Cordial Labelings of B_n and Linear Phenylene Graphs

By

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CIIT/FA17-RMT-001/LHR

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I Yusra Amjad, **CIIT/FA17-RMT-001/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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DEDICATION

To

My Mother

ACKNOWLEDGEMENT

In the name of Allah, the Most Gracious and Most Merciful, all the praises to Allah for the strengths and His blessing in completing this thesis. I feel much pleasure in expressing my heartiest gratitude to my supervisor Dr. M. Faisal Nadeem for his dynamic supervision and whose inspiring attitude made it very easy to undertake this project. It was only through their guidance that my efforts were fruitful. I express my sincerest gratitude to my institution and a special thanks to the Head of Department of Mathematics for providing us with very good research facilities, required for the accomplishment of our goals. My deepest appreciations are extended to my beloved Parents for my success, is really the fruit of their devoted prayers and encouragement. And a special thanks for the person who motivated me to achieve my goals.

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ABSTRACT

Cordial Labelings of B_n and Linear phenylene Graphs

Graph theory is one of the important branch of mathematics. Mathematicians consider it useful in light of the fact that it has wide range of applications in different areas such as in computer science, sociology, engineering, communication networks, radar tracking and much more see [2, 3, 13].It has risen as the most important branch of mathematics.

In graph theory there are different types of labeling. In this thesis our work is focused on cordial labelings of B_n graph and linear phenylene graph.

First chapter includes introduction. In this chapter we basically discussed the basic definitions related to the graph and studied different types of graphs including B_n graph and linear phenylene graph.

In second chapter we shed light on various types of labelings introduced by renowned scholars. We have discussed cordial labelings and its several types.

In third chapter we have discussed some results for cordial labeling of B_n graph and linear phenylene graph.

In chapter four we find the results for edge product cordial labeling of B_n graph and linear phenylene graph.

In chapter five we find the results for total edge product cordial labeling of B_n graph and linear phenylene graph.

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Chapter 1

Preliminaries and Introduction

1 Introduction

Graph theory is the study that deals with different types of graphs. Euler was the one who laid the foundation of graph theory by solving the problem “Seven Bridges of Königsberg” in 1736 see [5]. He was given the title “father of graph theory” because of his diligent efforts towards solving the geometrical problems.

In this chapter, we briefly studied about the introduction of graphs. This includes some fundamental definitions of graphs and their different types.

1.1 Graph

A set which comprised of the finite set of vertices and finite set of edges (V,E) forms a *graph* G, where $V(G)$ is the vertex set and $E(G)$ is the edge set. Example is shown in Figure 1.

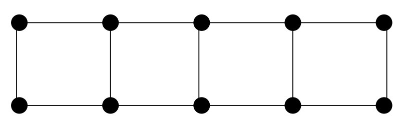

Figure 1: Graph

1.2 Edges

Edges are formed by connecting vertices with lines, these vertices are known as endpoints. Example is shown in Figure 2.

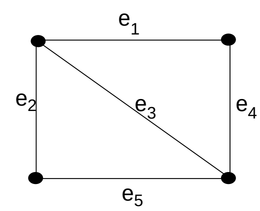

Figure 2: Edges $(e_1, e_2, e_3, e_4, e_5)$

1.3 Order of a Graph

The total vertices in a graph is regarded as *order of a graph* and it is symbolized by the notation $|V(G)|$. For example in Figure 3 the order of graph is 5 and mathematically it can be written as $|V(G)| = 5$.

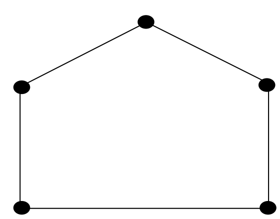

Figure 3: Order of a graph is 5

1.4 Size of a Graph

The total edges in a graph is regarded as *size of a graph* and it is symbolized by the notation $|E(G)|$. Example in Figure 4 shows that size of a graph is 4 and mathematically it can be written as $|E(G)| = 4$.

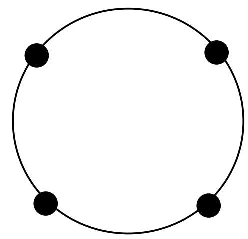

Figure 4: Size of a graph is 4

1.5 Vertex Degree

A vertex *u* that has number of edges incident to it, is regarded as *vertex degree* and the notation used to denote vertex degree is *deg(u)*. For example in Figure 5 every vertex of graph has degree 2.

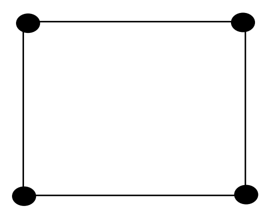

Figure 5: Degree of each Vertex is 2

1.6 Loop

If an edge is incident to a vertex and that vertex is adjacent to itself, then it is known as *loop*. The example of *loop* is shown in Figure 6.

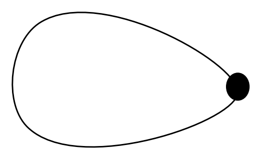

Figure 6: Loop

1.7 Simple Graph

Graph G that has no multiple edges or loops, then it is regarded as *simple graph*. Figure 7 shows the example of simple graph.

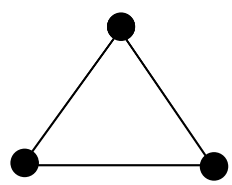

Figure 7: Simple Graph

1.8 Trivial Graph

A graph G that contains no edges and has a single vertex is regarded as *trivial graph* see Figure 8.

Figure 8: Trivial Graph

1.9 Non-trivial Graph

Graph G which is comprised of more than one vertex and also has edges, then it is regarded as *non-trivial graph* see Figure 9.

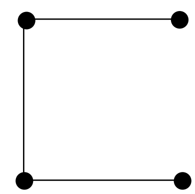

Figure 9: Non-Trivial Graph

1.10 Null Graph

If there exist vertices of zero degree in a graph G, then it is known as *null graph* see Figure 10.

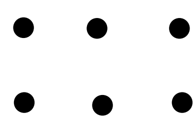

Figure 10: Null Graph

1.11 Connected Graph

A graph that contains at least one path between every two vertices, then it is said to be *connected graph* see Figure 11.

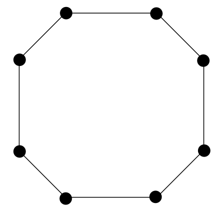

Figure 11: Connected Graph

1.12 Disconnected Graph

If there exists two nodes and no path in G has those nodes as endpoints, then the graph G is said to be *disconnected* see Figure 12.

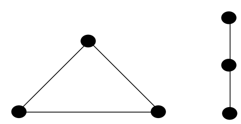

Figure 12: Disconnected Graph

1.13 Directed Graph

A graph that has edges associated with directions, then it is regarded as *directed graph* otherwise it is *undirected graph* see Figure 13.

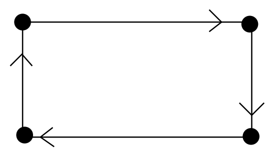

Figure 13: Directed Graph

1.14 Acyclic Graph

Graph G that contains no cycle, then it is said to be *acyclic graph* see Figure 14.

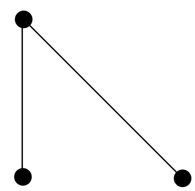

Figure 14: Acyclic Graph

1.15 Path

A graph G where edges are connected in such a fashion that no vertex is repeated, then it is regarded as a *path*. It is represented by P_m, and m represents the total vertices and total edges of the graph is $m-1$ see Figure 15.

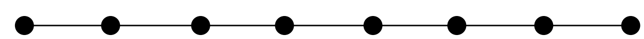

Figure 15: Path P_8

1.16 Cycle Graph

If the number of vertices connected in such a fashion that they form a closed chain then this leads to the formation of '*cycle graph*'. The notation used for *cycle graph* is C_m, and m represents the number of vertices see Figure 16.

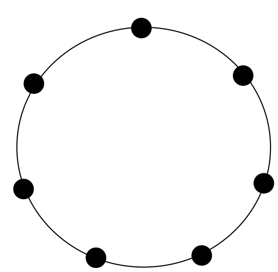

Figure 16: Cycle Graph C_7

1.17 Tree

Graph G which is connected as well as contains no cycle, then the graph is said to be *tree*. The notation used for *tree* is T_m, and m represents total vertices and, the total edges of the graph is $m-1$ see Figure 17.

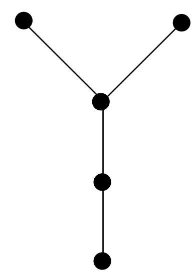

Figure 17: Tree Graph T_5

1.18 Complete Graph

Graph G that contains an edge between each pair of distinct vertices, then it is known as *complete graph*. The notation used to denote *complete graph* is K_r where r represents total vertices, and the size of the graph G is given by $\frac{r(r-1)}{2}$ see Figure 18.

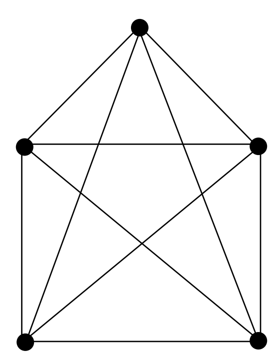

Figure 18: Complete Graph K_5

1.19 k-Partite Graph

If vertices of graph G are divided into k disjoint sets and no pair of vertices within the same set are adjacent, then the graph is said to be *k-partite graph*. If $k = 2$ then the graph G is known as bipartite graph see Figure 19.

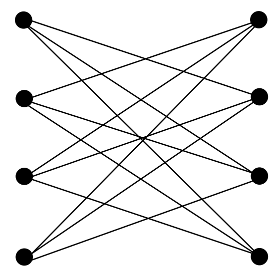

Figure 19: Bipartite Graph

1.20 Complete Bipartite Graph

If a graph G has vertices that are partitioned into two sets U and V where each vertex of U is connected with each vertex of V and no vertices within the same set are adjacent. If $|U| = r$ and $|V| = s$ then it is represented as $K_{r,s}$. The graph order is given by $r+s$ and size of the graph is rs see Figure 20.

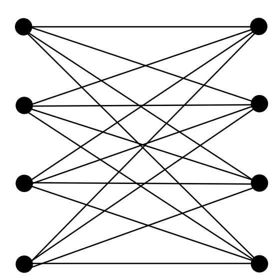

Figure 20: Complete Bipartite Graph $K_{4,4}$

1.21 Linear Phenylene Graph

Linear phenylene graph is obtained by a chain of alternate copies of hexagon and squares, where $V(G) = \{x_i, y_i; 1 \leq i \leq n\}$ is a set of vertices and $E(G) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{y_i y_{i+1}; 1 \leq i \leq n-1\} \cup \{x_1 y_1\} \cup \{x_{3i} y_{3i}; 1 \leq i \leq \frac{n}{3}\} \cup \{x_{3i+1} y_{3i+1}; 1 \leq i \leq \frac{n}{3} - 1\}$ is a set of edges. see Figure 21.

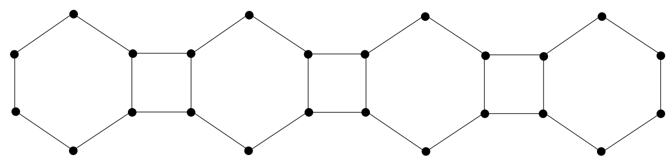

Figure 21: Linear Phenylene Graph

1.22 B_n Graph

B_n graph is a planar graph acquired by making ladder $L_n \simeq P_n \times P_2$ by vertices $V(G) = \{x_i, y_i, z_i; 1 \leq i \leq n\}$ and edges $E(G) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{y_i y_{i+1}; 1 \leq i \leq n-1\} \cup \{z_i z_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{x_{i+1} y_i; 1 \leq i \leq n-1\} \cup \{y_i z_i; 1 \leq i \leq n\}$ see Figure 22.

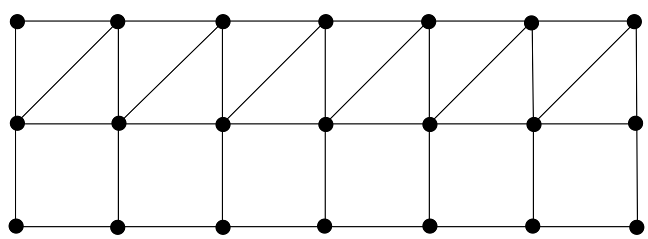

Figure 22: B_7 Graph

Chapter 2

Literature Review

2 Labeling

Graph is a structure that consists of vertices and edges. The assignment of the labels to the edges and vertices of the graph under particular conditions is known as labeling. The origin of labeling of the graph is associated with Rosa. He derived the concept of α-labeling and β-labeling, for detailed study see [14]. In 1972 β-labeling was renamed as graceful labeling by Golomb. Graham and Sloane proposed the concept of 'harmonious labeling', for detailed study see [7]. There are different types of labelings in graph theory such as queen labeling, graceful labeling, cordial labeling for details see [6]. Here our work is focused on cordial labelings of graphs.

2.1 Graph Labeling

Graph labeling is regarded as the assignment of labels to the edges or vertices or both, under specified conditions.

Edges and vertices are elements of the graph. Edges and vertices are used as a domain of the graph. By confining the domain in many ways we get different types of labeling.

2.1.1 Vertex Labeling

The assignment of the labels to the vertices of the graph is known as *vertex labeling* see Figure 23.

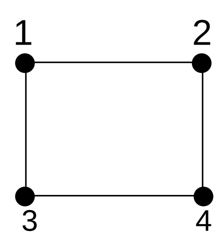

Figure 23: Vertex Labeling

2.1.2 Edge Labeling

The assignment of labels to the graph edges is known as *edge labeling* see Figure 24.

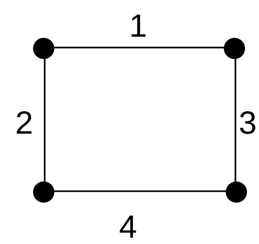

Figure 24: Edge Labeling

2.1.3 Total Labeling

The assignment of the labels to both vertices and edges of the graph, is known as *total labeling* see Figure 25.

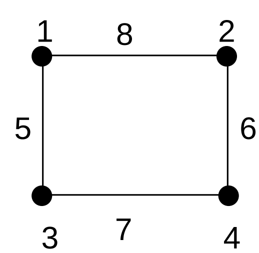

Figure 25: Total Labeling

Cahit introduced cordial labeling in 1987 see [4]. He illustrated that cordial labeling is a weakest version of harmonious labeling as well as graceful labeling. Ramanjaneyulu et al. proposed some results regarding to cordial labeling on planar graphs see [9].

2.2 Cordial Labeling

"The assignment of the labels $\{0, 1\}$ to the set of vertices, and the edges are assigned with labels as the absolute difference of the labels at their endpoints $|f(u) - f(v)|$. The labeling is said to be *cordial* if it satisfies the following two conditions.

