Cosmological Analysis of Reconstructed Einstein-Aether Models

By

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CIIT/SP17-RMT-007/LHR

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Declaration

I, Irfan Ullah Malik with registration number **CIIT/SP17-RMT-007/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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It is certified that Irfan Ullah Malik with CIIT/SP17-RMT-007/LHR has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore campus and the work fulfills the requirement for award of MS degree.

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DEDICATED TO

My Family

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Abstract

In this thesis, we reconstruct various solutions for the accelerated universe in the Einstein-Aether theory of gravity. For this purpose, we obtain the effective density and pressure for Einstein-Aether theory. We reconstruct the Einstein-Aether model parameter by comparing its energy density with various holographic dark energy models such as Tsallis, Rényi and Sharma-Mittal holographic dark energy models. For this reconstruction, we use two forms of scale factor, power-law and exponential forms. The cosmological analysis of underlying scenario has been done by exploring different cosmological parameters. This includes equation of state parameter, squared speed of sound and evolutionary equation of state parameter via graphical representation. We obtain some favorable results for some model parametric values.

Chapter 1

Introduction

The branch of astronomy that includes the study of whole of the universe structure is classified as cosmology. More precisely, cosmology is an approach towards the origin, nature, physical features as well as ultimate fate of the universe. Moreover, different laws of physics regarding to the advancement in universe are studied in cosmology. As Kepler gave the proposals about the motion of planets in his laws of planetary motion, but Newton offered the actual mechanism for Kepler's law. So, he gave the law of universe gravitation which resolved the irregularities between the planets occurred due to gravitational interaction.

In modern cosmology, Einstein adopted a new approach in 1917, in which he used cosmological constant in his field equations and gave the idea of static universe. After the era of Einstein, many scientists took interest in cosmology in which de Sitter, Karl Schwarzschild and Eddington are well known. The static idea of universe proposed by Einstein was no longer accepted and new idea of accelerating universe in 1922 was given by Friedmannn by rejecting the Einstein's theory. Difference between these two ideas causes long term debates among the scientists. At last these two different ideas end up when Hubble discover the novae in andromeda galaxy in 1923. The distance of this novae shown by the Hubble was far away even from the Milky Way's boundaries. The amazing results were found when the two searching teams high-Z Supernova and Supernova cosmology project were sent separately into the space. These results showed the universe as the expending universe with acceleration.

The current status of our universe has been obtained under the study of various observations and these observations involve study of luminosity type Ia Supernova (SNIa) [1]-[4] and cosmic Microwave Background

(CMB)[5]-[9]. Through their observations, it came to know that our universe is accelerating at present time. The source which is responsible for the accelerating universe is known as dark energy (DE), which is one of the unsolved mysteries [10]-[13]. Although there are many prospects through which the expansion of the universe has been discussed. But one of the major candidate which is responsible for the accelerating universe is the DE, whose abundance can be seen clearly that this candidate contains 68.3 percent of the total energy of the universe and other helping hand is of the dark matter (DM) which is about 26.8 percent. We can also confirm that our universe is accelerating through the analysis of WMAP data [14, 15]. Despite of prolonged discussion on this great persona and interesting matter of today's cosmology, still we have got limited knowledge regarding to DE. Although there are different types of DE, but one of the attractive and simple candidate for DE is the cosmological constant Λ.

Another idea to understand the DE is the equation of state (EoS). Different types of DE eras can be obtained from values of EoS parameter. If its value is in between $-\frac{1}{3}$ and -1, then it is classified as quintessence type of DE and for the value less than -1, then DE is known as phantom type and if value of EoS parameter is -1 then DE is given the name as vacuum type DE. Instantly major interest has been turned on for analyzing the DE by holographic DE (HDE) model [16]-[18]. The way to the issue of DE occurs from the holographic principle [19].

There are also many theories of gravity in the modified form, which make our access to examine the accelerated scenario of the universe. For this situation, enormous acceleration would emerge not from the DE as a substance but quite from the elements of modified gravity [20]. The modified theory DGP brane-world model [21] is one of the simplest theory. The $f(R)$ theory with the Ricci scalar curvature is also one of the modified theory explaining the acceleration scenario of the universe [22]-[24]. Other helping hands which include $f(G)$, $f(T)$, $f(R,T)$ and $f(R,G)$ modified theories [25]-[31] etc.

Here we talk about another important theory proposed by Jacobson et al. [32, 33], said to be Einstein-Aether theory. Under this theory many solutions regarding to the accelerating nature of universe have been examined [34]. The generalization of this theory has developed by Zlosnik et al. [35, 36]. Now a days, this theory is under consideration and lot of work is being done in generalized Einstein-Aether theory [37]-[43]. By considering the special type of Lagrangian density of Aether field in the Einstein-Aether theories, the possibility of Aether theory to be act like a DE model is studied in detail. For this special Aether field which is chosen as candidate for DE, has been discovered the limitations from observational information. It has been shown by Meng et al. [44, 45] that DE may be generated by only Einstein-Aether theory.

Motivated by the work of Pasqua et al. [46], We discuss the Einstein-Aether theory through the remodeling of Einstein-Hilbert action. For this theory, we briefly talk about the modified Friedmannn equations and hence from these equations we obtain the effective density and pressure for Einstein-Aether theory. Our present goal is to reconstruct the Einstein-Aether theory by using some DE models. We check the viability of our constructed models. In the presence of free function $F(K)$ with some restrictions on it, we treat the affective density and pressure as DE. Taking into account some modified HDE models which are Tsallis (THDE), Rényi (RHDE) and Sharma-Mittal

(SMHDE) HDE models, we have formed the unknown function $F(K)$ in the background of Einstein-Aether theory (where K is proportional to (H^2) by considering the some scale factor forms). We organize the thesis in the following format:

- In chapter 2, we provide the basic definitions and formalism to understand the thesis work.
- Chapter 3 is devoted to reconstruction of some HDE models in t Einstein-Aether theory of gravity. We construct some cosmological parameters taking two scale factors.
- In the last chapter, we summarize the results.

Chapter 2

Preliminaries

This chapter has the basic concepts for understanding the thesis work. The basic definitions are very useful for the calculations.

2.1 Cosmology

It is a human nature that makes him always curious about the unexpected results in the universe. A view of universe always wonders a human, when he observes different planets including stars, moon and the sun. His observations are not confined only at large scale of the universe, but also at a small scale of the universe. A human classified himself into the different groups of biologists, chemists and astrologist. Biologists observe our universe at microscopic level. More precisely, they study about plants, humans and animals along with the variations produced by different organisms. Chemists study the physical characteristics of substance. Hence they study nano particles while astronomers study heavenly bodies like stars, planets and galaxies. They observe the universe at macroscopic level. This implies the cosmology deals our universe on both, microscopic and macroscopic level. Thus, "the branch of astronomy that includes the study of whole of the universe structure is classified as cosmology". Generally speaking, cosmology is an approach towards the origin, nature, physical feature as well as ultimate fate of the universe.

2.2 Cosmic Acceleration

Although, cosmology is being considered an interesting subject among the scientists for a long time. But modern cosmology started in 1917, when Einstein gave an idea of static universe. But on the other hand, Friedmann gave an idea of accelerated universe by rejecting an idea of Einstein. Finally, two individual teams high-Z Supernova and Supernova cosmology project were directed into the space with the goal to gain a knowledge about the universe. Their collected data surprised the cosmologist by showing that universe is expanding. After a lot of effort it was one of the great success. Hence a noble prize was given to three of the members of this mission for their unexpected achievement. The current status of our universe has been obtained under the study of various observations and these observations involve study of SNIa, CMB, large scale structure, Sloan Digital Sky Survey, the integrated Sachs Wolfe effect and gravitational lensing. Through their study, it came to know that our universe is accelerating at present time. The source which is responsible for the accelerating universe is known as DE. Despite of a prolonged discussion on a DE, still our knowledge is limited about it.

2.3 Dark Matter and Dark Energy

Matter as we know it, atoms, stars and the galaxies, planets and trees, rocks and dust. This matter accounts 4.9 percent of the known universe. About 26.8 percent is DM and 68.3 percent is the DE, both of which are invisible. This is kind of strange because it suggests that every thing we experience is really only a tiny fraction of reality. We really have no clue, what DM and DE are, or how they work. But we are pretty sure they exist though.

Dark matter is the stuff that makes it possible for galaxies to exist. When we calculated why the universe is structured the way it is, it quickly became clear that there's only just not enough matter. It is made up of a complicated exotic particles that do not interact with light and matter in a way we expect. There are two categories of DM. One is baryonic DM and second is non-baryonic DM. As stars release the electromagnetic radiations, which cause the photons to spread in every direction, hence through this emission, astronomers can identify some DM. This is said to be baryonic matter. But it holds only 4.2 percent of the total DM. Another helping hand is of non-baryonic matter, which contributes 22.6 percent. As no any type of light with any wavelength is scattered or absorbed by this matter so, it is impossible to identify it directly.

