(\sqrt{\frac{2(b_2 - c_2\nu)}{d_2}}(sn\xi) + \sqrt[3]{\frac{2\beta_1(b_2 - c_2\nu)}{d_2}}\frac{1}{sn\xi})e^{i\phi} \tag{3.3.2}$$

For cosine function,

2)$P = -m^2; R = 1 - m^2; F = cn\xi$

$$\psi_1 = (\sqrt{\frac{2m^2(c_1\nu - b_1)}{d_1}}(cn\xi) + \sqrt[3]{\frac{2\alpha_1(1 - m^2)(b_1 - c_1\nu)}{d_1}}\frac{1}{cn\xi})e^{i\phi} \tag{3.3.3}$$

and

$$\psi_2 = (\sqrt{\frac{2m^2(c_2\nu - b_2)}{d_2}}(cn\xi) + \sqrt[3]{\frac{2\beta_1(1-m^2)(b_2 - c_2\nu)}{d_2}}\frac{1}{cn\xi})e^{i\phi} \qquad (3.3.4)$$

For different elliptic functions,

3)$P = -1; R = m^2 - 1; F = dn\xi$

$$\psi_1 = (\sqrt{\frac{2(c_1\nu - b_1)}{d_1}}(dn\xi) + \sqrt[3]{\frac{2\alpha_1(m^2-1)(b_1 - c_1\nu)}{d_1}}\frac{1}{dn\xi})e^{i\phi} \qquad (3.3.5)$$

$$\psi_2 = (\sqrt{\frac{2(c_2\nu - b_2)}{d_2}}(dn\xi) + \sqrt[3]{\frac{2\beta_1(1-m^2)(b_2 - c_2\nu)}{d_2}}\frac{1}{dn\xi})e^{i\phi} \qquad (3.3.6)$$

4)$P = 1 - m^2; R = -m^2; F = nc\xi$

$$\psi_1 = (\sqrt{\frac{2(1-m^2)(b_1 - c_1\nu)}{d_1}}(nc\xi) + \sqrt[3]{\frac{2\alpha_1 m^2(c_1\nu - b_1)}{d_1}}\frac{1}{nc\xi})e^{i\phi} \qquad (3.3.7)$$

$$\psi_2 = (\sqrt{\frac{2(1-m^2)(b_2 - c_2\nu)}{d_2}}(nc\xi) + \sqrt[3]{\frac{2\beta_1 m^2(c_2\nu - b_2)}{d_2}}\frac{1}{nc\xi})e^{i\phi} \qquad (3.3.8)$$

5)$P = m^2 - 1; R = -1; F = nd\xi$

$$\psi_1 = (\sqrt{\frac{2(m^2-1)(b_1 - c_1\nu)}{d_1}}(nd\xi) + \sqrt[3]{\frac{2\alpha_1(c_1\nu - b_1)}{d_1}}\frac{1}{nd\xi})e^{i\phi} \qquad (3.3.9)$$

$$\psi_2 = (\sqrt{\frac{2(m^2-1)(b_2-c_2\nu)}{d_2}}(nd\xi) + \sqrt[3]{\frac{2\beta_1(c_2\nu-b_2)}{d_2}}\frac{1}{nd\xi})e^{i\phi} \tag{3.3.10}$$

6) $P = -m^2(1-m^2); R = 2m^2-1; F = sd\xi$

$$\psi_1 = (\sqrt{\frac{2m^2(1-m^2)(c_1\nu-b_1)}{d_1}}(sd\xi) + \sqrt[3]{\frac{2\alpha_1(2m^2-1)(b_1-c_1\nu)}{d_1}}\frac{1}{sd\xi})e^{i\phi} \tag{3.3.11}$$

$$\psi_2 = (\sqrt{\frac{2m^2(1-m^2)(c_2\nu-b_2)}{d_2}}(sd\xi) + \sqrt[3]{\frac{2\beta_1(2m^2-1)(b_2-c_2\nu)}{d_2}}\frac{1}{sd\xi})e^{i\phi} \tag{3.3.12}$$