(1) The number of vertices assigned with label 0 and the number of vertices assigned with label 1 must be differ at most by 1 such that $|v_f(0) - v_f(1)| \leq 1$.

(2) The number of edges assigned with label 0 and the number of edges assigned with label 1 must be differ at most by 1 such that $|e_f(0) - e_f(1)| \leq 1$" [4]. See Figure 26 for cordial labeling of C_3.

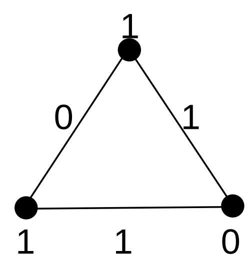

Figure 26: Cordial labeling

Sundaram et al. proposed the concept of product cordial labeling and total product cordial labeling in 2004 and 2006 see [10, 11]. He also concluded some results on total product cordial labeling see [12]. Vaidya and Barasara introduced results on product cordial graphs see [17, 18].

2.3 Product Cordial Labeling

"The assignment of the labels $\{0, 1\}$ to the set of vertices, and edges are assigned with labels as the product of the labels $f(u)f(v)$ at their endpoints. The labeling is said to be *product cordial* if it satisfies the following two conditions.

(1) The number of vertices assigned with label 0 and the number of vertices assigned with

label 1 must be differ at most by 1 such that $|v_f(0) - v_f(1)| \leq 1$.

(2) The number of edges assigned with label 0 and the number of edges assigned with label 1 must be differ at most by 1 such that $|e_f(0) - e_f(1)| \leq 1$" [10]. See Figure 27 for product cordial labeling of C_3.

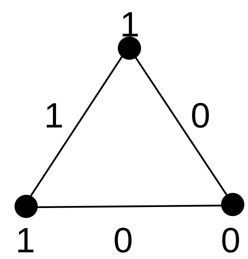

Figure 27: Product cordial labeling

2.4 Total Product Cordial labeling (TPC)

"The assignment of the labels $\{0,1\}$ to the set of vertices, and the edges are assigned with labels as the product of the labels $f(u)f(v)$ at their endpoints. The labeling is said to be *total product cordial* if it satisfies the given condition. The number of vertices and edges assigned with label 0 and the number of vertices and edges assigned with label 1 must be differ at most by 1 such that $|sum(0) - sum(1)| \leq 1$"[11]. See Figure 28 for TPCL of C_3.

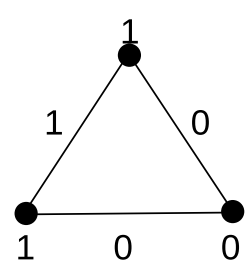

Figure 28: Total product cordial labeling

2.5 k-Total Product Cordial Labeling (k-TPC)

"The assignment of the labels $\{0, 1, \ldots, k\}$ to the set of vertices, and the edges are assigned with labels as $f(u)f(v)(\bmod k)$. The labeling is said to be *k-total product cordial* if it satisfies $|sum(x) - sum(y)| \leq 1$"[8]. See Figure 29 and Figure 30 for k-TPC labeling of C_3 and C_5 respectively.

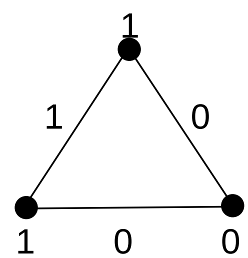

Figure 29: k-TPC Labeling

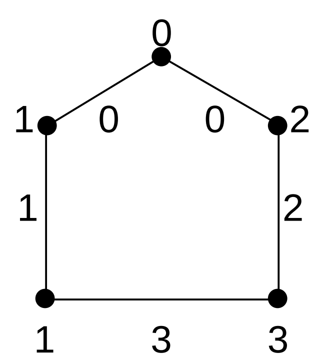

Figure 30: 3-TPC labeling

Later, by extending the idea of cordial labeling Vaidya and Barasara proposed edge product cordial labeling and total edge product cordial labeling in 2012 and 2013 respectively see [15, 16].

2.6 Edge Product Cordial Labeling (EPC)

"The assignment of the labels $\{0,1\}$ to the edges, and the vertices are assigned as the product of the edges incident to vertex v, the labeling is said to be *edge product cordial labeling* if it satisfies the following conditions.

(1) The number of vertices assigned with label 0 and the number of vertices assigned with label 1 must be differ at most by 1 such that $|v_f(0) - v_f(1)| \leq 1$.

(2) The number of edges assigned with label 0 and the number of edges assigned with label 1 must be differ at most by 1 such that $|e_f(0) - e_f(1)| \leq 1$" [15]. Figure 31 shows the EPC labeling of C_3.

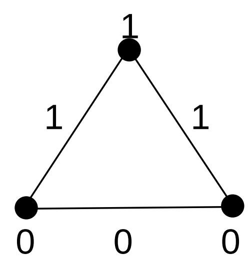

Figure 31: Edge product cordial labeling of C_3

2.7 k-Edge Product Cordial Labeling (k-EPC)

"The assignment of the labels $\{0,1,\ldots,k-1\}$ to the set of edges, and the vertices are assigned with labels as the product of the edges incident to vertex v and the labeling is said to be *k-edge product cordial labeling* if it satisfies $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(a) - e_f(b)| \leq 1$". See Figure 32 and Figure 33 for k-EPC labeling of C_3 and P_4 respectively.

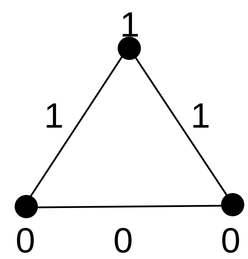

Figure 32: 2-EPC Labeling

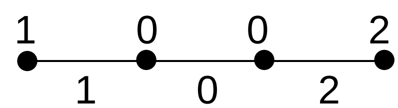

Figure 33: 3-EPC Labeling

2.8 Total Edge Product Cordial Labeling (TEPC)

“The assignment of the labels $\{0,1\}$ to the edges, and the vertices are labeled as the product of the edges incident to vertex v and if it satifies the condition $|sum(0)-sum(1)| \leq 1$ then the labeling is said to be *total edge product cordial labeling*” [16]. See Figure 34 for TEPC labeling of C_3.

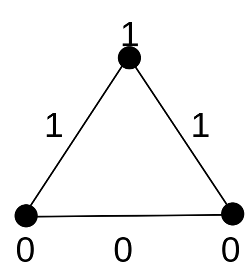

Figure 34: Total edge product cordial labeling of C_3

Azaizeh introduced another version of cordial labelings, known as *k-total edge product cordial labeling* in 2015 see [1].

2.9 k-Total Edge Product Cordial Labeling(k-TEPC)

"The assignment of the labels $\{0,1,\ldots,k-1\}$ to the edges, and the vertices are labeled as the product of the edges incident to vertex v such that
$h^* \equiv h(e_1),h(e_2)\ldots h(e_n)(\bmod k)$ and satisfies the condition $|sum(x) - sum(y)| \leq 1$ for $x,y \in \{0,1,\ldots,k-1\}$, then it is regarded as *k-total edge product cordial labeling*"[1]. See Figure 35 and Figure 36 for k-TEPC labeling of C_3 and P_8 respectively.

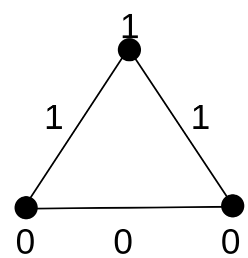

Figure 35: 2-TEPC Labeling

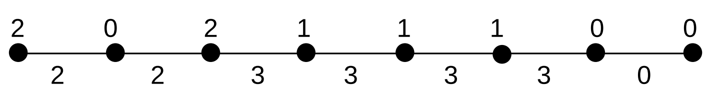

Figure 36: 4-TEPC Labeling

2.9.1 Known results

Theorem 2.1. *[4] "All trees T_m are cordial graphs."*

Theorem 2.2. *[4] " A complete graph K_n is cordial iff $n \leq 3$."*

Theorem 2.3. *[4] "All complete bipartite graphs $K_{m,n}$ are cordial."*

Theorem 2.4. *[16] "The cycle C_n is a total edge product cordial graph except for $n \neq 4$."*

Chapter 3

Cordial Labeling of B_n and Linear Phenylene Graphs

3 Cordial Labeling of B_n and Linear Phenylene Graphs

This chapter is about the cordial labeling of B_n and linear phenylene graphs. Here we have deduced some results regarding cordial labeling of B_n graph and linear phenylene graph.

Theorem 3.1. *The linear phenylene graph PH_n is cordial for $n = 6t$ where $t \geq 2$.*

Proof. Let G be a graph of n vertices with vertex set $V(G) = \{x_i, y_i; 1 \leq i \leq 3t\}$ and edge set $E(G) = \{x_i x_{i+1}; 1 \leq i \leq 3t-1\} \cup \{y_i y_{i+1}; 1 \leq i \leq 3t-1\} \cup \{x_{3i} y_{3i}; 1 \leq i \leq t\} \cup \{x_{3i+1} y_{3i+1}; 1 \leq i \leq t-1\} \cup \{x_1 y_1\}$.

Let $n = 6t$ where $t \geq 2$

Following are the vertices of the graph:

$f(x_1) = f(x_2) = f(x_5) = f(x_6) = 0$

$f(x_{3i+4}) = 0; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-4}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(x_{3i+5}) = 0; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-5}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(x_{3i+6}) = 0; \quad for \quad 1 \leq i \leq t-2, \quad and \quad i = 2,4,\ldots,3t-2$

$f(x_3) = f(x_4) = 1$

$f(x_{3i+1}) = 1; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-1}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(x_{3i+2}) = 1; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-2}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(x_{3i+3}) = 1; \quad for \quad 1 \leq i \leq t-1, \quad and \quad i = 2,4,\ldots,3t-2$

$f(y_1) = f(y_3) = 0$

$f(y_{3i+1}) = 0; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-1}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(y_{3i+2}) = 0; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-2}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(y_{3i+3}) = 0; \quad for \quad 1 \leq i \leq t-1, \quad and \quad i = 2,4,...,3t-2$

$f(y_2) = f(y_4) = f(y_5) = f(y_6) = 1$

$f(y_{3i+4}) = 1; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-4}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(y_{3i+5}) = 1; \quad for \quad 1 \leq i \leq \lfloor \frac{3t-5}{3} \rfloor, \quad and \quad i = 2,4,\ldots,3t-2$

$f(y_{3i+6}) = 1; \quad for \quad 1 \leq i \leq t-2, \quad and \quad i = 2,4,\ldots,3t-2$

Edges of the graph are defined as follows:

$f(x_1 x_2) = f(x_3 x_4) = f(x_5 x_6) = 0$

$f(x_{3i+4}x_{3i+5}) = 0; \quad for \qquad 1 \leq i \leq \left\lfloor \frac{3t-5}{3} \right\rfloor$

$f(x_{3i+5}x_{3i+6}) = 0; \quad for \qquad 1 \leq i \leq t-2$

$f(x_2x_3) = f(x_4x_5) = 1$

$f(x_{3i+3}x_{3i+4}) = 1; \quad for \qquad 1 \leq i \leq \left\lfloor \frac{3t-4}{3} \right\rfloor$

$f(y_{3i+1}y_{3i+2}) = 0; \quad for \qquad 1 \leq i \leq \left\lfloor \frac{3t-2}{3} \right\rfloor$

$f(y_{3i+2}y_{3i+3}) = 0; \quad for \qquad 1 \leq i \leq t-1$

$f(y_1y_2) = f(y_2y_3) = 1$

$f(y_{3i}y_{3i+1}) = 1; \quad for \qquad 1 \leq i \leq \left\lfloor \frac{3t-1}{3} \right\rfloor$

$f(x_1y_1) = f(x_4y_4) = 0$

$f(x_{3i}y_{3i}) = 1; \quad for \qquad 1 \leq i \leq t$

$f(x_{3i+4}y_{3i+4}) = 1; \quad for \qquad 1 \leq i \leq t-1$

We obtain $v_f(0) = v_f(1) = 3t$ and $e_f(0) = e_f(1) = 4t-1$.The difference between the 0 labeled vertices and 1 labeled vertices is 0. Similarly, the difference between the edges labeled 0 and the edges labeled 1 is 0. So the graph is cordial.

In Figure 37 for $t = 2$ and $n = 12$, we get $v_f(0) = v_f(1) = 6$, $e_f(0) = e_f(1) = 7$

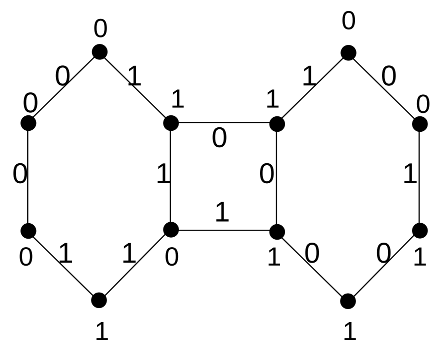

Figure 37: Cordial labeling of linear phenylene graph PH_{12}

In Figure 38 for $t = 3$ and $n = 18$, we get $v_f(0) = v_f(1) = 9$, $e_f(0) = e_f(1) = 11$

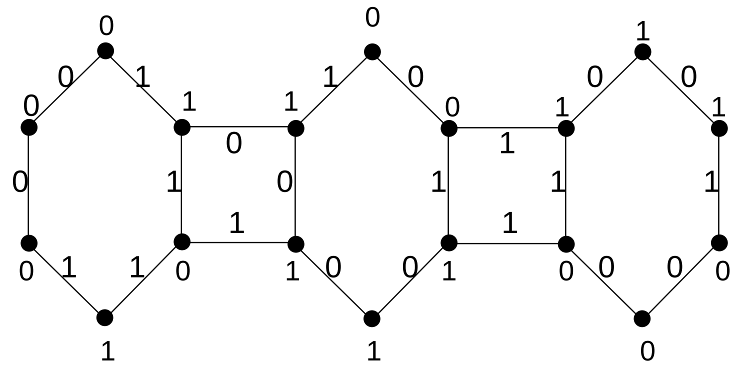

Figure 38: Cordial labeling of linear phenylene graph PH_{18}

In Figure 39 for $t = 4$ and $n = 24$, we get $v_f(0) = v_f(1) = 12$, $e_f(0) = e_f(1) = 15$

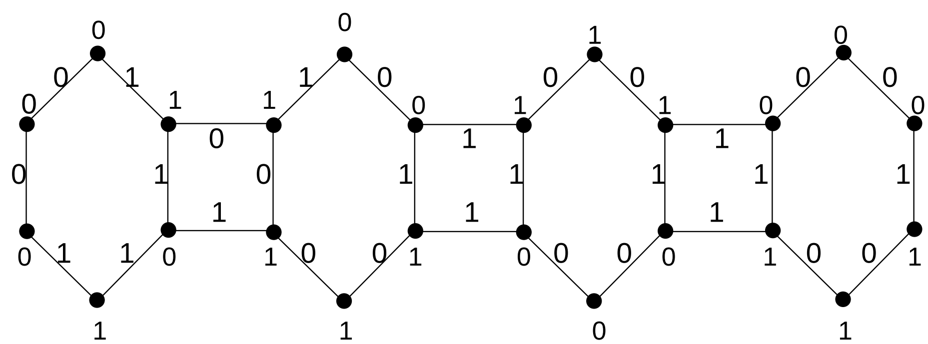

Figure 39: Cordial labeling of linear phenylene graph PH_{24}

In Figure 40 for $t = 5$ and $n = 30$, we get $v_f(0) = v_f(1) = 15$, $e_f(0) = e_f(1) = 19$