Dark energy is even more strange and mysterious. We can not detect it and we can not taste it. But its effect can be seen very clearly. This DE acts like a force which pushes the things instead of pulling them which shows that expansion of the universe is accelerating. Although many observations have been made, but our knowledge about the DE is limited. The only fact which we know about the DE is that it is a repulsive force having negative pressure.

2.4 Cosmological Parameters

To understand the geometry of the universe, following are some basic cosmological parameters.

2.4.1 Scale Factor

The modern cosmology came into existence when Hubble discovered the novae in andromeda galaxy and showed its distance far away even from the Milky Way's boundaries, then it came to know that universe is not static and it is expanding with time. Hence, scale factor is the measure that how much the universe has expanded since given time. It is represented by $a(t)$. Since, the latest cosmic observations have shown that universe is accelerating so $a(t) > 0$.

2.4.2 Hubble Parameter

As our universe contains many of the astronomical objects and these objects are moving with different velocities at a distance from each other. So, we can evaluate the distance and velocity between these astronomical objects. The parameter which helps us to do so is known as Hubble parameter. Mathematically, it can be written as

$$v = H(t)r, \tag{2.4.1}$$

here r and v indicate the radius and the recessional velocity respectively, $H(t)$ is Hubble parameter depends upon time. Hubble parameter is defined by using scale factor as

$$H = \frac{\dot{a}(t)}{a(t)}, \tag{2.4.2}$$

where dot represents time derivative. If $\ddot{a}(t) > 0$, then it indicates accelerated expansion of the universe. If $\ddot{a}(t) < 0$ then decelerated expansion of the universe can be observed.

2.4.3 Equation of State Parameter

The EoS parameter is used to categorize the decelerated and accelerated phases of the universe. This parameter is defined as $\omega = \frac{p}{\rho}$ where p represents the pressure and ρ expresses the density of the universe. The decelerated phase of the universe involve radiation era $0 < \omega < \frac{1}{3}$, and cold dark matter era $\omega = 0$. The DE dominated phase has following eras:

- $\omega = -1 \quad \Rightarrow \quad$ cosmological constant
- $-1 < \omega < \frac{-1}{3} \quad \Rightarrow \quad$ quintessence
- $\omega < -1 \quad \Rightarrow \quad$ phantom

2.4.4 Squared Speed of Sound

To examine the behavior of DE models, there is another parameter which is known as squared speed of sound. It is denoted by v_s^2 and is calculated by the following formula

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}}. \tag{2.4.3}$$

The stability of the model can be checked by it. If its graph is showing negative values then we may say that model is unstable and in case of non-negative values of the graph, it represents the stable behavior of the model.

2.4.5 ω_Λ-ω'_Λ Plane

There are different DE models which have different properties. To examine their dynamical behavior, we use ω_Λ-ω'_Λ plane, where prime denotes the derivative with respect to $\ln a$ and subscript Λ indicates DE scenario. This method was developed by Caldwell and Linder [47] and dividing ω_Λ-ω'_Λ plane into two parts, thawing part $(\omega'_\Lambda > 0, \omega_\Lambda < 0)$ is the region where EoS parameter nearly evolves from -1, increases with time while its evolution parameter expresses positive behavior, and freezing part $(\omega'_\Lambda < 0, \omega_\Lambda < 0)$ is the evolution parameter for EoS parameter remains negative.

2.5 FRW Geometry

The Friedmann-Robertson-Walker (FRW) metric is based on the assumption of the homogeneity and isotropy of the universe which is approximately true on the large scales. This homogeneous and isotropic cosmological model is called the FRW geometry and is given by

$$ds^2 = dt^2 - a(t)^2\bigg(\frac{dr^2}{1-kr^2} + r^2\big(d\theta^2 + \sin^2\theta d\phi^2\big)\bigg), \tag{2.5.1}$$

where represents curvature which involves three cases as follow

- $k = -1$ represents the negative curvature (hyperboloid),
- $k = 0$ corresponds to no curvature (flat surface),
- $k = 1$ gives the positive curvature (for the 3-sphere).

2.6 Einstein-Aether Theory

As our universe is full with many of the natural occurring phenomenons. One of them is transfer of light from one place to another and second is how gravity acts. To explain these kinds of phenomenons, many of the physicists were used the concept of Aether in many of the theories. In modern physics, Aether indicates a physical medium that is spread homogeneously at each point of the universe. Hence, it was considered that it is a medium in space that helps light to travel in a vacuum. According to this concept, a particular static frame reference is provided by Aether and everything has absolute relative velocity in this frame. That is suitable for Newtonian dynamics extremely well. But, when Einstein performed different experiments on optics in his theory of relativity, then Einstein rejected this ambiguity. By saying "Aether" framework, we do not mean a mechanical medium naively, but rather a locally preferred state resting for each point of the spacetime in our physical evolutionary universe, determine by some hitherto unknown physics or its physical state to be specified by some physical conditions with its environment. When CMB was introduced, many of the people took it a modern form of Aether. Gasperini has popularized Einstein-Aether theories [48]. This theory is said to be covariant modification of general relativity in which unit time like vector field (aether) breaks the Lorentz Invariance (LI) to examine the gravitational and cosmological effects of dynamical preferred frame [49]. Following is the action of Einstein-Aether theory [50, 51].

$$S = \int d^4x\sqrt{-g}\bigg(\frac{R}{4\pi G} + L_{EA} + L_m\bigg), \tag{2.6.1}$$

where L_{EA} represents the Lagrangian density for the vector field and L_m indicates Lagrangian density of matter field. Further, g, R and G indicate determinant of the metric tensor $g^{\mu\nu}$, Ricci scalar and gravitational constant respectively. The Lagrangian density for vector field can be written as

$$\begin{aligned} L_{EA} &= \frac{M^2}{16\pi G}F(K) + \frac{1}{16\pi G}\lambda(A^a A_a + 1), && (2.6.2)\\ K &= M^{-2}K^{ab}_{cd}\nabla_a A^c \nabla_b A^d c && (2.6.3)\\ K^{ab}_{cd} &= c_1 g^{ab} g_{cd} + c_2\delta^a_c\delta^b_d + c_3\delta^a_d\delta^b_c, \quad a, b = 0, 1, 2, 3. && (2.6.4)\end{aligned}$$

where c_i is the dimensionless constants, M is a coupling constant parameter, λ is a Lagrangian multiplier and A^a is a contravariant vector. The function $F(K)$ is any arbitrary function of K. From (2.6.1), we get the Einstein field equations for this theory

$$\begin{aligned} G_{ab} &= T^{EA}_{ab} + 8\pi G T^m_{ab}, && (2.6.5)\\ \nabla_a\bigg(\frac{dF}{dK}J^a_b\bigg) &= 2\lambda A_b, && (2.6.6)\end{aligned}$$

where $J^a_b = -2K^{ad}_{bc}\nabla_d A^c$, T^{EA}_{ab} shows energy momentum-tensor for vector field and T^m_{ab} indicates energy-momentum tensor for mater field. These tensors are given as

$$
\begin{aligned}
T^m_{ab} &= (\rho + p)u_a ub + pg_{ab}, && (2.6.7)\\
T^{EA}_{ab} &= \frac{1}{2}\nabla_d\left((J^d_a A_b - J^d_a A_b - J_{(ab)}A^d)\frac{dF}{dK}\right) - Y_{(ab)}\frac{dF}{dK}\\
&+ \frac{1}{2}g_{ab}M^2F + \lambda A_a A_b, && (2.6.8)
\end{aligned}
$$

where p and ρ represent energy density and pressure of the matter respectively. Furthermore, u_a expresses the four-velocity vector of the fluid and given as $u_a = (1, 0, 0, 0)$ and A_a is time-like unitary vector and is defined as $A_a = (1, 0, 0, 0)$. Moreover Y_{ab} is defined as

$$
Y_{ab} = c_1\left((\nabla_d A_a)(\nabla^d A_b) - (\nabla_a A_d)(\nabla_a A^d)\right), \qquad (2.6.9)
$$

where indices (ab) show the symmetry.

Chapter 3

Cosmological Analysis of Reconstructed Einstein-Aether Models

In this chapter, we provide the field equations for Einstein-Aether gravity taking FRW spacetime. In this framework, we reconstruct different models using DE models such as THDE, RHDE and SMHDE for Hubble horizon. We discuss some cosmological parameters like, EoS parameter, EoS parameter with its evolutionary parameter and squared speed of sound to check the stability of reconstructed models.