7) $P = 1; R = 1-m^2; F = cs\xi$

$$\psi_1 = (\sqrt{\frac{2(b_1-c_1\nu)}{d_1}}(cs\xi) + \sqrt[3]{\frac{2\alpha_1(1-m^2)(b_1-c_1\nu)}{d_1}}\frac{1}{cs\xi})e^{i\phi} \tag{3.3.13}$$

$$\psi_2 = (\sqrt{\frac{2(b_2-c_2\nu)}{d_2}}(cs\xi) + \sqrt[3]{\frac{2\beta_1(1-m^2)(b_2-c_2\nu)}{d_2}}\frac{1}{cs\xi})e^{i\phi} \tag{3.3.14}$$

8) $P = 1; R = -m^2(1-m^2); F = ds\xi$

$$\psi_1 = (\sqrt{\frac{2(b_1-c_1\nu)}{d_1}}(nd\xi) + \sqrt[3]{\frac{2\alpha_1 m^2(1-m^2)(c_1\nu-b_1)}{d_1}}\frac{1}{ds\xi})e^{i\phi} \tag{3.3.15}$$

$$\psi_2 = \left(\sqrt{\frac{2(b_2 - c_2\nu)}{d_2}}(ds\xi) + \sqrt[3]{\frac{2\beta_1 m^2(1-m^2)(c_2\nu - b_2)}{d_2}}\frac{1}{ds\xi}\right)e^{i\phi} \qquad (3.3.16)$$

We can compute many triangular and periodic solutions if we take modulus $m \to 0$, if the modulus $m \to 1$ bell and kink type soliton solutions can be found, because if we take $m \to 0$, Jacobian elliptic functions degenerate to

$sn\xi \to sin\xi$, $cn\xi \to cos\xi$, $dn\xi \to 1$

if $m \to 1$, The jacobian -elliptic functions degenerate to the hyperbolic functions

$sn\xi \to tanh\xi$, $dn\xi \to sech\xi$, $cn\xi \to sech\xi$,

here sech and tanh are hyperbolic secant and hyperbolic tangent function, respectively.

3.4 Anti Cubic Law

For anti cubic law the functional F is given by,

$$F(u) = \mu_1 u^{-2} + \mu_2 u + \mu_3 u^2 \tag{3.4.1}$$

which transform the Eq. (3.1.12) and Eq. (3.1.13) into,

$$-\omega u_1 + a_1 \kappa u_2 - b_1 \kappa^2 u_1 + b_1 u_1'' + c_1 \kappa \omega u_1 - c_1 u_1'' \nu + d_1 {u_1}^{-3} + d_1 {u_1}^{3} + d_1 {u_1}^{5} + \kappa_1 u_2 = 0 \tag{3.4.2}$$

and

$$-\omega u_2 + a_2 \kappa u_1 - b_2 \kappa^2 u_2 + b_2 u_2'' + c_2 \kappa \omega u_2 - c_2 u_2'' \nu + d_2 {u_2}^{-3} + d_2 {u_2}^{3} + d_2 {u_2}^{5} + \kappa_2 u_1 = 0 \tag{3.4.3}$$

In Eq. (3.4.2) and Eq. (3.4.3) balancing the u'' with u^3, we get n = 1, so we assume the solution,

$$u_1 = \alpha_o + \alpha_1 F + \frac{\gamma_1}{F} \tag{3.4.4}$$

and

$$u_2 = \beta_o + \beta_1 F + \frac{\gamma_2}{F} \tag{3.4.5}$$

After solving the Eq. (3.4.2) we get the set of algebraic equations,

$$d_1 \eta_3 \alpha_1^7 = 0 \tag{3.4.6}$$

$$7d_1\eta_3\gamma_1\alpha_1^6+21\alpha_1^5d_1\eta_3\alpha_o^2-2\alpha_1^3c_1\upsilon P+\alpha_1^5d_1\eta_2+2\alpha_1^3b_1P=0 \tag{3.4.7}$$

$$42d_1\eta_3\alpha_o\gamma_1\alpha_1^5+35\alpha_1^4d_1\eta_3\alpha_o^3+5\alpha_1^4d_1\eta_2\alpha_o-4\alpha_1^2c_1\upsilon P\alpha_o+4\alpha_1^2b_1P\alpha_o=0 \tag{3.4.8}$$

$$-4\alpha_1c_1\upsilon R\alpha_o\gamma_1+42\alpha_1d_1\eta_3\gamma_1^5\alpha_o+4\alpha_1b_1R\alpha_o\gamma_1+35d_1\eta_3\alpha_o^3\gamma_1^4+5d_1\eta_2\alpha_o\gamma_1^4=0 \tag{3.4.9}$$