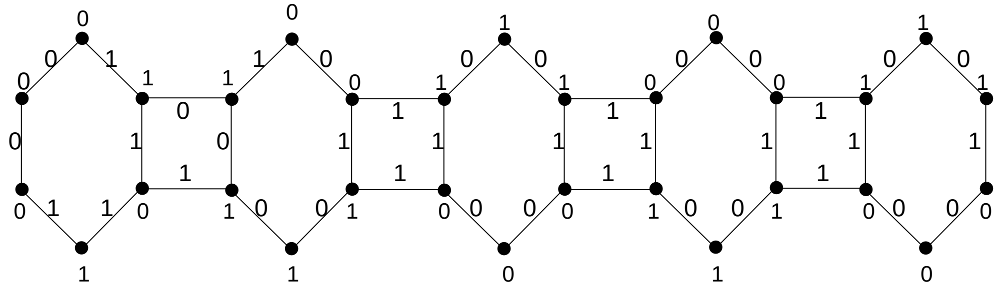

Figure 40: Cordial labeling of linear phenylene graph PH_{30}

In Figure 41 for $t = 6$ and $n = 36$, we get $v_f(0) = v_f(1) = 18$, $e_f(0) = e_f(1) = 23$

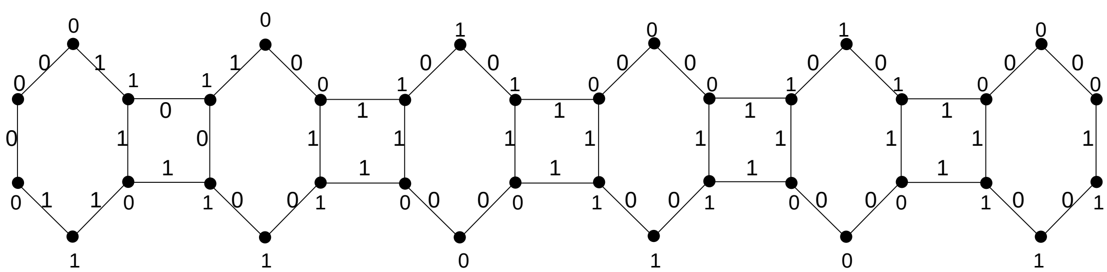

Figure 41: Cordial labeling of linear phenylene graph PH_{36}

□

Theorem 3.2. *The B_n graph is cordial for $n = 3t \;\; where \;\; t \geq 5$.*

Proof. Let $V(G) = \{x_i, y_i, z_i \,;\, 1 \leq i \leq t\}$ and $E(G) = \{x_i x_{i+1} \,;\, 1 \leq i \leq t-1\} \cup \{x_{i+1} y_i \,;\, 1 \leq i \leq t-1\} \cup \{y_i y i+1 \,;\, 1 \leq i \leq t-1\} \cup \{z_i z_{i+1} \,;\, 1 \leq i \leq t-1\} \cup \{y_i z_i \,;\, 1 \leq i \leq t\}$ be the vertex set and edge set respectively.

For $n = 3t \;\; ; \;\; t \geq 5$.

Following are the vertices defined:

$f(x_1) = 0 \;\; , \;\; f(x_2) = 1$

$f(x_i) = 0 \qquad for \qquad 3 \leq i \leq t$

$f(y_1 = 0$

$f(y_i) = 1 \qquad for \qquad 2 \leq i \leq t$

For even "t" z_i vertices are determined as:

$f(z_2) = 1$

$f(z_{t-1}) = 0 \quad when \quad t \equiv 0(mod\,4)$

$f(z_i) = 1 \quad when \quad i \equiv 0,3(mod\,4) \quad for \quad 1 \leq i \leq t$

$f(z_i) = 0 \quad when \quad i \equiv 1,2(mod\,4) \quad for \quad 1 \leq i \leq t$

For odd "t" z_i vertices are determined as:

$f(z_1) = 0$

$f(z_2) = 1$

$f(z_i) = 0 \quad for \quad i \equiv 1,2(mod\,4) \quad for \quad 1 \leq i \leq t$

$f(z_i) = 1 \quad for \quad i \equiv 0,3(mod\,4) \quad for \quad 1 \leq i \leq t$

Following are the edges defined:

$f(x_1x_2) = f(x_2x_3) = 1$

$f(x_ix_{i+1}) = 0 \quad for \quad 3 \leq i \leq t-1$

$f(y_1y_2) = 1$

$f(y_iy_{i+1}) = 0 \quad for \quad 2 \leq i \leq t-1$

$f(x_1y_1) = f(x_2y_2) = 0$

$f(x_iy_i) = 1 \quad for \quad 3 \leq i \leq t$

$f(x_{i+1}y_i) = 1 \quad for \quad 1 \leq i \leq t-1$

For odd "t" z_i edges are determined as:

$f(z_1z_2) = 1$

$f(z_2z_3) = 0$

$f(z_iz_{i+1}) = 1 \quad for \quad i \equiv 0,2(mod\,4) \quad where \quad 4 \leq i \leq t-1$

$f(z_iz_{i+1}) = 0 \quad for \quad i \equiv 1,3(mod\,4) \quad where \quad 4 \leq i \leq t-1$

$f(y_1z_1) = f(y_2z_2) = 0$

$f(y_iz_i) = 0 \quad for \quad i \equiv 0,3(mod\,4) \quad where \quad 3 \leq i \leq t$

$f(y_iz_i) = 1 \quad for \quad i \equiv 1,2(mod\,4) \quad where \quad 3 \leq i \leq t$

For even "t" z_i edges are determined as:

$f(z_2z_3) = 0$

$f(z_{t-1}z_t) = 1 \quad for \quad t \equiv 0(mod\,4)$

$f(z_i z_{i+1}) = 0 \quad for \quad i \equiv 1,3(mod\,4) \quad where \quad 3 \le i \le t-1$

$f(z_i z_{i+1}) = 1 \quad for \quad i \equiv 0,2(mod\,4) \quad where \quad 3 \le i \le t-1$

$f(y_1 z_1) = f(y_2 z_2) = 0$

$f(y_i z_i) = 0 \quad for \quad i \equiv 0,3(mod\,4) \quad where \quad 3 \le i \le t$

$f(y_i z_i) = 1 \quad for \quad i \equiv 1,2(mod\,4) \quad where \quad 3 \le i \le t$

$f(y_{t-1} z_t) = 1 \quad for \quad t \equiv 0(mod\,4)$

We obtained $v_f(0) = v_f(1) = \frac{3t}{2}$ vertices and $e_f(0) = e_f(1) = 3t-2$ edges when "t" is even, and $v_f(0) = \frac{3t-1}{2}$, $v_f(1) = \frac{3t+1}{2}$ vertices and $e_f(0) = e_f(1) = 3t-2$ edges when "t" is odd. So B_n graph is cordial.

For even "t":

In Figure 42 for $t = 6$ and $n = 18$, we get $v_f(0) = v_f(1) = 9$, $e_f(0) = e_f(1) = 16$

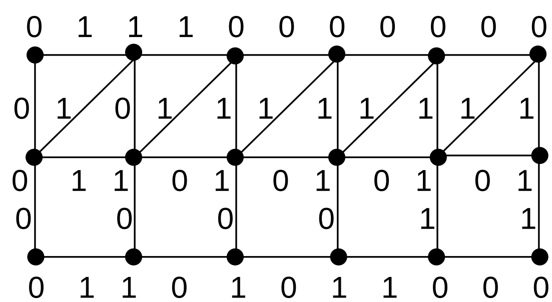

Figure 42: Cordial Labeling of B_{18} graph

In Figure 43 for $t = 8$ and $n = 24$, we get $v_f(0) = v_f(1) = 12$, $e_f(0) = e_f(1) = 22$

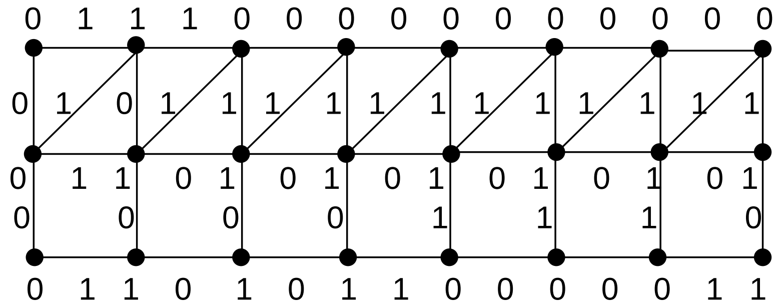

Figure 43: Cordial Labeling of B_{24} graph

In Figure 44 for $t = 10$ and $n = 30$, we get $v_f(0) = v_f(1) = 15$, $e_f(0) = e_f(1) = 28$

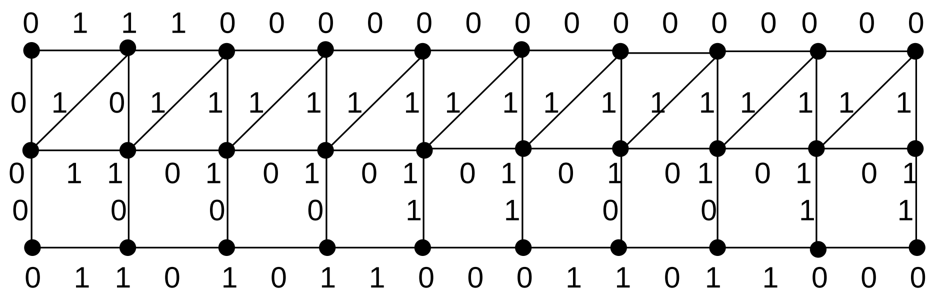

Figure 44: Cordial Labeling of B_{30} graph

In Figure 45 for $t = 12$ and $n = 36$, we get $v_f(0) = v_f(1) = 18$, $e_f(0) = e_f(1) = 34$

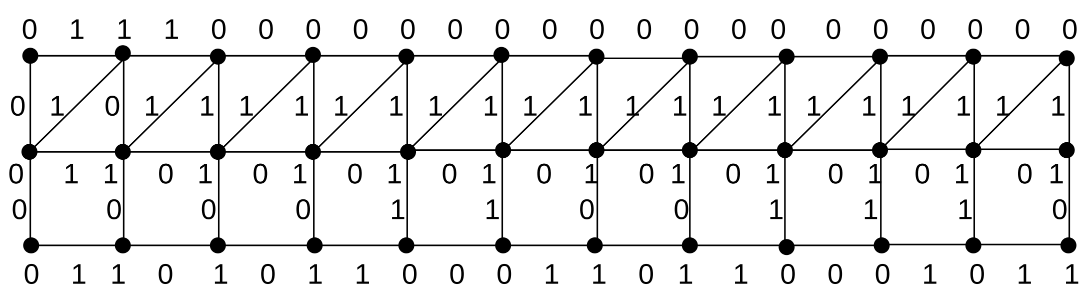

Figure 45: Cordial Labeling of B_{36} graph

For odd "t":

In Figure 46 for $t = 5$ and $n = 15$, we get $v_f(0) = 7$, $v_f(1) = 8$ and $e_f(0) = e_f(1) = 13$

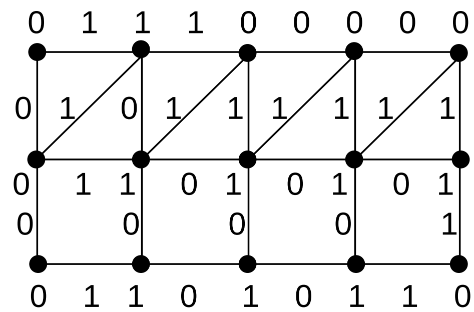

Figure 46: Cordial Labeling of B_{15} graph

In Figure 47 for $t = 7$ and $n = 21$, we get $v_f(0) = 10$, $v_f(1) = 11$ and $e_f(0) = e_f(1) = 19$

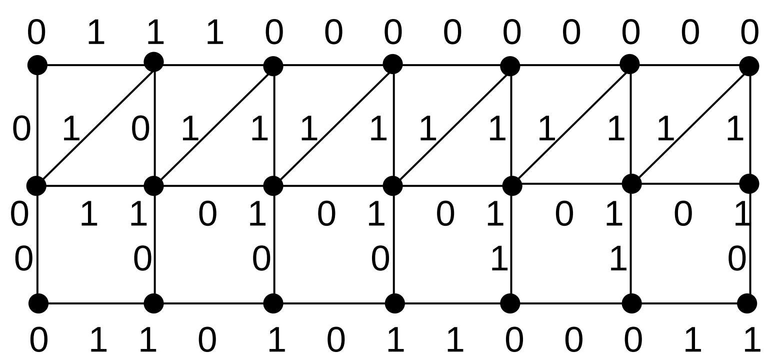

Figure 47: Cordial Labeling of B_{21} graph

In Figure 48 for $t = 9$ and $n = 27$, we get $v_f(0) = 13$, $v_f(1) = 14$ and $e_f(0) = e_f(1) = 25$

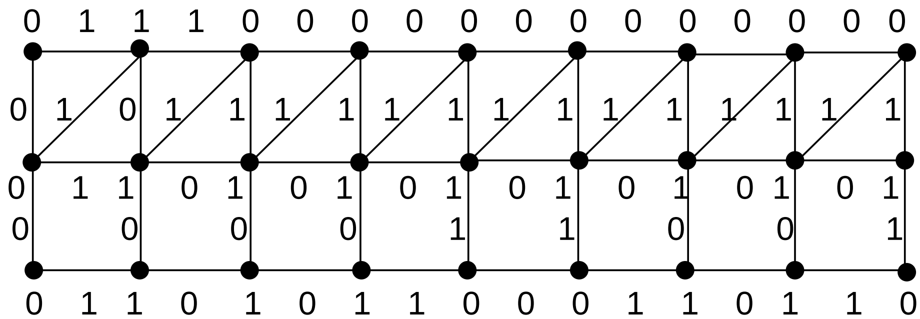

Figure 48: Cordial Labeling of B_{27} graph

In Figure 49 for $t = 11$ and $n = 33$, we get $v_f(0) = 16$, $v_f(1) = 17$ and $e_f(0) = e_f(1) = 31$

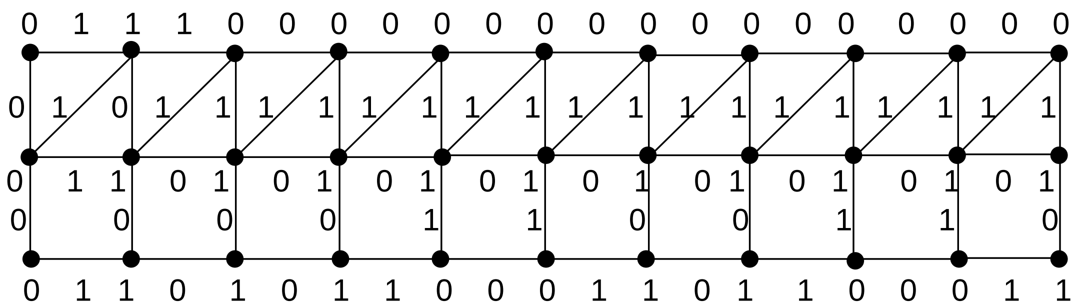

Figure 49: Cordial Labeling of B_{33} graph

□

Theorem 3.3. *Linear phenylene graph PH_n with pendant edges is cordial for $n = 12t$ where $t \geq 2$.*

Proof. Let $V(G) = \{x_i, y_i, u_i, v_i; 1 \leq i \leq 3t\}$ and $E(G) = \{x_ix_{i+1}; 1 \leq i \leq 3t-1\} \cup \{x_iu_i; 1 \leq i \leq 3t\} \cup \{y_iy_{i+1}; 1 \leq i \leq 3t-1\} \cup \{y_iv_i; 1 \leq i \leq 3t\} \cup \{x_1y_1\} \cup \{x_{3i}y_{3i}; 1 \leq i \leq t\} \cup \{x_{3i+1}y_{3i+1}; 1 \leq i \leq t-1\}$ be the vertex set and edge set respectively.