3.1 Field Equations

The Friedmann equations modified by the Einstein-Aether gravity are given as follows

$$\epsilon\left(\frac{F}{2K}-\frac{dF}{dK}\right)H^2+\left(H^2+\frac{k}{a^2}\right) = \left(\frac{8\pi G}{3}\right)\rho, \tag{3.1.1}$$

$$\epsilon\frac{d}{dt}\left(H\frac{dF}{dK}\right)+\left(-2\dot{H}+\frac{2k}{a^2}\right) = 8\pi G(p+\rho). \tag{3.1.2}$$

Here K becomes $K=\frac{3\epsilon H^2}{M^2}$, where ϵ is a constant parameter. We denote the effective energy density in Einstein-Aether gravity by ρ_{EA}, and the effective pressure in Einstein-Aether gravity by p_{EA}. So, we can rewrite Eqs.(3.1.1) and (3.1.2) as

$$\left(H^2+\frac{k}{a^2}\right) = \left(\frac{8\pi G}{3}\right)\rho+\frac{1}{3}\rho_{EA}, \tag{3.1.3}$$

$$\left(-2\dot{H}+\frac{2k}{a^2}\right) = 8\pi G(p+\rho)+(\rho_{EA}+p_{EA}), \tag{3.1.4}$$

where

$$\rho_{EA} = 3\epsilon H^2\left(\frac{dF}{dK}-\frac{F}{2K}\right), \tag{3.1.5}$$

$$p_{EA} = -3\epsilon H^2\left(\frac{dF}{dK}-\frac{F}{2K}\right)-\epsilon\left(\dot{H}\frac{dF}{dK}+H\frac{d\dot{F}}{dK}\right). \tag{3.1.6}$$

$$= \rho_{EA}-\frac{\dot{\rho}_{EA}}{3H}. \tag{3.1.7}$$

The EoS parameter for Einstein-Aether can be obtained by using Eqs.(3.5.1) and (3.1.6), which is given by

$$\omega_{EA} = \frac{p_{EA}}{\rho_{EA}} = -1 - \frac{\dot{H}\frac{dF}{dK} + H\frac{d\dot{F}}{dK}}{3H^2(\frac{dF}{dK} - \frac{F}{2K})}. \tag{3.1.8}$$

3.2 Scale Factors

As Einstein-Aether is one of the modified theory which may produce the accelerated expansion of the universe. By using this theory, we can reconstruct various well-known DE models. In order to do this, we take some modified HDE models such as THDE, RHDE and SMHDE models. Since $F(K)$ is a function that the Einstein-Aether theory contains, which can be determined by comparing the densities with the above DE models. For this purpose, we use some well-known forms of scale factor $a(t)$. We consider two forms of scale factors $a(t)$ in terms of power and exponential terms. These are

(i) Power-law form: $a(t) = a_0 t^m,\ m > 0$, where a_0 is a constant which indicates the value of scale factor at present-day [52]. From this scale factor, we get $H,\ \dot{H},\ K$ as follows

$$H = \frac{m}{t}, \quad \dot{H} = -\frac{m}{t^2}, \quad K = \frac{3\epsilon m^2}{M^2 t^2}. \tag{3.2.1}$$

(ii) Exponential form: $a(t) = e^{\alpha t^{\theta}}$ where α is a positive constant and θ lies between 0 and 1. This scale factor gives

$$H = \alpha\theta t^{\theta-1}, \quad \dot{H} = \alpha\theta(\theta-1)t^{\theta-2}, \quad K = \frac{3\epsilon\alpha^2\theta^2 t^{2(\theta-1)}}{M^2}. \tag{3.2.2}$$

3.3 Reconstruction from Tsallis Holographic Dark Energy Model

The energy density of THDE model is given by [53]

$$\rho_D = BL^{2\delta-4}, \tag{3.3.1}$$

where B is an unknown parameter. Taking into account Hubble radius as IR cutoff L, that is $L = \frac{1}{H}$, we have

$$\rho_D = BH^{-2\delta+4}. \tag{3.3.2}$$

In order to construct a DE model in the framework of Einstein-Aether gravity with THDE model, we compare the densities of both models, (i.e., $\rho_{EA} = \rho_D$). This yields

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{B}{3\epsilon}H^{-2\delta+2}, \tag{3.3.3}$$

which results the following form

$$F(K) = \frac{2BM^{-2\delta+2}K^{-\delta+2}}{(-2\delta+3)(3\epsilon)^{-\delta+2}} + C_1\sqrt{K}. \tag{3.3.4}$$

Power-law form of scale factor:

Using the expression of $F(K)$ along with Eq.(3.2.1) in (3.5.1), we obtain the energy density and pressure as

$$\begin{aligned}
\rho_{EA} &= \frac{m^2\left(3^\delta 2BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^\delta - 9\epsilon^2 C_1\right)}{6K^{\frac{3}{2}}t^2\epsilon}, && (3.3.5)\\
p_{EA} &= m\Bigg(4BM^{-2\delta}\epsilon^\delta\Bigg(-3^\delta K^{1-\delta}M^2(4-9m-2\delta+6m\delta) - 12M^2(-2 \\
&+ \delta)(-1+\delta)\left(\frac{m^2\epsilon}{M^2t^2}\right)^{1-\delta}\Bigg)(-3+2\delta)^{-1} + \Bigg(\frac{9(-1+6m)\epsilon^2}{K^{\frac{3}{2}}} + \frac{3\sqrt{3}}{m^4} \\
&\times M^4t^4\sqrt{\frac{m^2\epsilon}{M^2t^2}}\Bigg)C_1\Bigg)(36t^2\epsilon)^{-1}.
\end{aligned}$$

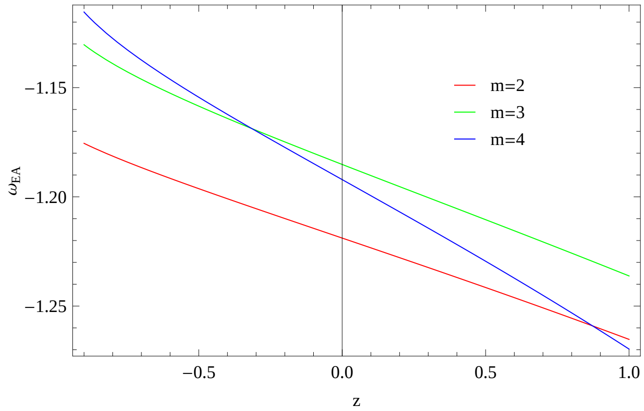

Figure 3.1: Plot of ω_{EA} versus z taking power-law scale factor for THDE model.

Using these expressions of energy density and pressure, we find the values of some cosmological parameters in the following. The EoS parameter takes the following form

$$\begin{aligned}\omega_{EA} &= K^{\frac{3}{2}}\bigg(4BM^{-2\delta}\epsilon^{\delta}\bigg(-3^{\delta}K^{1-\delta}M^2(4-9m-2\delta+6m\delta)-12M^2(-2\\ &+ \delta)(-1+\delta)\bigg(\frac{m^2\epsilon}{M^2t^2}\bigg)^{1-\delta}\bigg)(-3+2\delta)^1+\bigg(\frac{9(-1+6m)\epsilon^2}{K^{\frac{3}{2}}}+\frac{3\sqrt{3}M^4}{m^4}\\ &\times t^4\sqrt{\frac{m^2\epsilon}{M^2t^2}}\bigg)C_1\bigg)\bigg(6m\bigg(23^{\delta}BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^{\delta}-9\epsilon^2C_1\bigg)\bigg)^{-1}. \qquad (3.3.6)\end{aligned}$$

The derivative of EoS parameter with respect to $\ln a$ is given by

$$\begin{aligned}\omega'_{EA} &= K^{\frac{3}{2}}t\Bigg(\frac{96Bm^2M^{-2\delta}(1-\delta)(-2+\delta)(-1+\delta)\epsilon^{1+\delta}\left(\frac{m^2\epsilon}{M^2t^2}\right)^{-\delta}}{t^3(-3+2\delta)}+\bigg(-\frac{3\sqrt{3}}{m^2}\\ &\times \frac{M^2t\epsilon}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}}+\frac{12\sqrt{3}M^4t^3\sqrt{\frac{m^2\epsilon}{M^2t^2}}}{m^4}\bigg)C_1\Bigg)\bigg(6m^2\bigg(23^{\delta}BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^{\delta}-9\epsilon^2C_1\bigg)\bigg)^{-1}.\end{aligned} \qquad (3.3.7)$$

We plot EoS parameter versus z using the relation $t=\frac{1}{(1+z)^{\frac{1}{m}}}$ taking values of constants as $B=5,\ M=5,\ \delta=1.8,\ \epsilon=1$ and $C_1=2$. We plot ω_{EA} for three different values of scale factor parameter m as $m=2,\ 3,4$

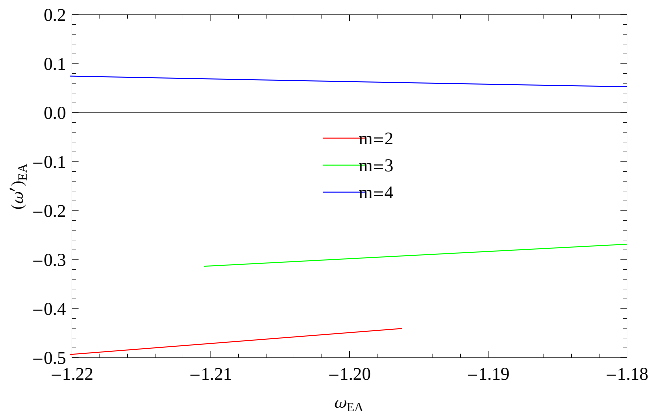

Figure 3.2: Plot of $\omega'_{EA} - \omega_{EA}$ taking power-law scale factor for THDE model.