$$d_1\eta_2\gamma_1^5+7\alpha_1d_1\eta_3\gamma_1^6+2\alpha_1b_1R\gamma_1^2-2\alpha_1c_1\upsilon R\gamma_1^2+21d_1\eta_3\alpha_o^2\gamma_1^5=0 \tag{3.4.10}$$

$$7d_1\eta_3\gamma_1^6\alpha_o=0 \tag{3.4.11}$$

$$d_1\eta_3\gamma_1^7=0 \tag{3.4.12}$$

Now solving the above equations simultaneously we get,

$$\gamma_1=0 \tag{3.4.13}$$

$$\alpha_0=\sqrt{\frac{2P(c_1\nu-b_1)}{21\alpha_1^2d_1\eta_3}-\frac{\eta_2}{21\eta_3}} \tag{3.4.14}$$

$$\alpha_1=\sqrt{\frac{4P(c_1\nu-b_1)}{5d_1(\eta_2+7\eta_3\alpha_0^2)}} \tag{3.4.15}$$

Hence Eq. (3.4.4) takes the form,

$$u_1 = \sqrt{\frac{2P(c_1\nu - b_1)}{21\alpha_1^2 d_1 \eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4P(c_1\nu - b_1)}{5d_1(\eta_2 + 7\eta_3\alpha_0^2)}}F \tag{3.4.16}$$

Following the same procedure we find, u_2, which is,

$$u_2 = \sqrt{\frac{2P(c_2\nu - b_2)}{21\beta_1^2 d_2 \eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4P(c_2\nu - b_2)}{5d_2(\eta_2 + 7\eta_3\beta_0^2)}}F \tag{3.4.17}$$

Eq. (3.4.16) and Eq. (3.4.17) leads the solution of the Eq. (3.1.3) which is,

$$\psi_1 = (\sqrt{\frac{2P(c_1\nu - b_1)}{21\alpha_1^2 d_1 \eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4P(c_1\nu - b_1)}{5d_1(\eta_2 + 7\eta_3\alpha_0^2)}}F)e^{i\phi} \tag{3.4.18}$$

and

$$\psi_2 = (\sqrt{\frac{2P(c_2\nu - b_2)}{21\beta_1^2 d_2 \eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4P(c_2\nu - b_2)}{5d_2(\eta_2 + 7\eta_3\beta_0^2)}}F)e^{i\phi} \tag{3.4.19}$$

These equation has many solution which are determined as under,

3.5 Cases

For sine function

1)$P = m^2; F = sn\xi$

$$\psi_1 = m(\sqrt{\frac{2(c_1\nu - b_1)}{21\alpha_1^2 d_1 \eta_3} - \frac{\eta_2}{21\eta_3}} + m\sqrt{\frac{4(c_1\nu - b_1)}{5d_1(\eta_2 + 7\eta_3\alpha_0^2)}}sn\xi)e^{i\phi} \tag{3.5.1}$$

$$\psi_2 = m(\sqrt{\frac{2(c_2\nu - b_2)}{21\beta_1^2 d_2 \eta_3} - \frac{\eta_2}{21\eta_3}} + m\sqrt{\frac{4(c_2\nu - b_2)}{5d_2(\eta_2 + 7\eta_3\beta_0^2)}}sn\xi)e^{i\phi} \tag{3.5.2}$$

For cosine function,

2)$P=-m^2; F=cn\xi$

$$\psi_1=(\sqrt{\frac{2m^2(b_1-c_1\nu)}{21\alpha_1^2 d_1\eta_3}-\frac{\eta_2}{21\eta_3}}+\sqrt{\frac{4m^2(b_1-c_1\nu)}{5d_1(\eta_2+7\eta_3\alpha_0^2)}}cn\xi)e^{i\phi} \tag{3.5.3}$$

and

$$\psi_2=(\sqrt{\frac{2m^2(b_2-c_2\nu)}{21\beta_1^2 d_2\eta_3}-\frac{\eta_2}{21\eta_3}}+\sqrt{\frac{4m^2(b_2c_2\nu)}{5d_2(\eta_2+7\eta_3\beta_0^2)}}cn\xi)e^{i\phi} \tag{3.5.4}$$