Let $n = 12t$ where $t \geq 2$.

Vertices of the graph are defined as follows:

$f(x_i) = 0; \quad for \quad i \equiv 0,1,2,5,6 (\bmod 9)$

$f(x_i) = 1; \quad for \quad i \equiv 3,4,7,8 (\bmod 9)$

$f(y_i) = 0; \quad for \quad i \equiv 0,3,5,8 (\bmod 9)$

$f(y_i) = 1; \quad for \quad i \equiv 1,2,4,6,7 (\bmod 9)$

$f(u_i) = 0; \quad for \quad 1 \leq i \leq 3t$

$f(v_i) = 1; \quad for \quad 1 \leq i \leq 3t$

Edges of the graph are defined as follows:

$f(x_i x_{i+1}) = 0; \quad for \quad i \equiv 0,1,3,5,7 (\bmod 9)$

$f(x_i x_{i+1}) = 1; \quad for \quad i \equiv 2,4,6,8 (\bmod 9)$

$f(y_i y_{i+1}) = 0; \quad for \quad i \equiv 1,6,8 (\bmod 9)$

$f(y_i y_{i+1}) = 1; \quad for \quad i \equiv 0,2,3,4,5,7 (\bmod 9)$

$f(x_i u_i) = 0; \quad for \quad i \equiv 0,1,2,5,6 (\bmod 9)$

$f(x_i u_i) = 1; \quad for \quad i \equiv 3,4,7,8 (\bmod 9)$

$f(y_i v_i) = 0; \quad for \quad i \equiv 1,2,4,6,7 (\bmod 9)$

$f(y_i v_i) = 1; \quad for \quad i \equiv 0,3,5,8 (\bmod 9)$

$f(x_i y_i) = 0; \quad for \quad i \equiv 0,4,7 (\bmod 9)$

$f(x_i y_i) = 1; \quad for \quad i \equiv 1,3,6 (\bmod 9)$

We obtained $v_f(0) = v_f(1) = 6t$ and $e_f(0) = e_f(1) = 7t - 1$ for vertices and edges respectively. The absolute difference between the 0 labeled vertices and 1 labeled vertices is 0. Similarly, the absolute difference between the 0 labeled edges and the 1 labeled edges is 0. So, the graph is cordial.

In Figure 50 for $t = 2$ and $n = 24$, we get $v_f(0) = v_f(1) = 12$, $e_f(0) = e_f(1) = 13$

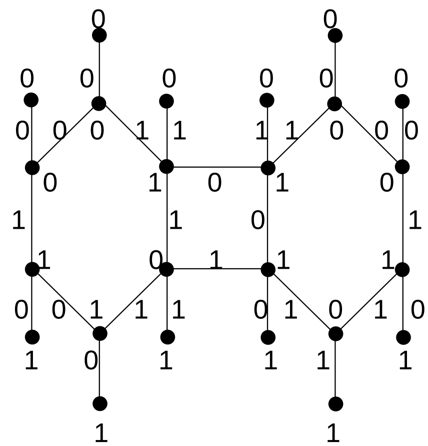

Figure 50: Cordial labeling of linear phenylene graph PH_{24} with pendant edges

In Figure 51 for $t = 3$ and $n = 36$, we get $v_f(0) = v_f(1) = 18$, $e_f(0) = e_f(1) = 20$

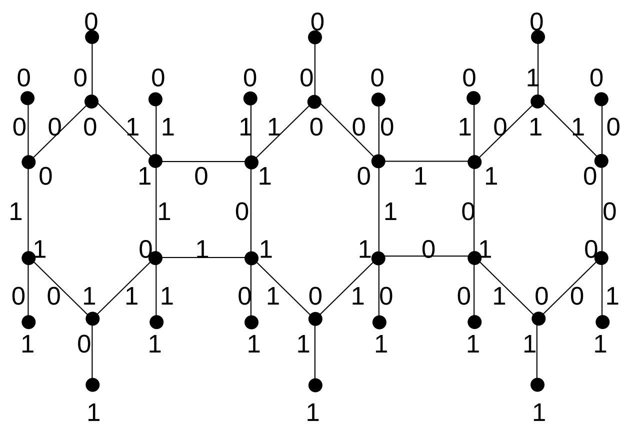

Figure 51: Cordial labeling of linear phenylene graph PH_{36} with pendant edges

In Figure 52 for $t = 4$ and $n = 48$, we get $v_f(0) = v_f(1) = 24$, $e_f(0) = e_f(1) = 27$

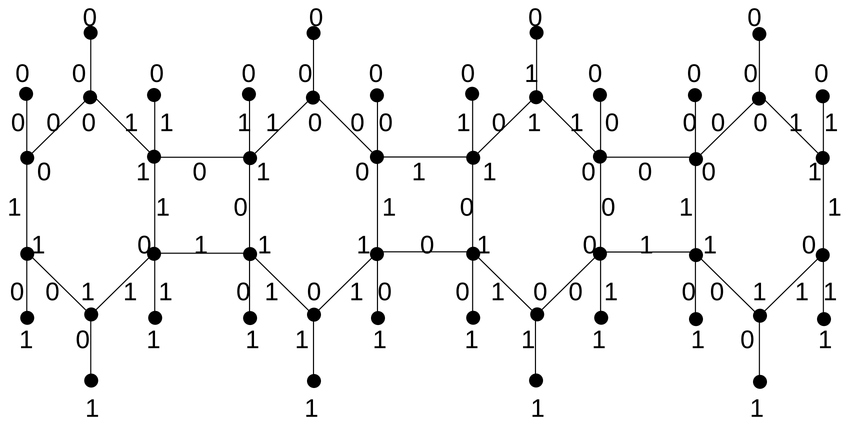

Figure 52: Cordial labeling of linear phenylene graph PH_{48} with pendant edges

In Figure 53 for $t = 5$ and $n = 60$, we get $v_f(0) = v_f(1) = 30$, $e_f(0) = e_f(1) = 34$

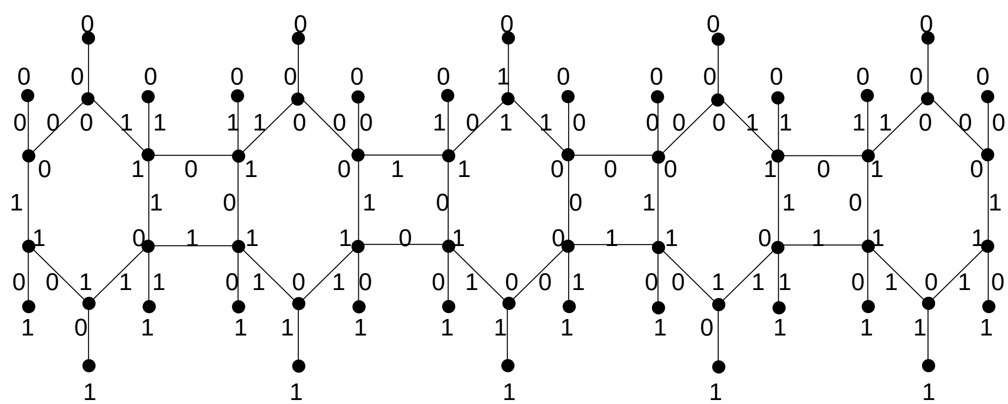

Figure 53: Cordial labeling of linear phenylene graph PH_{60} with pendant edges

In Figure 54 for $t = 6$ and $n = 72$, we get $v_f(0) = v_f(1) = 36$, $e_f(0) = e_f(1) = 41$

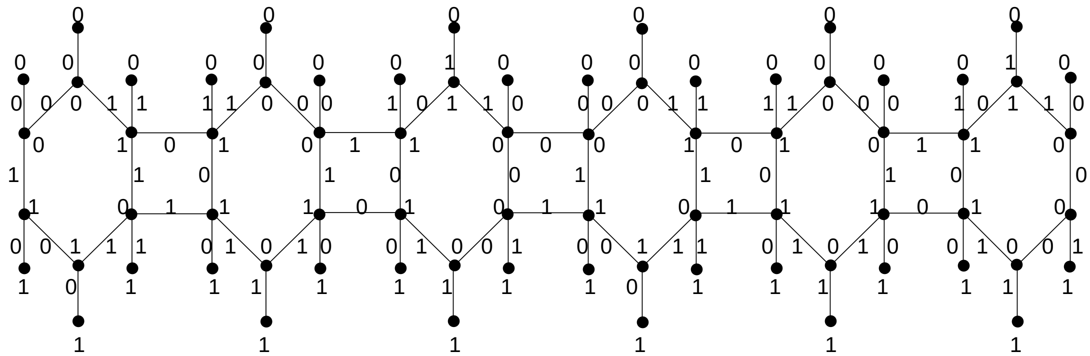

Figure 54: Cordial labeling of linear phenylene graph PH_{72} with pendant edges

$\square$

Theorem 3.4. *The B_n graph with pendant edges is cordial for $n = 5t$ where $t \geq 4$.*

Proof. Let $V(G) = \{x_i, y_i, z_i, u_i, v_i; 1 \leq i \leq t\}$ and $E(G) = \{x_i x_{i+1}; 1 \leq i \leq t-1\} \cup \{y_i y_{i+1}; 1 \leq i \leq t-1\} \cup \{x_i u_i; 1 \leq i \leq t\} \cup \{z_i z_{i+1}; 1 \leq i \leq t-1\} \cup \{x_i y_i; 1 \leq i \leq t\} \cup \{x_{i+1} y_i; 1 \leq i \leq t-1\} \cup \{z_i y_i; 1 \leq i \leq t\} \cup \{z_i v_i; 1 \leq i \leq t\}$ be the vertex set and edge set respectively.

For $n = 5t$ where $t \geq 4$.

Following are the vertices of the graph defined:

$f(x_1) = 0$

$f(x_i) = 1; \quad for \quad 2 \leq i \leq 5$

$f(x_i) = 0; \quad for \quad i \equiv 0 (mod\, 2) \quad where \quad 6 \leq i \leq t$

$f(x_i) = 1; \quad for \quad i \equiv 1 (mod\, 2) \quad where \quad 6 \leq i \leq t$

$f(y_2) = f(y_3) = f(y_6) = 1$

$f(y_1) = f(y_4) = f(y_5) = 0$

$f(y_i) = 0; \quad for \quad 7 \leq i \leq t$

$f(z_2) = 1$

$f(z_1) = f(z_3) = f(z_4) = f(z_5) = 0$

$f(z_i) = 1; \quad for \quad i \equiv 0 (mod\, 2) \quad where \quad 6 \leq i \leq t$

$f(z_i) = 0; \quad for \quad i \equiv 1 (mod\, 2) \quad where \quad 6 \leq i \leq t$

$f(u_i) = 0; \quad for \quad 1 \leq i \leq 6$

$f(u_i) = 1; \quad for \quad 7 \leq i \leq t$

$f(v_1) = f(v_3) = f(v_5) = 1$

$f(v_i) = 1; \quad for \quad i \equiv 0 (mod\, 2) \quad where \quad 2 \leq i \leq t$

$f(v_i) = 0; \quad for \quad i \equiv 1 (mod\, 2) \quad where \quad 6 \leq i \leq t$

Edges of the graph are defined below:

$f(x_i x_{i+1}) = 0; \quad for \quad 2 \leq i \leq 4$

$f(x_1 x_2) = 1$

$f(x_i x_{i+1}) = 1; \quad for \quad 5 \leq i \leq t-1$

$f(x_6 u_6) = 0$

$f(x_i u_i) = 1; \quad for \quad 2 \leq i \leq 5$

$f(x_i u_i) = 0; \quad for \quad i \equiv 1 (mod\, 2) \quad where \quad 7 \leq i \leq t$

$f(x_i u_i) = 1; \quad for \quad i \equiv 0 (mod\, 2) \quad where \quad 7 \leq i \leq t$

$f(x_i y_i) = 0; \quad for \quad 1 \leq i \leq 3$

$f(x_4 y_4) = f(x_6 y_6) = 1$

$f(x_i y_i) = 1; \quad for \quad i \equiv 1 (mod\, 2) \quad where \quad 5 \leq i \leq t$

$f(x_i y_i) = 0; \quad for \quad i \equiv 0 (mod\, 2) \quad where \quad 7 \leq i \leq t$

$f(x_3 y_2) = f(x_4 y_3) = f(x_7 y_6) = 0$

$f(x_2 y_1) = f(x_5 y_4) = 1$

$f(x_{i+1} y_i) = 0; \quad for \quad i \equiv 1 (mod\, 2) \quad where \quad 5 \leq i \leq t-1$

$f(x_{i+1} y_i) = 1; \quad for \quad i \equiv 0 (mod\, 2) \quad where \quad 7 \leq i \leq t-1$

$f(y_2 y_3) = f(y_4 y_5) = 0$

$f(y_1 y_2) = f(y_3 y_4) = f(y_5 y_6) = f(y_6 y_7) = 1$

$f(y_i y_{(i+1)}) = 0; \quad for \quad 7 \leq i \leq t-1$

$f(z_1 z_2) = f(z_2 z_3) = 1$

$f(z_3 z_4) = f(z_4 z_5) = 0$

$f(z_i z_{i+1}) = 1; \quad for \quad 5 \leq i \leq t-1$

$f(y_1 z_1) = f(y_2 z_2) = f(y_4 z_4) = f(y_6 z_6) = 0$

$f(y_3 z_3) = 1$

$f(y_i z_i) = 1; \quad for \quad i \equiv 0 (mod\, 2) \quad where \quad 7 \leq i \leq t$

$f(y_i z_i) = 0; \quad for \quad i \equiv 1 (mod\, 2) \quad where \quad 5 \leq i \leq t$

$f(z_1 v_1) = f(z_3 v_3) = f(z_4 v_4) = f(z_5 v_5) = 1$

$f(z_2 v_2) = 0$

$f(z_i v_i) = 0; \quad for \quad 6 \leq i \leq t$

We obtained $v_f(0) = v_f(1) = \frac{5t}{2}$ and $e_f(0) = e_f(1) = 4t-2$ where $t \geq 4$ vertices and edges respectively for even case, and $v_f(0) = \frac{5t+1}{2}$, $v_f(1) = \frac{5t-1}{2}$ vertices and $e_f(0) = e_f(1) = 4t-2$ edges where $t \geq 4$ for odd case. So the graph B_n for $n = 5t$, $t \geq 4$ is cordial.