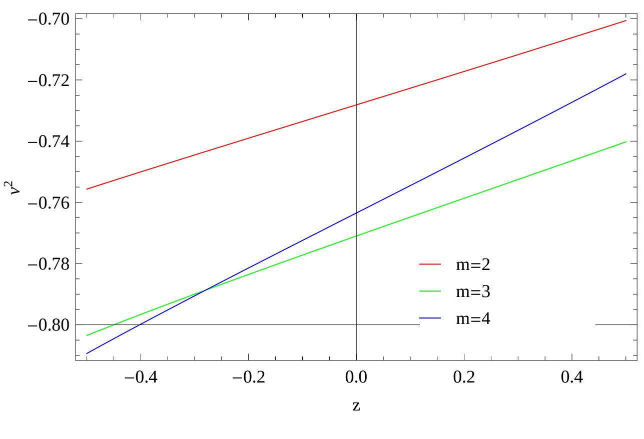

Figure 3.3: Plot of v_s^2 versus z taking power-law scale factor for THDE model.

as shown in Figure 3.1. All the three trajectories represent the phantom behavior of the universe related to redshift parameter. In Figure 3.2, we plot $\omega'_{EA}-\omega EA$ plane taking same values of the parameters for $-1 \leq z \leq 1$. For $m = 2$ and 3, the evolving EoS parameter shows negative behavior with respect to negative EoS parameter which indicates the freezing region of the universe. The trajectory of ω'_{EA} for $m = 4$ represents the positive behavior for negative EoS parameter and expresses the evolving universe in thawing region of the universe.

Also the squared speed of sound in the underlying scenario becomes

$$\begin{aligned} v_s^2 &= \Bigg(\bigg(\frac{m^2\epsilon}{M^2t^2}\bigg)^{-\delta}\bigg(8Bm^4\epsilon^\delta\bigg(-12K^{\frac{3}{2}+\delta}m^2(-2+\delta)^2(-1+\delta)\epsilon+3^\delta K^{\frac{5}{2}}M^2 \\ &\times\ t^2(4-9m-2\delta+6m\delta)\bigg(\frac{m^2\epsilon}{M^2t^2}\bigg)^\delta\bigg)+3K^\delta M^{2\delta}t^2(-3+2\delta)\bigg(\frac{m^2\epsilon}{M^2t^2}\bigg)^\delta \\ &\times\ \bigg(6(1-6m)m^4\epsilon^2+\sqrt{3}K^{\frac{3}{2}}M^4t^4\sqrt{\frac{m^2\epsilon}{M^2t^2}}\bigg)C_1\bigg)\Bigg)\Big/\bigg(12m^5t^2(-3+2\delta) \\ &\times\ \bigg(-23^\delta BK^{\frac{5}{2}}M^2\epsilon^\delta+9K^\delta M^{2\delta}\epsilon^2C_1\bigg)\bigg). \end{aligned} \tag{3.3.8}$$

Figure 3.3 shows the plot of v_s^2 versus z to check the behavior of Einstein-Aether model for THDE and power-law scale factor for same values of parameters. The trajectories represent the negative behavior of the model which indicated the instability of the model.

<u>**Exponential form of scale factor:**</u>

Following the same steps for exponential form of scale factor, we get energy density and pressure as

$$\rho_{EA} = \frac{t^{-2+2\theta}\alpha^2\theta^2\bigg(23^\delta BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^\delta-9\epsilon^2C_1\bigg)}{6K^{\frac{3}{2}}\epsilon}, \tag{3.3.9}$$

$$p_{EA} = \frac{1}{12\epsilon}t^{-2+\theta}\alpha\theta\Bigg(\frac{2t^\theta\alpha\theta\bigg(-23^\delta BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^\delta+9\epsilon^2C_1\bigg)}{K^{\frac{3}{2}}}-\epsilon^2(-1$$

$$
\begin{aligned}
&+ \quad \theta)\Bigg(8BM^{-2\delta}(-2+\delta)\epsilon^{-2+\delta}\Bigg(3^{\delta}K^{1-\delta}M^{2}-6M^{2}(-1+\delta)\Bigg(\frac{t^{-2+2\theta}}{M^{2}}\\
&\times \quad \alpha^{2}\epsilon\theta^{2}\Bigg)^{1-\delta}\Bigg)\Bigg(3(-3+2\delta)\Bigg)^{-1}+\Bigg(-\frac{3}{K^{\frac{3}{2}}}+\frac{\sqrt{\frac{t^{-2+2\theta}\alpha^{2}\epsilon\theta^{2}}{M^{2}}}}{\alpha^{4}\epsilon^{2}\theta^{4}}\sqrt{3}M^{4}\\
&\times \quad t^{4-4\theta}\Bigg)C_{1}\Bigg)\Bigg).
\end{aligned}
$$

Now by using above density and pressure, we obtain the EoS parameter and its derivative for Einstein-Aether gravity as follows

$$
\begin{aligned}
\omega_{EA} &= \Bigg(K^{\frac{3}{2}}t^{-\theta}\Bigg(\frac{2t^{\theta}\alpha\theta\Bigg(-23^{\delta}BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^{\delta}+9\epsilon^{2}C_{1}\Bigg)}{K^{\frac{3}{2}}}-\epsilon^{2}(-1+\theta)\\
&\times \quad \Bigg(8BM^{-2\delta}(-2+\delta)\epsilon^{-2+\delta}\Bigg(3^{\delta}K^{1-\delta}M^{2}-6M^{2}(-1+\delta)\Bigg(\frac{t^{-2+2\theta}\alpha^{2}}{M^{2}}\\
&\times \quad \epsilon\theta^{2}\Bigg)^{1-\delta}\Bigg)\Bigg(3(-3+2\delta)\Bigg)^{-1}+\Bigg(-\frac{3}{K^{\frac{3}{2}}}+\frac{\sqrt{3}M^{4}t^{4-4\theta}\sqrt{\frac{t^{-2+2\theta}\alpha^{2}\epsilon\theta^{2}}{M^{2}}}}{\alpha^{4}\epsilon^{2}\theta^{4}}\Bigg)\\
&\times \quad C_{1}\Bigg)\Bigg)\Bigg)/\Bigg(2\alpha\theta\Bigg(23^{\delta}BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^{\delta}-9\epsilon^{2}C_{1}\Bigg)\Bigg),\\
\omega'_{EA} &= \frac{1}{6\alpha^{2}\theta\Bigg(23^{\delta}BK^{\frac{5}{2}-\delta}M^{2-2\delta}\epsilon^{\delta}-9\epsilon^{2}C_{1}\Bigg)}K^{\frac{3}{2}}t^{-2(1+\theta)}(-1+\theta)\Bigg(8BK^{-\delta}\\
&\times \quad M^{-2\delta}(-2+\delta)\epsilon^{\delta}\Bigg(\frac{t^{-2+2\theta}\alpha^{2}\epsilon\theta^{2}}{M^{2}}\Bigg)^{-\delta}\Bigg(-6K^{\delta}t^{2\theta}\alpha^{2}(-1+\delta)\epsilon(2+2\delta\\
&\times \quad (-1+\theta)-\theta)\theta+3^{\delta}KM^{2}t^{2}\Bigg(\frac{t^{-2+2\theta}\alpha^{2}\epsilon\theta^{2}}{M^{2}}\Bigg)^{\delta}\Bigg)\Bigg(-3+2\delta\Bigg)^{-1}-3\\
&\times \quad t^{2-4\theta}\Bigg(3t^{4\theta}\alpha^{4}\epsilon^{2}\theta^{5}+\sqrt{3}K^{\frac{3}{2}}M^{4}t^{4}(3-4\theta)\sqrt{\frac{t^{-2+2\theta}\alpha^{2}\epsilon\theta^{2}}{M^{2}}}\Bigg)C_{1}\Bigg(K^{\frac{3}{2}}\alpha^{4}\\
&\times \quad \theta^{5}\Bigg)^{-1}\Bigg).
\end{aligned}
$$