For different elliptic functions,

3)$P=-1; F=dn\xi$

$$\psi_1=(\sqrt{\frac{2(b_1-c_1\nu)}{21\alpha_1^2 d_1\eta_3}-\frac{\eta_2}{21\eta_3}}+\sqrt{\frac{4(b_1-c_1\nu)}{5d_1(\eta_2+7\eta_3\alpha_0^2)}}dn\xi)e^{i\phi} \tag{3.5.5}$$

$$\psi_2=(\sqrt{\frac{2(b_2-c_2\nu)}{21\beta_1^2 d_2\eta_3}-\frac{\eta_2}{21\eta_3}}+\sqrt{\frac{4(b_2-c_2\nu)}{5d_2(\eta_2+7\eta_3\beta_0^2)}}dn\xi)e^{i\phi} \tag{3.5.6}$$

4)$P=1-m^2; F=nc\xi$

$$\psi_1=(\sqrt{\frac{2(1-m^2)(c_1\nu-b_1)}{21\alpha_1^2 d_1\eta_3}-\frac{\eta_2}{21\eta_3}}+\sqrt{\frac{4(1-m^2)(c_1\nu-b_1)}{5d_1(\eta_2+7\eta_3\alpha_0^2)}}nc\xi)e^{i\phi} \tag{3.5.7}$$

$$\psi_2=(\sqrt{\frac{2(1-m^2)(c_2\nu-b_2)}{21\beta_1^2 d_2\eta_3}-\frac{\eta_2}{21\eta_3}}+\sqrt{\frac{4(1-m^2)(c_2\nu-b_2)}{5d_2(\eta_2+7\eta_3\beta_0^2)}}nc\xi)e^{i\phi} \tag{3.5.8}$$

5) $P = m^2 - 1; F = nd\xi$

$$\psi_1 = (\sqrt{\frac{2(m^2-1)(c_1\nu - b_1)}{21\alpha_1^2 d_1\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4(m^2-1)(c_1\nu - b_1)}{5d_1(\eta_2+7\eta_3\alpha_0^2)}}nd\xi)e^{i\phi} \quad (3.5.9)$$

$$\psi_2 = (\sqrt{\frac{2(m^2-1)(c_2\nu - b_2)}{21\beta_1^2 d_2\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4(m^2-1)(c_2\nu - b_2)}{5d_2(\eta_2+7\eta_3\beta_0^2)}}nd\xi)e^{i\phi} \quad (3.5.10)$$

6) $P = -m^2(1-m^2); R = 2m^2 - 1; F = sd\xi$

$$\psi_1 = (\sqrt{\frac{2m^2(1-m^2)(b_1 - c_1\nu)}{21\alpha_1^2 d_1\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4m^2(1-m^2)(b_1 - c_1\nu)}{5d_1(\eta_2+7\eta_3\alpha_0^2)}}sd\xi)e^{i\phi} \quad (3.5.11)$$

$$\psi_2 = (\sqrt{\frac{2m^2(1-m^2)(b_2 - c_2\nu)}{21\beta_1^2 d_2\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4m^2(1-m^2)(b_2 - c_2\nu)}{5d_2(\eta_2+7\eta_3\beta_0^2)}}sd\xi)e^{i\phi} \quad (3.5.12)$$

7) $P = 1; F = cs\xi$

$$\psi_1 = (\sqrt{\frac{2(c_1\nu - b_1)}{21\alpha_1^2 d_1\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4(c_1\nu - b_1)}{5d_1(\eta_2+7\eta_3\alpha_0^2)}}cs\xi)e^{i\phi} \quad (3.5.13)$$

$$\psi_2 = (\sqrt{\frac{2(c_2\nu - b_2)}{21\beta_1^2 d_2\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4(c_2\nu - b_2)}{5d_2(\eta_2+7\eta_3\beta_0^2)}}cs\xi)e^{i\phi} \quad (3.5.14)$$

$8)P = 1; F = ds\xi$

$$\psi_1 = (\sqrt{\frac{2(c_1\nu - b_1)}{21\alpha_1^2 d_1\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4(c_1\nu - b_1)}{5d_1(\eta_2 + 7\eta_3\alpha_0^2)}} ds\xi)e^{i\phi} \tag{3.5.15}$$

$$\psi_2 = (\sqrt{\frac{2(c_2\nu - b_2)}{21\beta_1^2 d_2\eta_3} - \frac{\eta_2}{21\eta_3}} + \sqrt{\frac{4(c_2\nu - b_2)}{5d_2(\eta_2 + 7\eta_3\beta_0^2)}} ds\xi)e^{i\phi} \tag{3.5.16}$$

We can compute many triangular and periodic solutions if we take modulus $m \to 0$, if the modulus $m \to 1$ bell and kink type soliton solutions can be found, because if we take $m \to 0$, Jacobian elliptic functions degenerate to

$sn\xi \to sin\xi$, $cn\xi \to cos\xi$, $dn\xi \to 1$

if $m \to 1$, The jacobian -elliptic functions degenerate to the hyperbolic functions

$sn\xi \to tanh\xi$, $dn\xi \to sech\xi$, $cn\xi \to sech\xi$,

here sech and tanh are hyperbolic secant and hyperbolic tangent function, respectively.