For even 't':

In Figure 55 for $t = 4$ and $n = 20$, we get $v_f(0) = v_f(1) = 10$, $e_f(0) = e_f(1) = 14$

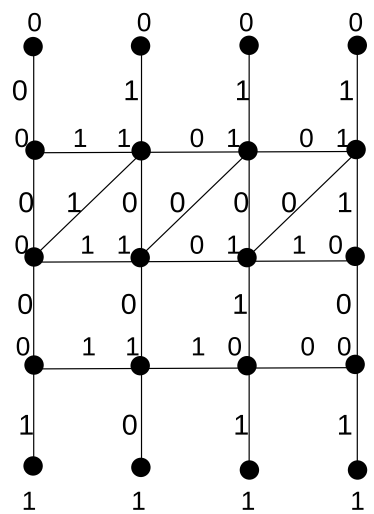

Figure 55: Cordial labeling of B_{20} graph

In Figure 56 for $t = 6$ and $n = 30$, we get $v_f(0) = v_f(1) = 15$, $e_f(0) = e_f(1) = 22$

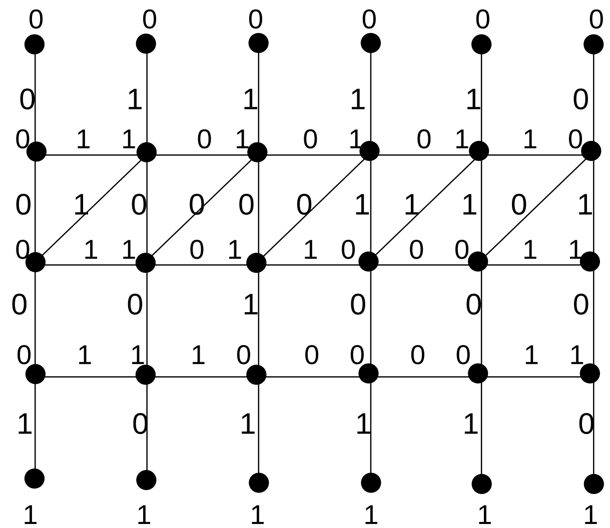

Figure 56: Cordial labeling of B_{30} graph

In Figure 57 for $t = 8$ and $n = 40$, we get $v_f(0) = v_f(1) = 20$, $e_f(0) = e_f(1) = 30$

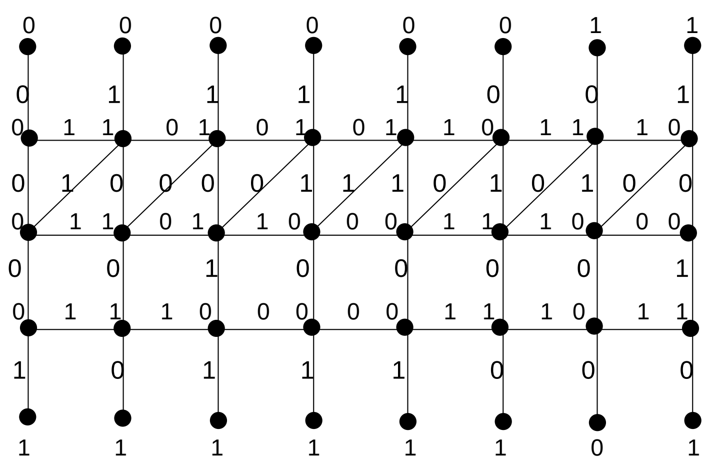

Figure 57: Cordial labeling of B_{40} graph

For odd 't':

In Figure 58 for $t = 5$ and $n = 25$, we get $v_f(0) = 13$, $v_f(1) = 12$ and $e_f(0) = e_f(1) =$ 18

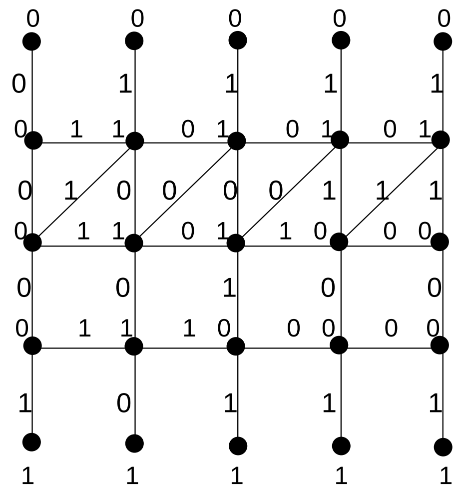

Figure 58: Cordial labeling of B_{25} graph

In Figure 59 for $t = 7$ and $n = 35$, we get $v_f(0) = 18$, $v_f(1) = 17$ and $e_f(0) = e_f(1) = 26$

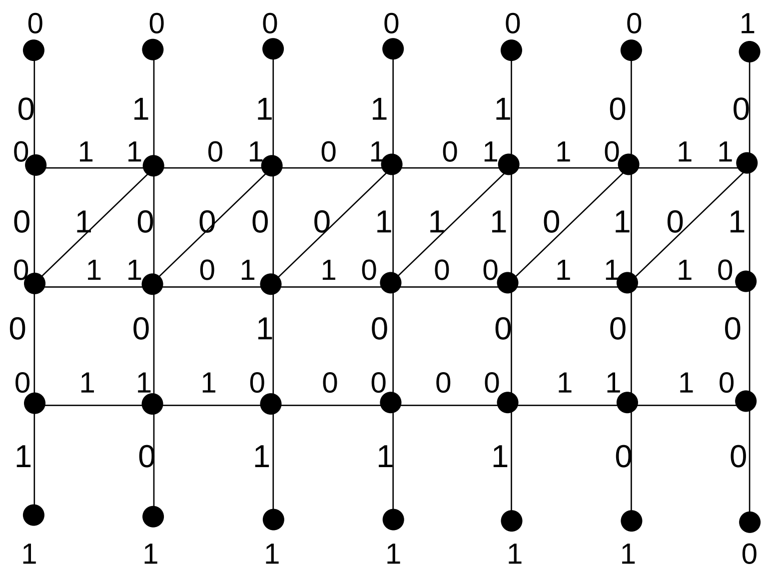

Figure 59: Cordial labeling of B_{35} graph

In Figure 60 for $t = 9$ and $n = 45$, we get $v_f(0) = 23$, $v_f(1) = 22$ and $e_f(0) = e_f(1) = 34$

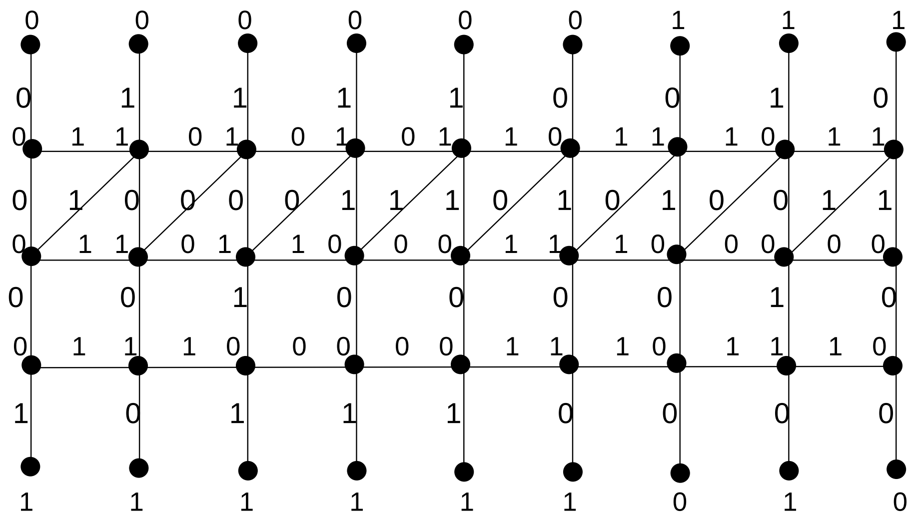

Figure 60: Cordial labeling of B_{45} graph

$\square$

Chapter 4

Edge Product Cordial Labeling of B_n and Linear Phenylene Graphs with Pendant Edges

4 Edge Product Cordial Labeling of B_n and Linear Phenylene Graphs with Pendant Edges

This chapter contains the results relating to the edge product cordial labeling of B_n graph and linear phenylene graph.

Theorem 4.1. *Linear phenylene graph PH_n with pendant edges is edge product cordial for $n = 12t$ where $t \geq 2$.*

Proof. Let $n = 12t$ where $t \geq 2$ then $V(G) = \{x_i, y_i, u_i, v_i; 1 \leq i \leq 3t\}$ and $E(G) = \{x_iu_i; 1 \leq i \leq 3t\} \cup \{x_ix_{i+1}; 1 \leq i \leq 3t-1\} \cup \{x_{3i}y_{3i}; 1 \leq i \leq t\} \cup \{x_{3i+1}y_{3i+1}; 1 \leq i \leq t-1\} \cup \{x_1y_1\} \cup \{y_iy_{i+1}; 1 \leq i \leq 3t-1\} \cup \{y_iv_i; 1 \leq i \leq 3t\}$ be the vertex-set and edge-set of the graph respectively.

Vertices of the graph are defined as follows:

$f(x_i) = 0; \quad for \quad 1 \leq i \leq 3t$

$f(u_i) = 1; \quad for \quad 1 \leq i \leq 3t$

$f(y_i) = 0; \quad for \quad 1 \leq i \leq 3t$

$f(v_i) = 1; \quad for \quad 1 \leq i \leq 3t$

And edges of the graph are defined as follows:

$f(x_ix_{i+1}) = 0; \quad for \quad i \equiv 0, 1 (mod\, 3) \quad where \quad 1 \leq i \leq 3t-1$

$f(x_ix_{i+1}) = 1; \quad for \quad i \equiv 2 (mod\, 3) \quad where \quad 2 \leq i \leq 3t-1$

$f(y_iy_{i+1}) = 0; \quad for \quad 1 \leq i \leq 3t-1$

$f(x_iu_i) = 1; \quad for \quad 1 \leq i \leq 3t$

$f(y_iv_i) = 1; \quad for \quad 1 \leq i \leq 3t$

$f(x_iy_i) = 0 \quad for \quad i \equiv 0, 1 (mod\, 3)$

We obtain $e_f(0) = e_f(1) = 7t - 1$ and $v_f(0) = v_f(1) = 6t$ where $t \geq 2$. So the graph is edge product cordial.

In Figure 61 for $t = 2$ and $n = 24$, we get $e_f(0) = e_f(1) = 13$, $v_f(0) = v_f(1) = 12$.

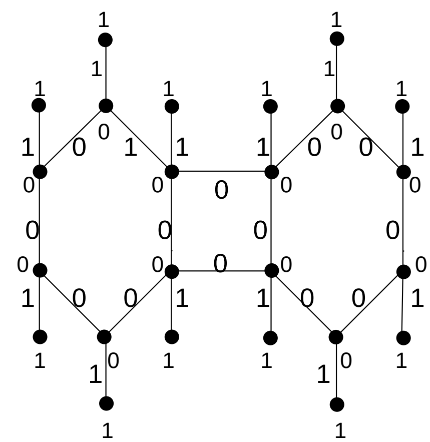

Figure 61: EPC labeling of linear phenylene graph PH_{24}

In Figure 62 for $t = 3$ and $n = 36$, we get $e_f(0) = e_f(1) = 20$, $v_f(0) = v_f(1) = 18$

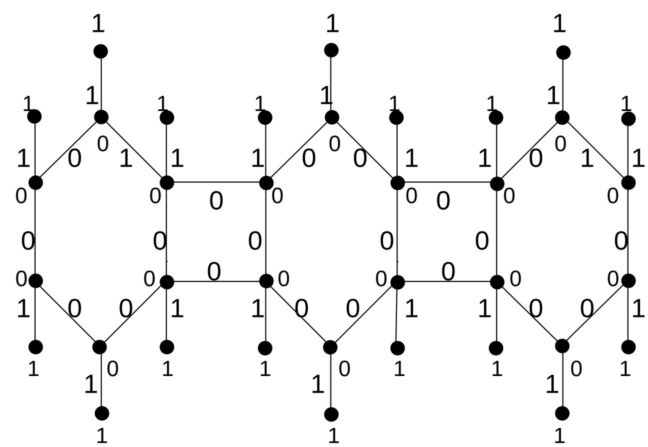

Figure 62: EPC labeling of linear phenylene graph PH_{36}

In Figure 63 for $t = 4$ and $n = 48$, we get $e_f(0) = e_f(1) = 27$, $v_f(0) = v_f(1) = 24$

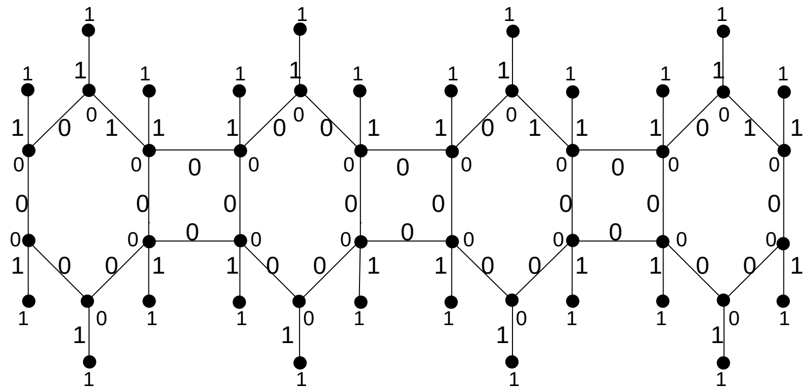

Figure 63: EPC labeling of linear phenylene graph PH_{48}

In Figure 64 for $t = 5$ and $n = 60$, we get $e_f(0) = e_f(1) = 34$, $v_f(0) = v_f(1) = 30$