The squared speed of sound for second form of scale factor is given by

$$
\begin{aligned}
v_{s}^{2} &= K^{\frac{3}{2}}t^{-\theta}\Bigg(8BM^{-2\delta}\epsilon^{\delta}\Bigg(-6M^{2}(-2+\delta)(-1+\delta)(4+2\delta(-1+\theta)-3\theta)\\
&\times \quad \Bigg(\frac{t^{-2+2\theta}\alpha^{2}\epsilon\theta^{2}}{M^{2}}\Bigg)^{1-\delta}-3^{\delta}K^{1-\delta}M^{2}\Bigg((-2+\delta)(-2+\theta)+3t^{\theta}\alpha(-3+2\\
&\times \quad \delta)\theta\Bigg)\Bigg)(-3+2\delta)^{-1}+3t^{-4\theta}\Bigg(3t^{4\theta}\alpha^{4}\epsilon^{2}(-2+\theta)\theta^{4}+36t^{5\theta}\alpha^{5}\epsilon^{2}\theta^{5}+\sqrt{3}
\end{aligned}
$$

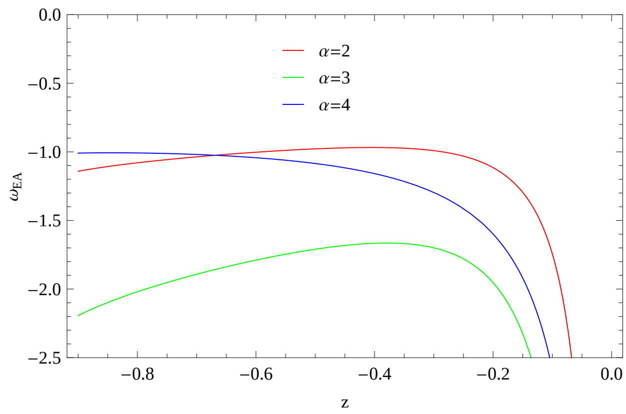

Figure 3.4: Plot of ω_{EA} versus z taking exponential scale factor for THDE model.

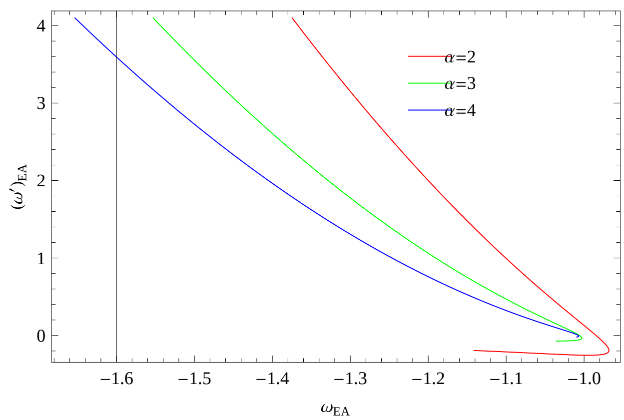

Figure 3.5: Plot of $\omega'_{EA} - \omega_{EA}$ taking exponential scale factor for THDE model.

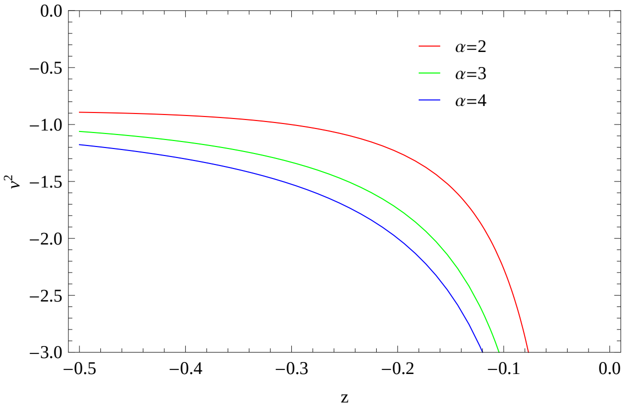

Figure 3.6: Plot of v_s^2 versus z taking exponential scale factor for THDE model.

$$\times \quad K^{\frac{3}{2}} M^4 t^4 \sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}(-1+2\theta)\Big) C_1 (K^{\frac{3}{2}}\alpha^4\theta^4)^{-1}\Big)\Big(12\alpha\theta\Big(23^{\delta} B$$
$$\times \quad K^{\frac{5}{2}-\delta} M^{2-2\delta}\epsilon^{\delta} - 9\epsilon^2 C_1\Big)\Big).$$

Figure 3.4 represents the graph of EoS parameter versus z for exponential form of scale factor taking $B = 5 = M,\ \delta = 1.8,\ \epsilon = 1, C_1 = -0.5,\ \theta = 0.5$ and scale factor parameter $\alpha = 2,\ 3,\ 4$. This parameter represents the phantom behavior of the universe for $\alpha = 3$ and after a transition from quintessence to phantom era for $\alpha = 2$. For $\alpha = 4$, the trajectory of the EoS parameter corresponds to the ΛCDM model $\omega_{EA} = -1$. In Figure 3.4, the graph is plotted between ω'_{EA} and ω_{EA}. The graph represents initially freezing region and then indicates the thawing region of the evolving universe. As we increase the value of α, the trajectories indicate the thawing region only. However, the graph of v_s^2 versus z as shown in Figure 3.6 shows the unstable behavior.

3.4 Reconstruction from R$\acute{e}$nyi Holographic Dark Energy Model

The energy density of RHDE model is [54].

$$\rho_D = \frac{3C^2L^{-2}}{8\pi\left(1+\frac{\delta\pi}{H^2}\right)}. \tag{3.4.1}$$

For Hubble horizon, it takes the form

$$\rho_D = \frac{3C^2H^2}{8\pi\left(1+\frac{\delta\pi}{H^2}\right)}. \tag{3.4.2}$$

Now we compare the Einstein-Aether model energy density with the RHDE model density (i.e., $\rho_{EA} = \rho_D$) in order to get reconstructed equation,

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{C^2}{8\pi\epsilon(1+\frac{\delta\pi}{H^2})}. \tag{3.4.3}$$

The solution of this equation is given by

$$F(K) = \frac{C^2}{4\pi M\epsilon}\left(KM - \sqrt{3K\delta\epsilon\pi}\arctan\left(\frac{M\sqrt{K}}{\sqrt{3\epsilon\pi\delta}}\right)\right) + C_2\sqrt{K}. \tag{3.4.4}$$

<u>Power-law form of scale factor:</u>

Inserting all the corresponding values in Eqs.(3.5.1) and (3.1.6), we get density and pressure of Einstein-Aether gravity model as follows

$$\begin{aligned}
\rho_{EA} &= \frac{1}{8\pi t^2}3C^2m^2\left(1 - \frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\left(KM^2+3\delta\epsilon\pi\right)}\right). \\
p_{EA} &= \frac{1}{24\pi t^5}C^2m\left(9mt^3\left(-1+\frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\left(KM^2+3\delta\epsilon\pi\right)}\right) - \frac{1}{M^4}\epsilon\left(3M^3\right.\right. \\
&\times\ t^3\left(M\left(-2+\frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\left(KM^2+3\delta\epsilon\pi\right)}\right) + \frac{\mathrm{ArcTan}\left(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta\epsilon\pi}}\right)}{K}\sqrt{3}\right.
\end{aligned}$$

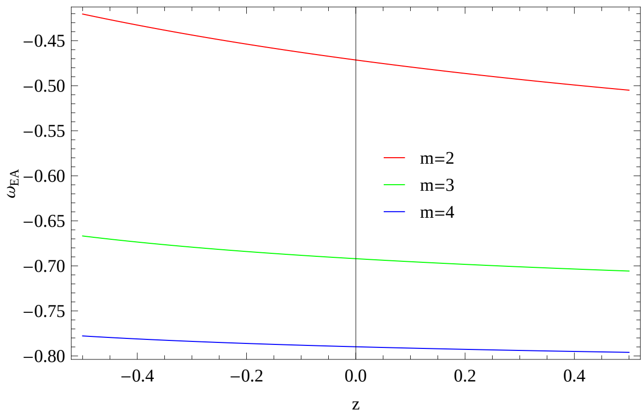

Figure 3.7: Plot of ω_{EA} versus z taking power-law scale factor for RHDE model.