3.6 Gausson soliton in dual-core fibers with inter-modal dispersion

3.7 The Model

The mathematical illustration for decoupled NLSE of optical solitons in dual core fibers with inter modal dispersion is given as,

$$i(\frac{\partial\psi_1}{\partial t}+a_1\frac{\partial\psi_2}{\partial x})+b_1\frac{\partial^2\psi_1}{\partial x_2}+c_1\frac{\partial^2\psi_1}{\partial x\partial t}+d_1F(|\psi_1|^2)\psi_1+k_1\psi_2=0 \tag{3.7.1}$$

$$i(\frac{\partial\psi_2}{\partial t}+a_2\frac{\partial\psi_1}{\partial x})+b_2\frac{\partial^2\psi_2}{\partial x_2}+c_2\frac{\partial^2\psi_2}{\partial x\partial t}+d_2F(|\psi_2|^2)\psi_2+k_2\psi_1=0 \tag{3.7.2}$$

In the above coupled model ψ_1 and ψ_2 represents the field envelopes, x is the propagation co-ordinate, a_j are group velocities , and b_j are group velocity dispersion. While c_j and k_j represents spatio-temporal dispersion and linear coupling coefficients respectively, for $j=1,2$. The functional F is real-valued algebraic function.

$$\psi_j(x,t)=P_je^{\iota\phi}, where j=1,2 \tag{3.7.3}$$

and the phase component $\phi(x,t)$ is defined as

$$\phi=-\kappa x+\omega t+\theta \tag{3.7.4}$$

By using Eq. (3.7.3) and its derivatives along with Eq. (3.7.4) and Eq. (3.7.5) into Eq. (4.7.1) and Eq. (4.7.2) and decomposing into real and imaginary parts,The real part equations for two components are

$$-\omega P_1+\kappa a_1 P_2+b_1\left(\frac{\partial^2 P_1}{\partial x^2}-\kappa^2 P_1\right)+c_1\left(\kappa\omega P_1+\frac{\partial^2 P_1}{\partial x\partial t}\right)+d_1 F({P_1}^2)P_1+\kappa_1 P_2=0 \tag{3.7.5}$$

and

$$-\omega P_2+\kappa a_2 P_1+b_2\left(\frac{\partial^2 P_2}{\partial x^2}-\kappa^2 P_2\right)+c_2\left(\kappa\omega P_2+\frac{\partial^2 P_2}{\partial x\partial t}\right)+d_2 F({P_2}^2)P_2+\kappa_2 P_1=0 \tag{3.7.6}$$

The imaginary part equations for the two components lead to the velocity of the solitons as,

$$v=\frac{a_1+\omega c_1-2\kappa b_1}{1-c_1\kappa} \tag{3.7.7}$$

and

$$v=\frac{a_2+\omega c_2-2\kappa b_2}{1-c_2\kappa} \tag{3.7.8}$$

whenever $1\neq c_1\kappa$ and $1\neq c_2\kappa$

3.8 Log law

For log law nonlinearity the functional F is given by,

$$F(P)=ln(P) \tag{3.8.1}$$

By using log law in Eq. (3.7.6) and Eq. (3.7.7), and obtained the following equations,

$$-\omega P_1+\kappa a_1 P_2+b_1(\frac{\partial^2 P_1}{\partial x^2}-\kappa^2 P_1)+c_1(\kappa\omega P_1+\frac{\partial^2 P_1}{\partial x\partial t})+d_1 ln({P_1}^2)P_1+\kappa_1 P_2=0 \tag{3.8.2}$$

and

$$-\omega P_2+\kappa a_2 P_1+b_2(\frac{\partial^2 P_2}{\partial x^2}-\kappa^2 P_2)+c_2(\kappa\omega P_2+\frac{\partial^2 P_2}{\partial x\partial t})+d_2 ln({P_2}^2)P_2+\kappa_2 P_1=0 \tag{3.8.3}$$

The starting hypothesis for log law nonlinearity is given by the Gausson,

$$P(x,t)=Ae^{-\tau^2} \tag{3.8.4}$$

And

$$\tau=B(x-vt) \tag{3.8.5}$$

Where A is the amplitude of the Gausson, B is inverse width of the Gausson, and v is the velocity of the Gausson.