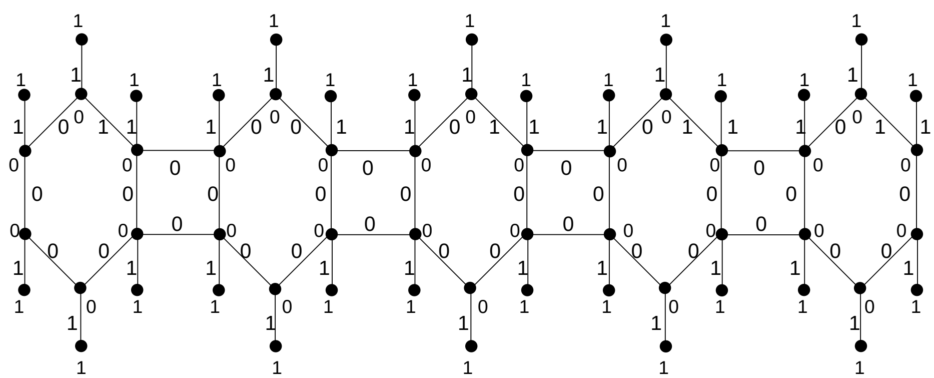

Figure 64: EPC labeling of linear phenylene graph PH_{60}

In 65 for $t = 6$ and $n = 72$, we get $e_f(0) = e_f(1) = 41$, $v_f(0) = v_f(1) = 36$

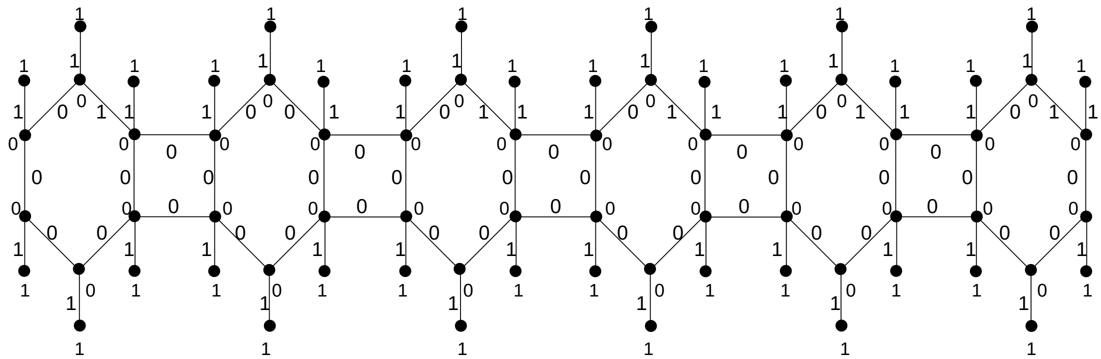

Figure 65: EPC labeling of linear phenylene graph PH_{72}

□

Theorem 4.2. *The B_n graph with pendant edges is edge product cordial for $n = 5t$ where $t \geq 4$.*

Proof. let $V(G) = \{x_i, yi, z_i, v_i, w_i; 1 \leq i \leq t\}$ and $E(G) = \{x_ix_{i+1}; 1 \leq i \leq t-1\} \cup \{x_iv_i; 1 \leq i \leq t\} \cup \{x_iy_i; 1 \leq i \leq t\} \cup \{x_{i+1}yi; 1 \leq i \leq t-1\} \cup \{y_iy_{i+1}; 1 \leq i \leq t-1\} \cup \{y_iz_i; 1 \leq i \leq t\} \cup \{z_iz_{i+1}; 1 \leq i \leq t-1\} \cup \{z_iw_i; 1 \leq i \leq t\}$ be the vertex set and edge set respectively.

For $n = 5t$ where $t \geq 4$.

Edges of the graph are defined as follow:

$f(x_iv_i) = 1; \quad for \quad 1 \leq i \leq t$

$f(x_ix_{i+1}) = 0; \quad for \quad 1 \leq i \leq t-1$

$f(x_iy_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(x_{i+1}y_i) = 0; \quad for \quad 1 \leq i \leq t-1$

$f(y_iy_{i+1}) = 0; \quad for \quad 1 \leq i \leq 4$

$f(y_iy_{i+1}) = 1; \quad for \quad i \equiv 1(mod\, 2) \quad where \quad 5 \leq i \leq t-1$

$f(y_iy_{i+1}) = 0; \quad for \quad i \equiv 0(mod\, 2) \quad where \quad 5 \leq i \leq t-1$

$f(y_iz_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(z_1z_2) = 1$

$f(z_iz_{i+1}) = 0; \quad for \quad 2 \leq i \leq \lceil \frac{t-1}{2} \rceil$

$f(z_iz_{i+1}) = 1; \quad for \quad \lceil \frac{t-1}{2} \rceil + 1 \leq i \leq t-1$

$f(z_i w_i) = 1; \quad for \quad 1 \leq i \leq t$

Vertices of the graph are defined as follows:

$f(v_i) = 1; \quad for \quad 1 \leq i \leq t$

$f(w_i) = 1; \quad for \quad 1 \leq i \leq t$

$f(x_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(y_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(z_1) = 1; \quad for \quad i = 1$

$f(z_i) = 0; \quad for \quad 2 \leq i \leq \lceil \frac{t-1}{2} \rceil + 1$

$f(z_i) = 1; \quad for \quad \lceil \frac{t-1}{2} \rceil + 2 \leq i \leq t$

We obtained $v_f(0) = v_f(1) = \frac{5t}{2}$ and $e_f(0) = e_f(1) = 4t - 2$ where $t \geq 4$ vertices and edges for even number of vertices. And $v_f(0) = \frac{5t-1}{2}$, $v_f(1) = \frac{5t+1}{2}$ vertices and $e_f(0) = e_f(1) = 4t - 2$ edges where $t \geq 4$ for odd number of vertices.

For even 't':

In Figure 66 for $t = 4$ and $n = 20$, we have $v_f(0) = v_f(1) = 10$, $e_f(0) = e_f(1) = 14$

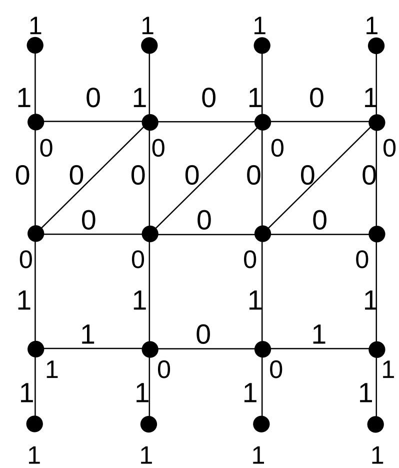

Figure 66: EPC labeling of B_{20} graph

In Figure 67 for $t = 6$ and $n = 30$, we have $v_f(0) = v_f(1) = 15$, $e_f(0) = e_f(1) = 22$

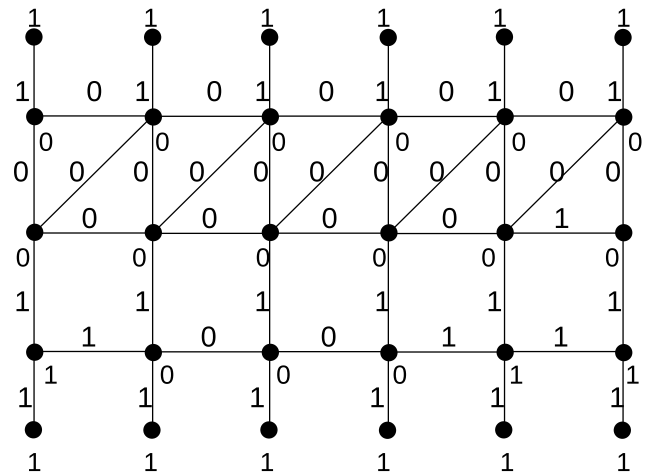

Figure 67: EPC labeling of B_{30} graph

In Figure 68 for $t = 8$ and $n = 40$, we have $v_f(0) = v_f(1) = 20$, $e_f(0) = e_f(1) = 30$

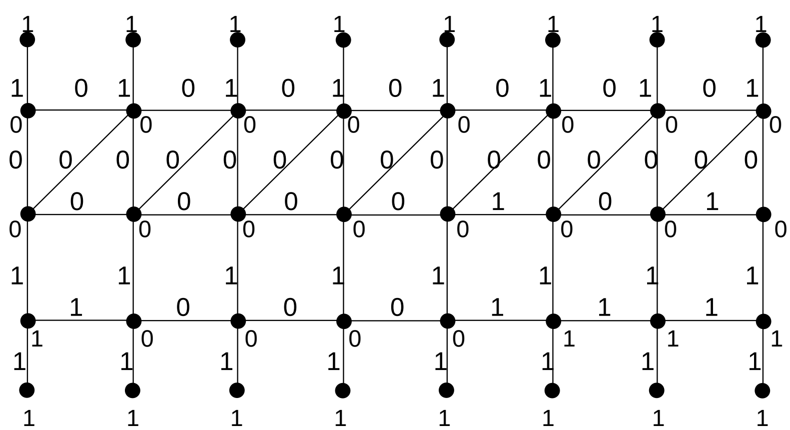

Figure 68: EPC labeling of B_{40} graph

For odd 't':

In Figure 69 for $t = 5$ and $n = 25$, we have $v_f(0) = 12$, $v_f(1) = 13$ and $e_f(0) = e_f(1) = 18$

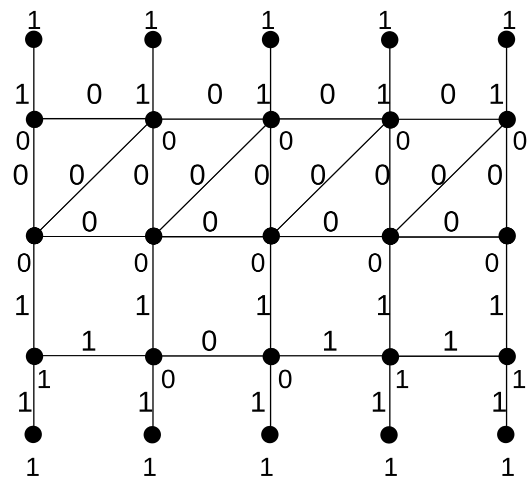

Figure 69: EPC labeling of B_{25} graph

In Figure 70 for $t = 7$ and $n = 35$, we have $v_f(0) = 17$, $v_f(1) = 18$ and $e_f(0) = e_f(1) = 26$

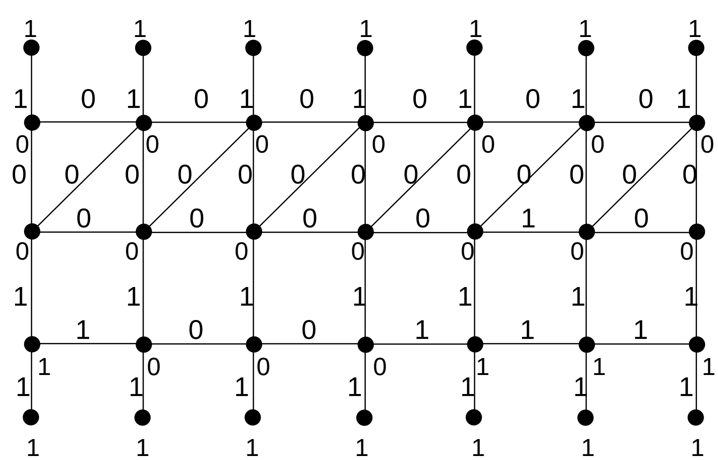

Figure 70: EPC labeling of B_{35} graph

In Figure 71 for $t = 9$ and $n = 45$, we have $v_f(0) = 22$, $v_f(1) = 23$ and $e_f(0) = e_f(1) = 34$

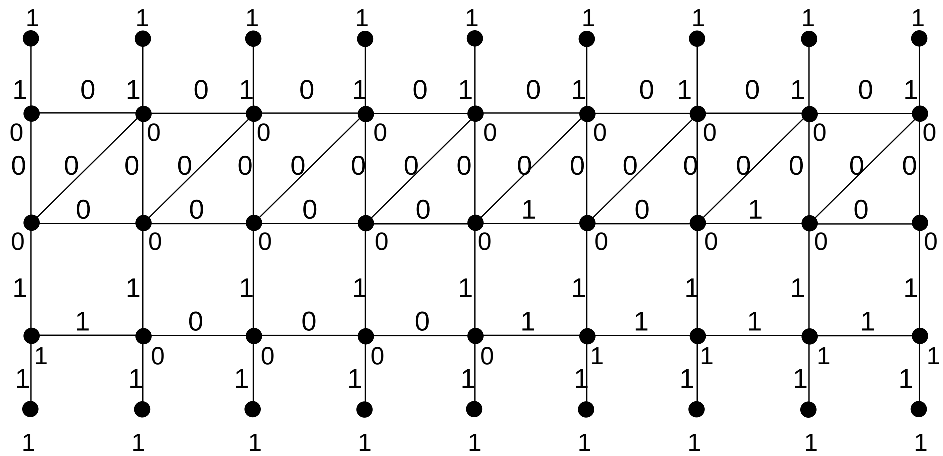

Figure 71: EPC labeling of B_{45} graph

□

Chapter 5

Total Edge Product Cordial Labeling of B_n and Linear Phenylene Graphs

5 Total Edge Product Cordial Labeling of B_n and Linear Phenylene Graphs

Here we have deduced the results relating to the total edge product cordial labeling of graphs.

Theorem 5.1. *The linear phenylene graph PH_n is total edge product cordial for $n = 6t$.*

Proof. Let $V(G) = \{x_i, y_i; 1 \leq i \leq 3t\}$ be the vertices and $E(G) = \{x_i x_{i+1}; 1 \leq i \leq 3t - 1\} \cup \{y_i y_{i+1}; 1 \leq i \leq 3t-1\} \cup \{x_{3i} y_{3i}; 1 \leq i \leq t\} \cup \{x_{3i+1} y_{3i+1}; 1 \leq i \leq t-1\} \cup \{x_1 y_1\}$ be the edges of the graph.

For $n = 6t$ where $t \geq 1$.