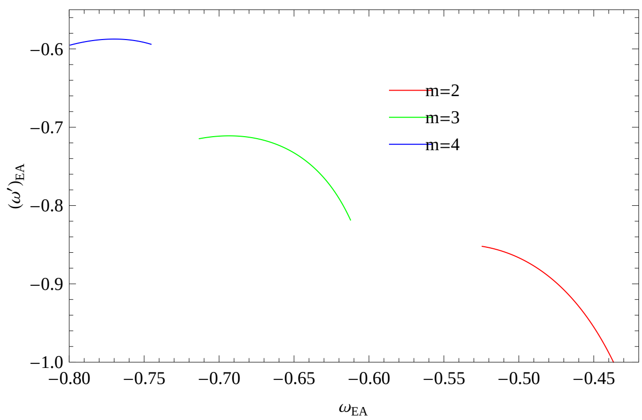

Figure 3.8: Plot of $\omega'_{EA} - \omega_{EA}$ taking power-law scale factor for RHDE model.

$$\begin{aligned}
\times &\ \sqrt{K\delta\epsilon\pi} - \frac{C_2}{\sqrt{K}}\Big)(\epsilon)^{-1} + \Bigg(m^2\Bigg(- 3m^2t\delta^2\epsilon^2\pi^2\Big(m^2 - t^2\delta\pi\Big) - 3 \\
\times &\ m\delta^{3/2}\epsilon^2\pi^{\frac{3}{2}}\Big(m^2 + t^2\delta\pi\Big)^2 \mathrm{ArcTan}\Bigg(\frac{m}{t\sqrt{\delta}\sqrt{\pi}}\Bigg) + \sqrt{3}Mt\sqrt{\delta\epsilon\pi}m\epsilon \\
\times &\ \sqrt{\frac{\delta\pi}{M^2t^2}}\Big(m^2 + t^2\delta\pi\Big)^2 C_2\Bigg)\Bigg)/\Bigg(\Bigg(\frac{m^2\epsilon}{M^2t^2}\Bigg)^{3/2}\sqrt{\delta\epsilon\pi}\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}\Bigg(m^2 \\
+ &\ t^2\delta\pi\Bigg)^2\Bigg)\Bigg)\Bigg).
\end{aligned}$$

In this case, the EoS parameter takes the form

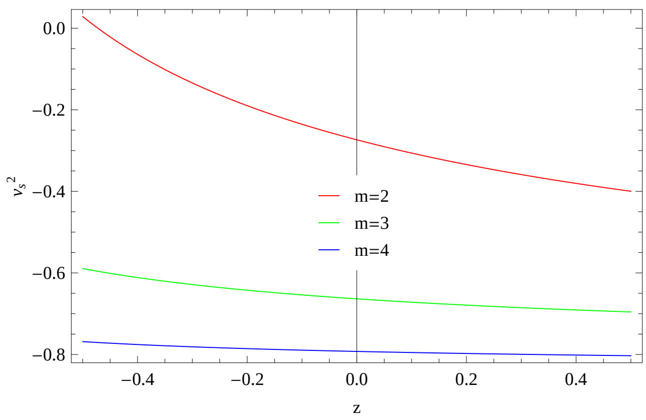

Figure 3.9: Plot of v_s^2 versus z taking power-law scale factor for RHDE model.

$$
\begin{aligned}
\omega_{EA} &= \Bigg(9mt^3\Bigg(-1+\frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)-\frac{1}{M^4}\epsilon\Bigg(3M^3t^3\Bigg(M\Bigg(-2\\
&+ \frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)+\frac{\sqrt{3}\sqrt{K\delta\epsilon\pi}\,\mathrm{ArcTan}\Big(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta\epsilon\pi}}\Big)}{K}-\frac{C_2}{\sqrt{K}}\Bigg)\\
&\times (\epsilon)^{-1}+\Bigg(m^2\Bigg(-3m^2t\delta^2\epsilon^2\pi^2\Big(m^2-t^2\delta\pi\Big)-3m\delta^{\frac{3}{2}}\epsilon^2\pi^{\frac{3}{2}}\Big(m^2+t^2\\
&\times \delta\pi\Big)^2\mathrm{ArcTan}\Big(\frac{m}{t\sqrt{\delta}\sqrt{\pi}}\Big)+\sqrt{3}Mt\sqrt{\delta\epsilon\pi}\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}\Big(m^2+t^2\delta\pi\Big)^2\\
&\times C_2\Bigg)\Bigg)\Bigg/\Bigg(\Bigg(\frac{m^2\epsilon}{M^2t^2}\Bigg)^{\frac{3}{2}}\sqrt{\delta\epsilon\pi}\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}\Big(m^2+t^2\delta\pi\Big)^2\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(9t^3\Bigg(m\\
&- \frac{3m\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)\Bigg),
\end{aligned}
$$

and ω'_{EA} is given as follows

$$
\omega'_{EA} = 3\Bigg(6-\frac{4\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}\sqrt{\delta\epsilon\pi}}+m\Bigg(-9+\frac{27\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)+\sqrt{\delta\epsilon\pi}
$$

$$
\begin{aligned}
&\times \Bigg(-\frac{9\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)} + \frac{4m^2\sqrt{\frac{m^2\epsilon}{M^2t^2}}\Big(m^4+4m^2t^2\delta\pi+t^4\delta^2\pi^2\Big)}{\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}\Big(m^2+t^2\delta\pi\Big)^3}\Bigg) \\
&+ \frac{4t\sqrt{\delta}\sqrt{\pi}\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}\mathrm{ArcTan}\Big(\frac{m}{t\sqrt{\delta}\sqrt{\pi}}\Big)}{m\sqrt{\frac{m^2\epsilon}{M^2t^2}}\sqrt{\delta\epsilon\pi}} - \frac{3\sqrt{3}\sqrt{K\delta\epsilon\pi}\mathrm{ArcTan}\Big(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta\epsilon\pi}}\Big)}{KM}\Bigg) \\
&+ \frac{\Bigg(9-\frac{4\sqrt{3}\sqrt{K}}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}}\Bigg)C_2}{\sqrt{K}M}\Bigg(9m\Bigg(m-\frac{3m\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)\Bigg)^{-1}.
\end{aligned}
$$

The expression for squared speed of sound turns out as

$$
\begin{aligned}
v_s^2 &= -3\Bigg(-4+\frac{\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}\sqrt{\delta\epsilon\pi}} + m\Bigg(6-\frac{18\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg) + \sqrt{\delta\epsilon\pi} \\
&\times \Bigg(\frac{6\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)} - \frac{m^2\sqrt{\frac{m^2\epsilon}{M^2t^2}}\Big(m^4+4m^2t^2\delta\pi+11t^4\delta^2\pi^2\Big)}{\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}\Big(m^2+t^2\delta\pi\Big)^3}\Bigg) \\
&- \frac{t\sqrt{\delta}\sqrt{\pi}\sqrt{\frac{m^2\delta\epsilon^2\pi}{M^2t^2}}\mathrm{ArcTan}\Big(\frac{m}{t\sqrt{\delta}\sqrt{\pi}}\Big)}{m\sqrt{\frac{m^2\epsilon}{M^2t^2}}\sqrt{\delta\epsilon\pi}} + \frac{2\sqrt{3}\sqrt{K\delta\epsilon\pi}\mathrm{ArcTan}\Big(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta\epsilon\pi}}\Big)}{KM}\Bigg) \\
&+ \frac{\Bigg(-6+\frac{\sqrt{3}\sqrt{K}}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}}\Bigg)C_2}{\sqrt{K}M}\Bigg(18\Bigg(m-\frac{3m\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)\Bigg)^{-1}.
\end{aligned}
$$

The plot of EoS parameter is shown in Figure 3.7 with respect to z. All the trajectories of EoS parameter represent the quintessence phase of the universe. Figure 3.8 shows the graph of $\omega_{EA}-\omega'_{EA}$ plane for same range of z. The trajectories of ω'_{EA} describe the negative behavior for all $\omega_{EA}<0$ give the freezing region of the universe. To check the stability of the underlying model, Figure 3.9 shows the unstable behavior of the model. However, for $m=2$, we get some stable points for $z<-0.475$.

Exponential form of scale factor:

Taking into account second scale factor Eq.(3.1.6) along with $F(K)$, we get the following energy density and pressure

$$\rho_{EA} = \frac{3C^2t^{-2+2\theta}\alpha^2\theta^2\Big(-3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}+\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)\Big)}{8\sqrt{K}\pi\Big(KM^2+3\delta\epsilon\pi\Big)} \quad (3.4.5)$$

$$\begin{aligned}
p_{EA} &= \frac{1}{24\pi}C^2t^{-3+\theta}\alpha\theta\Bigg(9t^{1+\theta}\alpha\theta\Bigg(-1+\frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)-\frac{1}{M^2}\epsilon(-1\\
&+ \theta)\Bigg(3Mt\Bigg(M\Bigg(2-\frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)-\frac{\mathrm{ArcTan}\Big(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta\epsilon\pi}}\Big)}{K}\\
&\times \sqrt{3}\sqrt{K\delta\epsilon\pi}+\frac{C_2}{\sqrt{K}}\Bigg)(\epsilon)^{-1}+\Bigg(\delta\pi\Bigg(3t^{\theta}\alpha\delta^{\frac{3}{2}}\epsilon^2\theta\pi^{\frac{3}{2}}\Bigg(t^{1+\theta}\alpha\sqrt{\delta}\theta\sqrt{\pi}\\
&\times \Big(t^{2\theta}\alpha^2\theta^2-t^2\delta\pi\Big)+\Big(t^{2\theta}\alpha^2\theta^2+t^2\delta\pi\Big)^2\mathrm{ArcTan}\Bigg(\frac{t^{-1+\theta}\alpha\theta}{\sqrt{\delta}\sqrt{\pi}}\Bigg)\Bigg)\\
&- \sqrt{3}Mt\sqrt{\delta\epsilon\pi}\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}\Big(t^{2\theta}\alpha^2\theta^2+t^2\delta\pi\Big)^2C_2\Bigg)\Bigg)\Big/\Bigg(\alpha\theta\\
&\times \sqrt{\frac{t^{-2+2\theta}\epsilon}{M^2}}(\delta\epsilon\pi)^{\frac{3}{2}}\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}\Big(t^{2\theta}\alpha^2\theta^2+t^2\delta\pi\Big)^2\Bigg)\Bigg)\Bigg). \quad (3.4.6)
\end{aligned}$$