Now

$$(P)_t=(2ABv\tau)e^{-\tau^2} \tag{3.8.6}$$

Also,

$$(P)_x=(-2AB\tau)e^{-\tau^2} \tag{3.8.7}$$

and

$$(P)_{xx} = (4AB^2\tau^2 - 2AB^2)e^{-\tau^2} \tag{3.8.8}$$

we can also find,

$$(P)_{xt} = (2AB^2\nu - 4AB^2\nu\tau)e^{-\tau^2} \tag{3.8.9}$$

Now substituting the values of Eq. (3.8.6) to (3.8.9) into Eq. (3.8.2) and (3.8.3), we get the following equations,

$$\begin{aligned} -\omega A + \kappa a_1 A + 4b_1 AB^2\tau^2 - 2b_1 AB^2 - b_1 A\kappa^2 + c_1 A\omega\kappa + \\ 2c_1 AB^2\nu - 4c_1 AB^2\nu\tau + d_1 AlnA^2 - 2d_1 A\tau^2 + k_1 A = 0 \end{aligned} \tag{3.8.10}$$

and

$$\begin{aligned} -\omega A + \kappa a_2 A + 4b_2 AB^2\tau^2 - 2b_2 AB^2 - b_2 A\kappa^2 + c_2 A\omega\kappa + \\ 2c_2 AB^2\nu - 4c_2 AB^2\nu\tau + d_2 AlnA^2 - 2d_2 A\tau^2 + k_2 A = 0 \end{aligned} \tag{3.8.11}$$

Now linear independent function are τ^j for $j = 0, 1, 2, 3, 4, ...$, By setting their respective coefficients to zero we get,

$$\omega_1 = b_1(-2B_1^2 - \kappa_1^2) + d_1 lnA^2 \tag{3.8.12}$$

and

$$B_1 = \sqrt{\frac{d_1}{2b_1}} \tag{3.8.13}$$

$$\kappa_1 = \frac{-2B_1^2 v}{\omega_1} \tag{3.8.14}$$

$$c_1 = 0 \tag{3.8.15}$$

Similarly for Eq. (3.8.11) we get the following results,

$$\omega_2 = b_2(-2B_2^2 - \kappa_2^2) + d_2 lnA^2 \tag{3.8.16}$$

and,

$$B_2 = \sqrt{\frac{d_2}{2b_2}} \tag{3.8.17}$$

$$\kappa_2 = \frac{-2B_2^2 v}{\omega_2} \tag{3.8.18}$$

$$c_2 = 0 \tag{3.8.19}$$

Hence the Gausson solution is given as,

$$\psi_1(x,t) = Ae^{-B_1^2(x-vt)^2} e^{i(-\kappa+\omega_1 t+\theta_1)} \tag{3.8.20}$$

and

$$\psi_2(x,t) = Ae^{-B_2^2(x-vt)^2}e^{i(-\kappa+\omega_2 t+\theta_2)} \tag{3.8.21}$$

Chapter 4

Conclusions

4.1 Conclusions

In this dissertation, First we apply extended F-expansion method to solve the LPD model using three nonlinear medium, which are Kerr-law, parabolic law and anti cubic nonlinearity. Using extended F-expansion method, we have derived many exact Jacobian elliptic periodic solutions, which includes triangular periodic solutions (when the modulus $m = \rightarrow 0$) and soliton solutions (when the modulus $m \rightarrow 1$). This method is very useful and gives reliable soliton solutions than the existing schemes.

Secondly we have used the extended F-expansion method to solve the dimensionless form of the coupled NLSE with Kerr-Law and anti cubic nonlinearity. We have obtained many exact Jacobian elliptic periodic solutions including bell and kink type soliton solutions. Finally we used the log law nonlinearity to find Gausson Soliton in dual-core fibers with inter-modal dispersion. This scheme is very helpful and gives reliable and useful to find Gausson soliton and these results are encouraging towards future goals.

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