Vertices of the graph are determined as:

$f(x_i) = 1 \quad for \quad i \equiv 0 (mod\, 3) \quad where \quad 1 \leq i \leq 3t$

$f(x_i) = 0 \quad for \quad i \equiv 1,2 (mod\, 3) \quad where \quad 1 \leq i \leq 3t$

$f(y_1) = 1$

$f(y_3) = 0$

$f(y_i) = 1 \quad for \quad i \equiv 0 (mod\, 3) \quad where \quad 2 \leq i \leq 3t$

$f(y_i) = 0 \quad for \quad i \equiv 1,2 (mod\, 3) \quad where \quad 2 \leq i \leq 3t$

Following are the edges of the graph defined:

$f(x_i x_{i+1}) = 0 \quad for \quad i \equiv 1 (mod\, 3) \quad where \quad 1 \leq i \leq 3t-1$

$f(x_i x_{i+1}) = 1 \quad for \quad i \equiv 0,2 (mod\, 3) \quad where \quad 2 \leq i \leq 3t-1$

$f(y_i y_{i+1}) = 0 \quad for \quad i \equiv 1 (mod\, 3) \quad where \quad 1 \leq i \leq 3t-1$

$f(y_i y_{i+1}) = 1 \quad for \quad i \equiv 0,2 (mod\, 3) \quad where \quad 2 \leq i \leq 3t-1$

$f(x_1 y_1) = 1$

$f(x_i y_i) = 1 \quad for \quad i \equiv 0 (mod\, 3) \quad where \quad 1 \leq i \leq t$

$f(x_i y_i) = 0 \quad for \quad i \equiv 1 (mod\, 3) \quad where \quad 1 \leq i \leq t-1$

We obtained $sum(0) = sum(1) = 7t - 1$.

In Figure 72 for $t = 1$ and $n = 6$, we get $sum(0) = sum(1) = 6$

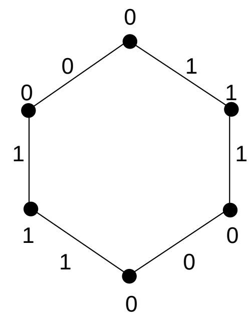

Figure 72: TEPC Labeling of PH_6 Linear phenylene graph

In Figure 73 for $t = 2$ and $n = 12$, we get $sum(0) = sum(1) = 13$

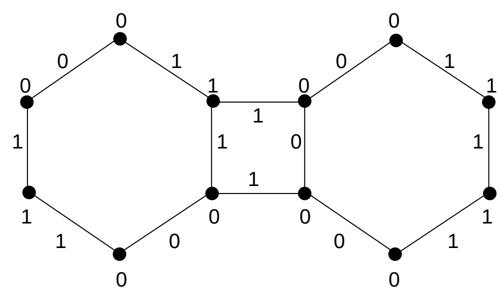

Figure 73: TEPC Labeling of PH_{12} Linear phenylene graph

In Figure 74 for $t = 3$ and $n = 18$, we get $sum(0) = sum(1) = 20$

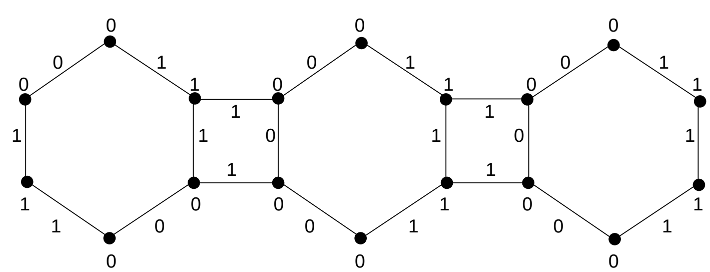

Figure 74: TEPC Labeling of PH_{18} Linear phenylene graph

In Figure 75 for $t = 4$ and $n = 24$, we get $sum(0) = sum(1) = 27$

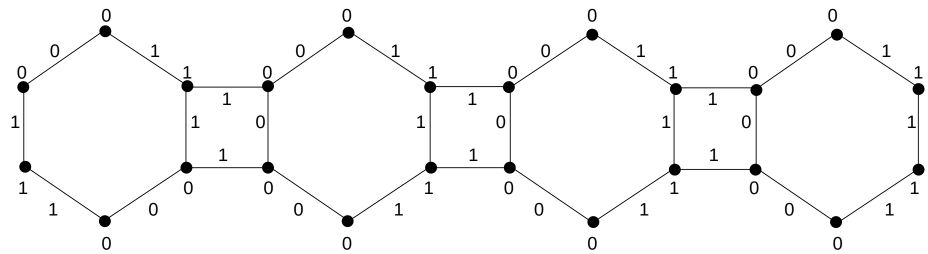

Figure 75: TEPC Labeling of PH_{24} Linear phenylene graph

□

Theorem 5.2. *The B_n graph is total edge product cordial for $n = 3t$ where $t \leq 2$.*

Proof. Let $V(G) = \{x_i, y_i, z_i; 1 \leq i \leq t\}$ and $E(G) = \{x_ix_{i+1}; 1 \leq i \leq t-1\} \cup \{y_iy_{i+1}; 1 \leq i \leq t-1\} \cup \{z_iz_{i+1}; 1 \leq i \leq t-1\} \cup \{x_iy_i; 1 \leq i \leq t\} \cup \{y_iz_i; 1 \leq i \leq t\} \cup \{x_{i+1}y_i; 1 \leq i \leq t-1\}$ be the vertex set and edge set of the graph.

For $n = 3t$ where $t \leq 2$.

Following are the vertices defined:

$f(x_i) = 1 \quad for \quad 1 \leq i \leq t$

$f(y_i) = 0 \quad for \quad 1 \leq i \leq t$

$f(z_i) = 0 \quad for \quad 1 \leq i \leq t$

Edges are determined as follows:

$f(x_ix_{i+1}) = 1 \quad for \quad 1 \leq i \leq t-1$

$f(y_1y_2) = 1$

$f(y_iy_{i+1}) = 1 \quad for \quad i \equiv 0(mod\,2) \quad where \quad 2 \leq i \leq t-1$

$f(y_iy_{i+1}) = 0 \quad for \quad i \equiv 1(mod\,2) \quad where \quad 2 \leq i \leq t-1$

$f(x_{i+1}y_i) = 1 \quad for \quad 1 \leq i \leq t-1$

$f(x_iy_i) = 1 \quad for \quad 1 \leq i \leq t$

$f(y_iz_i) = 0 \quad for \quad 1 \leq i \leq t$

$f(z_iz_{i+1}) = 0 \quad for \quad 1 \leq i \leq t-1$

We obtained $sum(0) = sum(1) = \frac{9t-4}{2}$ for even case, and $sum(0) = \frac{9t-5}{2}$ and $sum(1) = \frac{9t-3}{2}$ for odd case.

For even 't':

In Figure 76 for $t = 2$ and $n = 6$, we get $sum(0) = sum(1) = 7$

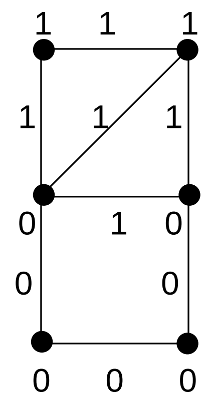

Figure 76: TEPC Labeling of B_6

In Figure 77 for $t = 4$ and $n = 12$, we get $sum(0) = sum(1) = 16$

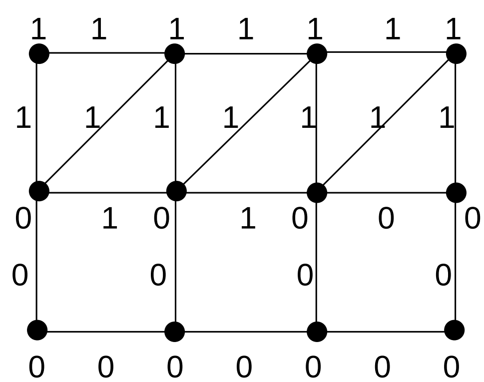

Figure 77: TEPC Labeling of B_{12}

In Figure 78for $t = 6$ and $n = 18$, we get $sum(0) = sum(1) = 25$

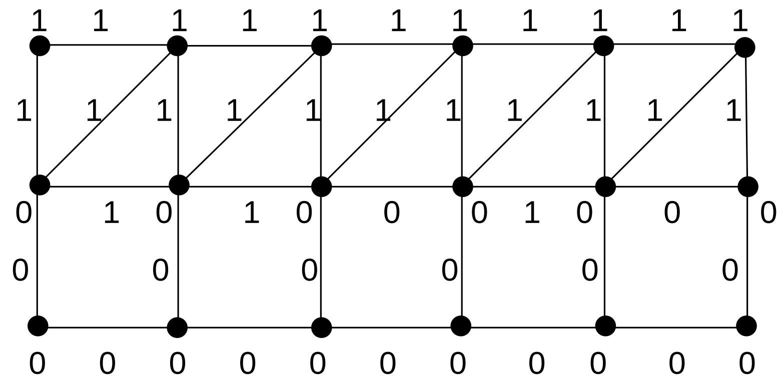

Figure 78: TEPC Labeling of B_{18}

For odd 't':

In Figure 79for $t = 3$ and $n = 9$, we get $sum(1) = 12$, $sum(0) = 11$

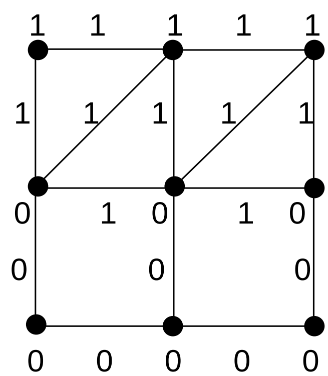

Figure 79: TEPC Labeling of B_9

In Figure 80 for $t = 5$ and $n = 15$, we get $sum(1) = 21$, $sum(0) = 20$

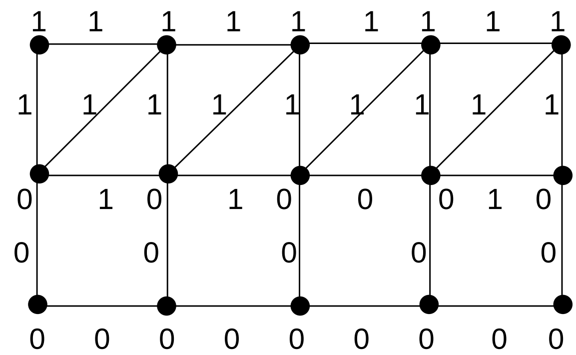

Figure 80: TEPC Labeling of B_{15}

In Figure 81 for $t = 7$ and $n = 21$, we get $sum(1) = 30$, $sum(0) = 29$

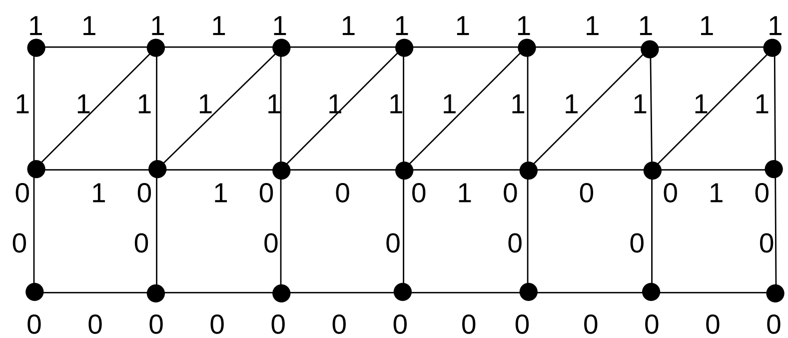

Figure 81: TEPC Labeling of B_{21}

$\square$

Theorem 5.3. *The B_n graph with pendant edges is total edge product cordial for $n = 5t$ where $t \geq 4$.*

Proof. Let $n = 5t$ where $t \geq 4$.

Edges of the graph are defined as follow:

$f(x_i v_i) = 1; \quad for \quad 1 \leq i \leq t$

$f(x_i x_{i+1}) = 0; \quad for \quad 1 \leq i \leq t-1$

$f(x_i y_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(x_{i+1}y_i) = 0; \quad for \quad 1 \leq i \leq t-1$

$f(y_iy_{i+1}) = 0; \quad for \quad 1 \leq i \leq 4$

$f(y_iy_{i+1}) = 1; \quad for \quad i \equiv 1(mod\,2) \quad where \quad 5 \leq i \leq t-1$

$f(y_iy_{i+1}) = 0; \quad for \quad i \equiv 0(mod\,2) \quad where \quad 5 \leq i \leq t-1$

$f(y_iz_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(z_1z_2) = 1$

$f(z_iz_{i+1}) = 0; \quad for \quad 2 \leq i \leq \lceil \frac{t-1}{2} \rceil$

$f(z_iz_{i+1}) = 1; \quad for \quad \lceil \frac{t-1}{2} \rceil + 1 \leq i \leq t-1$

$f(z_iw_i) = 1; \quad for \quad 1 \leq i \leq t$

Vertices of the graph are defined as follows:

$f(v_i) = 1; \quad for \quad 1 \leq i \leq t$

$f(w_i) = 1; \quad for \quad 1 \leq i \leq t$

$f(x_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(y_i) = 0; \quad for \quad 1 \leq i \leq t$

$f(z_1) = 1; \quad for \quad i = 1$

$f(z_i) = 0; \quad for \quad 2 \leq i \leq \lceil \frac{t-1}{2} \rceil + 1$

$f(z_i) = 1; \quad for \quad \lceil \frac{t-1}{2} \rceil + 2 \leq i \leq t$

We obtained $sum(0) = sum(1) = \frac{13t-4}{2}$ where $t \geq 4$ for even number of vertices. And $sum(0) = \frac{13t-5}{2}$, $sum(1) = \frac{13t-3}{2}$ where $t \geq 4$ for odd number of vertices.