The EoS parameter is obtained from above energy density and pressure. This parameter with its derivative are given by

$$\begin{aligned}
\omega_{EA} &= \Bigg(\sqrt{K}t^{-1-\theta}\Big(KM^2+3\delta\epsilon\pi\Big)\Bigg(9t^{1+\theta}\alpha\theta\Bigg(-1+\frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)\\
&- \frac{1}{M^2}\epsilon(-1+\theta)\Bigg(3Mt\Bigg(M\Bigg(2-\frac{3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{\sqrt{K}\Big(KM^2+3\delta\epsilon\pi\Big)}\Bigg)-\sqrt{3}\sqrt{K\delta\epsilon\pi}\\
&\times \frac{\mathrm{ArcTan}\Big(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta\epsilon\pi}}\Big)}{K}+\frac{C_2}{\sqrt{K}}\Bigg)(\epsilon)^{-1}+\Bigg(\delta\pi\Bigg(3t^{\theta}\alpha\delta^{\frac{3}{2}}\epsilon^2\theta\pi^{\frac{3}{2}}\Bigg(t^{1+\theta}\alpha\sqrt{\delta}
\end{aligned}$$

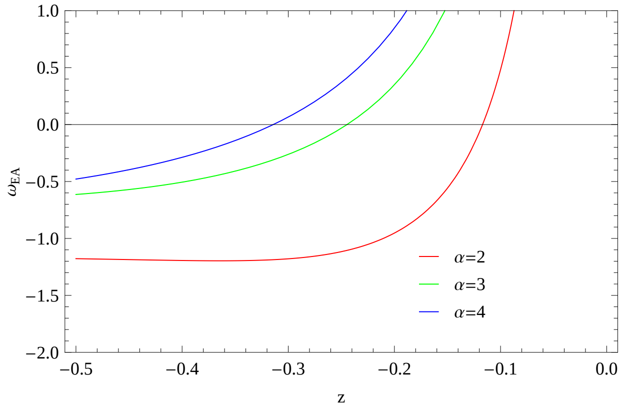

Figure 3.10: Plot of ω_{EA} versus z taking exponential scale factor for RHDE model.

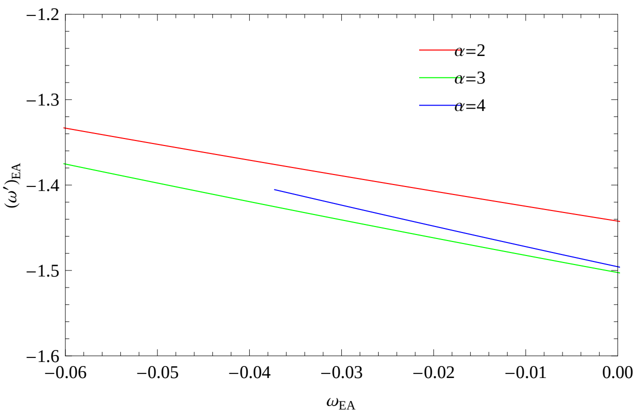

Figure 3.11: Plot of $\omega'_{EA} - \omega_{EA}$ taking exponential scale factor for RHDE model.

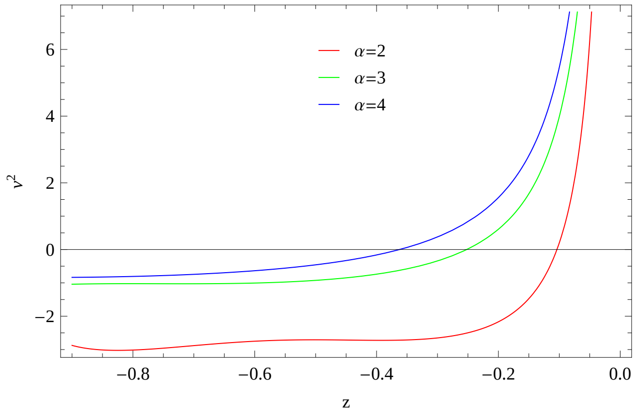

Figure 3.12: Plot of v_s^2 versus z taking exponential scale factor for RHDE model.

$$
\begin{aligned}
&\times\ \theta\sqrt{\pi}\Big(t^{2\theta}\alpha^2\theta^2 - t^2\delta\pi\Big) + \Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^2 \mathrm{ArcTan}\Big(\frac{t^{-1+\theta}\alpha\theta}{\sqrt{\delta}\sqrt{\pi}}\Big)\Big) \\
&-\ \sqrt{3}Mt\sqrt{\delta\epsilon\pi}\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}\Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^2 C_2\Big)\Big)/\Big(\alpha\theta\sqrt{\epsilon} \\
&\times\ \sqrt{\frac{t^{-2+2\theta}}{M^2}}(\delta\epsilon\pi)^{\frac{3}{2}}\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}\Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^2\Big)\Big)\Big)\Big)/\Big(9 \\
&\times\ \alpha\theta\Big(-3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi} + \sqrt{K}\Big(KM^2 + 3\delta\epsilon\pi\Big)\Big)\Big). \qquad (3.4.7)
\end{aligned}
$$

$$
\begin{aligned}
\omega'_{EA} &= \Big(t^{-5\theta}(-1+\theta)\Big(KM^2 + 3\delta\epsilon\pi\Big)\Big(3\Big(-\frac{3\sqrt{K}Mt^{3\theta}\alpha^3\theta^4\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{KM^2 + 3\delta\epsilon\pi} \\
&+\ \frac{1}{\epsilon^2\Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^3}KM^3t^{2+\theta}\alpha\theta\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}\Big(2t^{6\theta}\alpha^6\epsilon\theta^7\alpha\theta \\
&\times\ \sqrt{\frac{t^{-2+2\theta}\epsilon}{M^2}} + t^6\delta^2\pi^2\Big(2\delta\epsilon\theta\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}\pi + (1-2\theta)\sqrt{\delta\epsilon\pi}\alpha\epsilon\theta \\
&\times\ \sqrt{\frac{t^{-2+2\theta}\delta\pi}{M^2}}\Big) + 2t^{4+2\theta}\alpha^2\delta\theta^2\pi\Big(3\delta\epsilon\theta\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}\pi - 4(-1 \\
&+\ \theta)\sqrt{\delta\epsilon\pi}\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}\Big) + t^{2+4\theta}\alpha^4\theta^4\Big(6\delta\epsilon\theta\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}\pi \\
&+\ (-1+2\theta)\sqrt{\delta\epsilon\pi}\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}\Big)\Big) - \delta^{3/2}\pi^{3/2}\Big(\sqrt{3}t^{3\theta}\alpha^3\epsilon^{3/2}\theta^4 \\
&\times\ \sqrt{K\delta\epsilon\pi}\mathrm{ArcTan}\Big(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta}\sqrt{\epsilon}\sqrt{\pi}}\Big) + KM^3t^3(1-2\theta)\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}
\end{aligned}
$$

$$
\begin{aligned}
&\times \quad \sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}\text{ArcTan}\left(\frac{t^{-1+\theta}\alpha\theta}{\sqrt{\delta}\sqrt{\pi}}\right)\Bigg)((\delta\epsilon\pi)^{3/2})^{-1}\Bigg) + \alpha\theta\Bigg(3 \\
&\times \quad \sqrt{K}t^{3\theta}\alpha^2\epsilon\theta^3 - \sqrt{3}KM^2t^{2+\theta}\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}(-1+2\theta)\Bigg)C_2(\epsilon)^{-1}\Bigg) \\
&\times \quad \Bigg)/\Bigg(9\sqrt{K}M\alpha^5\theta^5\Bigg(-3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi} + \sqrt{K}\Bigg(KM^2 + 3\delta\epsilon\pi\Bigg)\Bigg)\Bigg) \qquad (3.4.8)
\end{aligned}
$$