For even ‘t’:

In Figure 82 for $t = 4$ and $n = 20$, we have $sum(0) = sum(1) = 24$

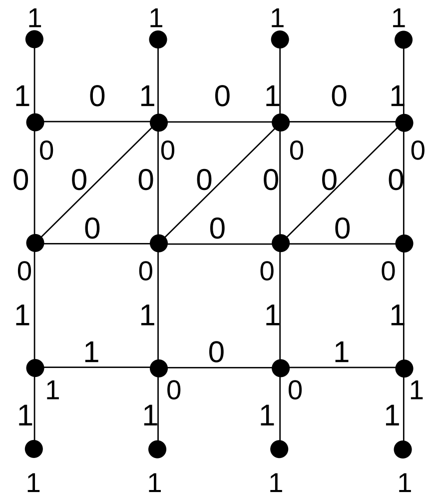

Figure 82: TEPC labeling of B_{20} graph

In Figure 83 for $t = 6$ and $n = 30$, we have $sum(0) = sum(1) = 37$

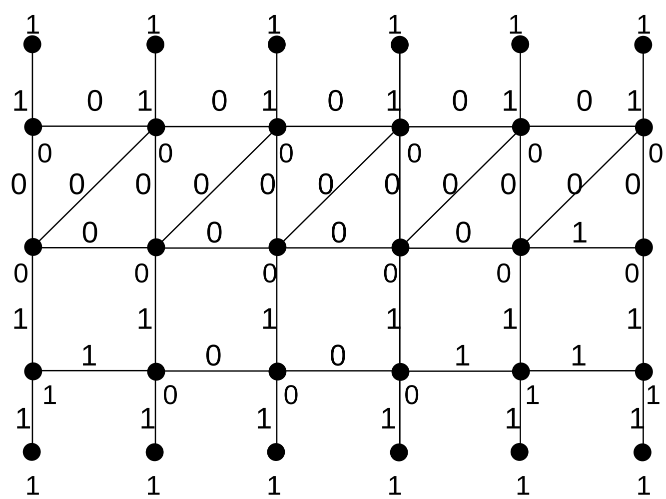

Figure 83: TEPC labeling of B_{30} graph

In Figure 84 for $t = 8$ and $n = 40$, we have $sum(0) = sum(1) = 50$

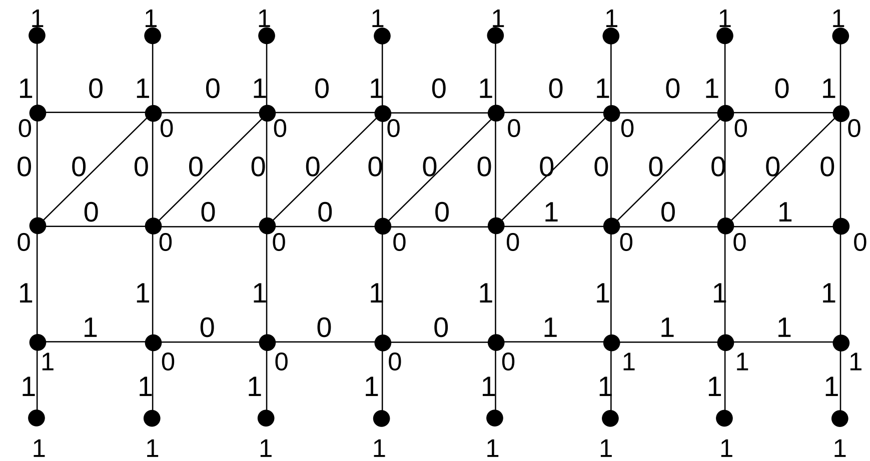

Figure 84: TEPC labeling of B_{40} graph

For odd 't':

In 85 for $t = 5$ and $n = 25$, we have $sum(0) = \frac{13t-5}{2}$, $sum(1) = 31$

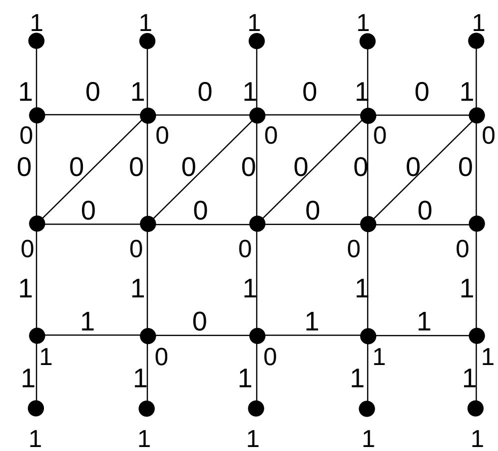

Figure 85: TEPC labeling of B_{25} graph

In Figure 86 for $t = 7$ and $n = 35$, we have $sum(0) = \frac{13t-5}{2}$, $sum(1) = 44$

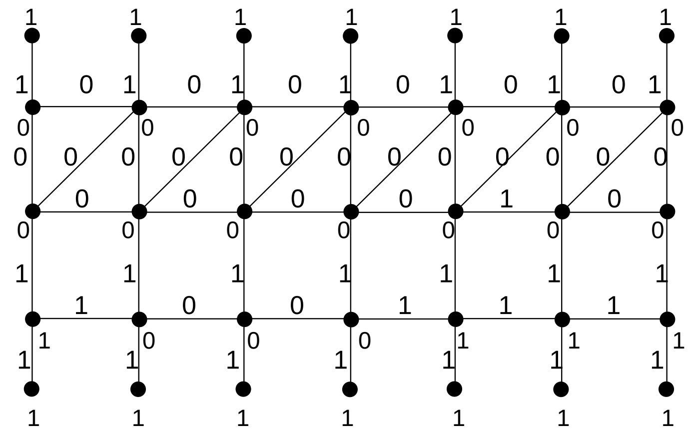

Figure 86: TEPC labeling of B_{35} graph

In Figure 87 for $t = 9$ and $n = 45$, we have $sum(0) = \frac{13t-5}{2}$, $sum(1) = 57$

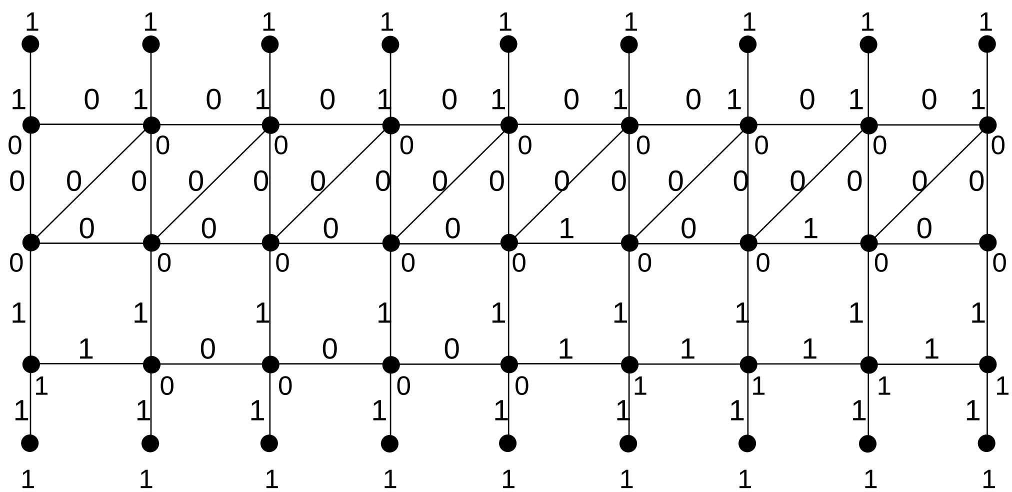

Figure 87: TEPC labeling of B_{45} graph

□

Theorem 5.4. *Linear phenylene graph PH_n with pendant edges is total edge product cordial for $n = 12t$ where $t \geq 2$.*

Proof. Let $n = 12t$ where $t \geq 2$ then $V(G) = \{x_i, y_i, u_i, v_i; 1 \leq i \leq 3t\}$ and $E(G) = \{x_iu_i; 1 \leq i \leq 3t\} \cup \{x_ix_{i+1}; 1 \leq i \leq 3t-1\} \cup \{x_{3i}y_{3i}; 1 \leq i \leq t\} \cup \{x_{3i+1}y_{3i+1}; 1 \leq i \leq t-1\} \cup \{x_1y_1\} \cup$

$\{y_iy_{i+1}; 1 \leq i \leq 3t-1\} \cup \{y_iv_i; 1 \leq i \leq 3t\}$ be the vertex-set and edge-set of the graph respectively.

Vertices of the graph are defined as follows:

$f(x_i) = 0; \quad for \quad 1 \leq i \leq 3t$

$f(u_i) = 1; \quad for \quad 1 \leq i \leq 3t$

$f(y_i) = 0; \quad for \quad 1 \leq i \leq 3t$

$f(v_i) = 1; \quad for \quad 1 \leq i \leq 3t$

And edges of the graph are defined as follows:

$f(x_ix_{i+1}) = 0; \quad for \quad i \equiv 0, 1 (mod\, 3) \quad where \quad 1 \leq i \leq 3t-1$

$f(x_ix_{i+1}) = 1; \quad for \quad i \equiv 2 (mod\, 3) \quad where \quad 2 \leq i \leq 3t-1$

$f(y_iy_{i+1}) = 0; \quad for \quad 1 \leq i \leq 3t-1$

$f(x_iu_i) = 1; \quad for \quad 1 \leq i \leq 3t$

$f(y_iv_i) = 1; \quad for \quad 1 \leq i \leq 3t$

$f(x_iy_i) = 0 \quad for \quad i \equiv 0, 1 (mod\, 3)$

We obtained $sum(0) = sum(1) = 13t-1$, where $t \geq 2$. So the graph is total edge product cordial.

In Figure 88 for $t = 2$ and $n = 24$, we have $sum(0) = sum(1) = 25$

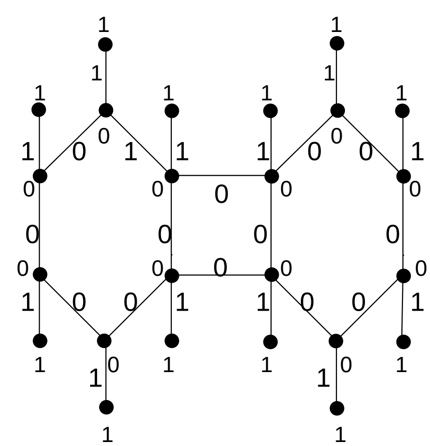

Figure 88: TEPC labeling of linear phenylene graph PH_{24}

In Figure 89 for $t = 3$ and $n = 36$, we have $sum(0) = sum(1) = 38$

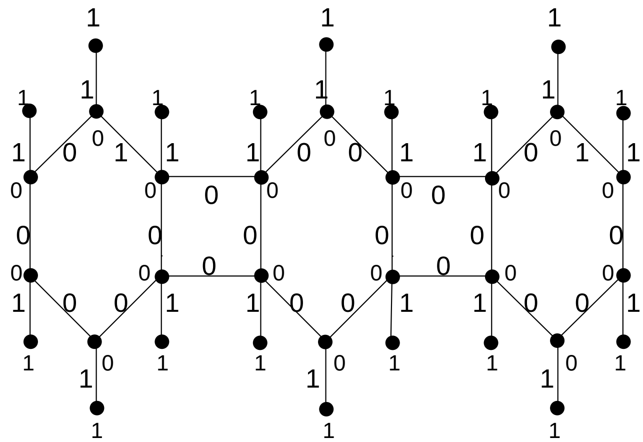

Figure 89: TEPC labeling of linear phenylene graph PH_{36}

In Figure 90 for $t = 4$ and $n = 48$, we have $sum(0) = sum(1) = 51$

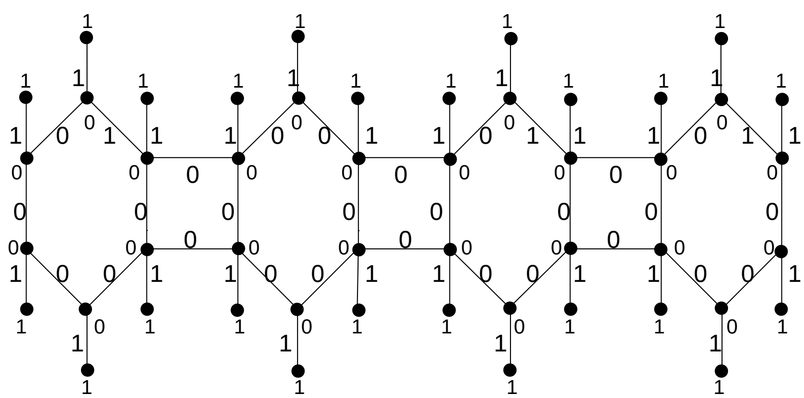

Figure 90: TEPC labeling of linear phenylene graph PH_{36}

In Figure 91 for $t = 5$ and $n = 60$, we have $sum(0) = sum(1) = 64$

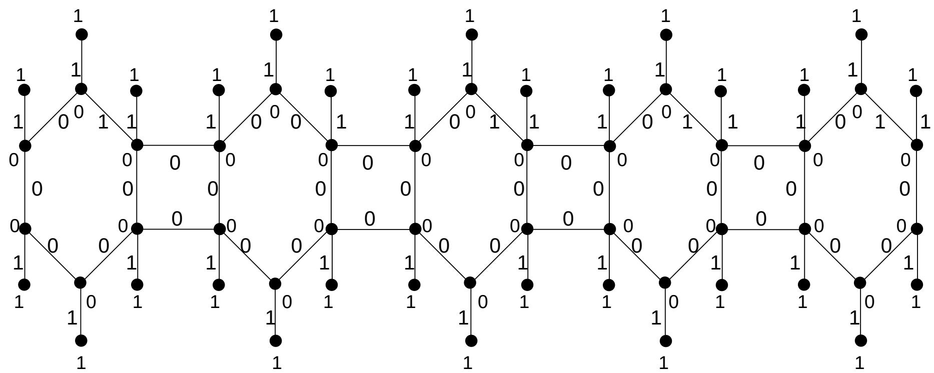

Figure 91: TEPC labeling of linear phenylene graph PH_{60}

In Figure 92 for $t = 6$ and $n = 72$, we have $sum(0) = sum(1) = 77$

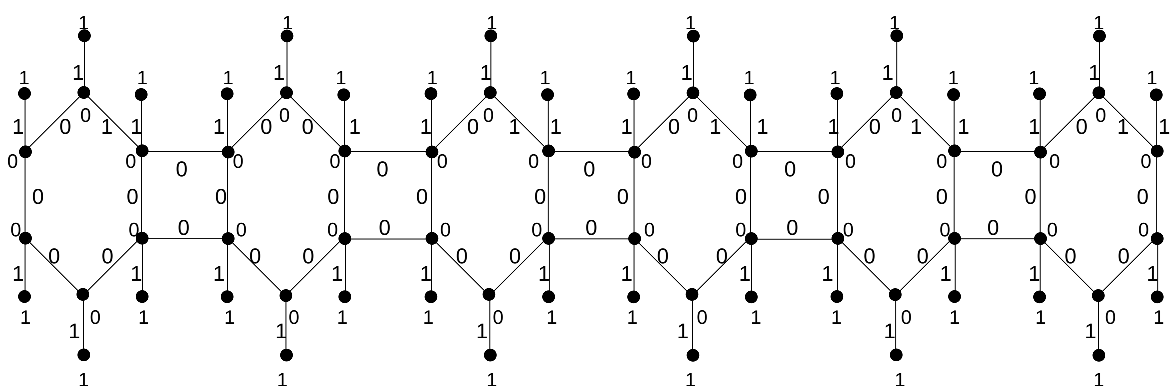

Figure 92: TEPC labeling of linear phenylene graph PH_{72}

$\square$

Chapter 6

Conclusion

6 Conclusion

In this thesis we worked on cordial labelings. We have proved the existence of different types of cordial labelings on B_n and linear phenylene graphs. We found that cordial labeling holds in B_n and linear phenylene graphs. We also concluded that B_n and linear phenylene graphs admits edge product cordial labeling and total edge product cordial labeling.

We believe that these evaluations of the graphs are exact and precise. These evaluations would help us in analysing cordial labelings of other graphs.

Chapter 7

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