The correspond expression for v_s^2 is given by

$$
\begin{aligned}
v_s^2 \quad &= \quad \Bigg(t^{-\theta}\Bigg(KM^2 + 3\delta\epsilon\pi\Bigg)\Bigg(3\Bigg(\frac{3\sqrt{K}M\Big(-2+\theta+6t^{\theta}\alpha\theta\Big)\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi}}{KM^2+3\delta\epsilon\pi} \\
&+ \quad KM\Bigg(4 - \frac{t^6\delta^3\pi^3\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}}{\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}\sqrt{\delta\epsilon\pi}\Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^3} + t^{6\theta}\alpha^6\theta^6\sqrt{\delta\epsilon\pi} \\
&\times \quad \frac{\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}}{M^2\left(\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}\right)^{3/2}\Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^3} + 8M^2t^6\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}} \\
&\times \quad (\delta\epsilon\pi)^{\frac{3}{2}}\frac{\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}}{\epsilon^3\Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^3} + 2\theta\Bigg(-1 - 3t^{\theta}\alpha - 4M^2t^6\alpha\theta(\delta\epsilon\pi)^{\frac{3}{2}} \\
&\times \quad \sqrt{\frac{t^{-2+2\theta}\epsilon}{M^2}}\frac{\sqrt{\frac{t^{-2+2\theta}\alpha^2\delta\epsilon^2\theta^2\pi}{M^2}}}{\epsilon^3\Big(t^{2\theta}\alpha^2\theta^2 + t^2\delta\pi\Big)^3}\Bigg)\Bigg) + Kt^{-1+\theta}\alpha\theta\sqrt{\delta\epsilon\pi}\alpha\epsilon\theta\sqrt{\frac{t^{-2+2\theta}\delta\pi}{M^2}} \\
&\times \quad \frac{\text{ArcTan}\left(\frac{t^{-1+\theta}\alpha\theta}{\sqrt{\delta}\sqrt{\pi}}\right)}{M\sqrt{\delta}\left(\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}\right)^{3/2}\sqrt{\pi}} + \sqrt{3}(-2+\theta)\sqrt{K\delta\epsilon\pi}\text{ArcTan}\left(\frac{\sqrt{K}M}{\sqrt{3}\sqrt{\delta\epsilon\pi}}\right)\Bigg) \\
&+ \quad \sqrt{K}\Bigg(6 - 3\theta - \frac{\sqrt{3}\sqrt{K}}{\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}}\Bigg)C_2\Bigg)\Bigg)/\Bigg(18\sqrt{K}M\alpha\theta\Bigg(-3\sqrt{\delta\epsilon\pi}\sqrt{K\delta\epsilon\pi} \\
&+ \quad \sqrt{K}\Bigg(KM^2 + 3\delta\epsilon\pi\Bigg)\Bigg)\Bigg). \qquad (3.4.9)
\end{aligned}
$$

Figure 3.10 represents the graph of EoS parameter versus z for RHDE model taking exponential form of the scale factor. For $\alpha = 2$, initially the trajectory expresses the transition from decelerated phase to accelerated phase and then crosses the phantom divide line and gives the phantom phase of the universe. For higher values of the α that is for $\alpha = 3, 4$, the trajectories of EoS parameter represents the quintessence phase. In Figure 3.11, we plot the graph of evolution parameter of EoS versus EoS parameter which gives the freezing region of the universe. Figure 3.12 shows the graph of v_s^2 for stability analysis of the model. Initially the graph gives the stability and then for decreasing z, the model becomes unstable. As we increase the value of α, the trajectories give more stable points.

3.5 Reconstruction from Sharma-Mittal Holographic Dark Eneryg Model

Sharma-Mittal introduced a two parametric entropy and is defined as [55]

$$S_{SM} = \frac{1}{1-r}\left((\Sigma_{i=1}^{n} P_i^{1-\delta})^{1-r/\delta} - 1\right), \tag{3.5.1}$$

where r is a new free parameter. The expression of SMHDE model for Hubble horizon is given by

$$\rho_D = \frac{3\epsilon H^4}{8\pi R}\left((1 + \frac{\delta\pi}{H^2})^{\frac{R}{\delta}} - 1\right). \tag{3.5.2}$$

By comparing the energy densities of SMHDE model and Einstein-Aether gravity model, we find

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{KM^2}{24\epsilon\pi R}\left(\left(1 + \frac{3\epsilon\pi\delta}{KM^2}\right)^{\frac{R}{\delta}} - 1\right), \tag{3.5.3}$$

which leads us to the following solution

$$F(K) = \frac{K^2M^2\left(-1 + \text{Hypergeometric2F1}(-\frac{3}{2}, -\frac{R}{\delta}, \frac{-1}{2}, \frac{-3\pi\delta\epsilon}{KM^2})\right)}{36\pi R\epsilon}. \tag{3.5.4}$$

Power-law form of scale factor:

For this scale factor, we obtain

$$\rho_{EA} = \frac{Km^2M^2\Bigg(-1+\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{\frac{R}{\delta}}\Bigg)}{8\pi Rt^2}. \quad (3.5.5)$$

$$\begin{aligned}
p_{EA} &= \frac{1}{72t^4}m\Bigg(\frac{1}{\pi R\Big(m^2+\pi t^2\delta\Big)}\Bigg(3m^2\Bigg(-8\Big(m^2+\pi t^2\delta\Big)\\
&+ 3\Big(1+\frac{\pi t^2\delta}{m^2}\Big)^{\frac{R}{\delta}}\Big(3m^2+\pi t^2(-2R+3\delta)\Big)\Bigg)\epsilon+KM^2\\
&\times t^2\Big(m^2+\pi t^2\delta\Big)\Bigg(-4+9m-3(-1+3m)\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{\frac{R}{\delta}}\Bigg)\\
&- \Big(m^2+\pi t^2\delta\Big)\Bigg(3m^2\epsilon\text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^2\delta}{m^2}\Big)\\
&- KM^2t^2\text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{3\pi\delta\epsilon}{KM^2}\Big)\Bigg)\Bigg)\\
&- \frac{12t^2\epsilon\Bigg(-3+\frac{\sqrt{3}\sqrt{K}}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}}\Bigg)C_3}{\sqrt{K}}\Bigg). \quad (3.5.6)
\end{aligned}$$

The cosmological parameters are given by

$$\begin{aligned}
\omega_{EA} &= \frac{1}{9KmM^2t^2\Bigg(-1+\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{\frac{R}{\delta}}\Bigg)}\pi R\Bigg(\frac{1}{\pi R\Big(m^2+\pi t^2\delta\Big)}\Bigg(3m^2\\
&\times \Bigg(-8\Big(m^2+\pi t^2\delta\Big)+3\Big(1+\frac{\pi t^2\delta}{m^2}\Big)^{\frac{R}{\delta}}\Big(3m^2+\pi t^2(-2R+3\delta)\Big)\Bigg)\\
&\times \epsilon+KM^2t^2\Big(m^2+\pi t^2\delta\Big)\Bigg(-4+9m-3(-1+3m)\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{\frac{R}{\delta}}\\
&\times \Bigg)-\Big(m^2+\pi t^2\delta\Big)\Bigg(3m^2\epsilon\text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^2\delta}{m^2}\Big)\\
&- KM^2t^2\text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{3\pi\delta\epsilon}{KM^2}\Big)\Bigg)\Bigg)-12t^2\epsilon\\
&\times \frac{\Bigg(-3+\frac{\sqrt{3}\sqrt{K}}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}}\Bigg)C_3}{\sqrt{K}}\Bigg), \quad (3.5.7)
\end{aligned}$$

$$\omega'_{EA} = m^4\epsilon\Bigg(-3\Big(1+\frac{\pi t^2\delta}{m^2}\Big)^{\frac{R}{\delta}}\Bigg(5m^4+2m^2\pi t^2(-3R+5\delta)+\pi^2t^4\Big(4R^2$$

$$
\begin{aligned}
&- \quad 10R\delta+5\delta^2\Big)\Big)-\Big(m^2+\pi t^2\delta\Big)^2\Big(-16\\
&+ \quad \text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^2\delta}{m^2}\Big)\Big)\Big)-4\sqrt{3}M^2\pi Rt^4\\
&\times \quad \Big(m^2+\pi t^2\delta\Big)^2\sqrt{\frac{m^2\epsilon}{M^2t^2}}C_3\Big(3Km^4M^2t^2\Big(m^2+\pi t^2\delta\Big)^2\Big(-1+\Big(1\\
&+ \quad \frac{3\pi\delta\epsilon}{KM^2}\Big)^{\frac{R}{\delta}}\Big)\Big), \qquad (3.5.8)
\end{aligned}
$$

$$
\begin{aligned}
v_s^2 \quad &= \quad \frac{1}{18K^{\frac{3}{2}}m^3M^2t^2\Big(m^2+\pi t^2\delta\Big)^2\Big(-1+\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}\Big)}\Big(\sqrt{K}m^2\\
&\times \quad \Big(3m^2\Big(-32\Big(m^2+\pi t^2\delta\Big)^2+3\Big(1+\frac{\pi t^2\delta}{m^2}\Big)^{R/\delta}\Big(11m^4+2m^2\pi t^2\\
&\times \quad (-5R+11\delta)+\pi^2t^4\Big(4R^2-14R\delta+11\delta^2\Big)\Big)\Big)\epsilon-2KM^2t^2\Big(m^2\\
&+ \quad \pi t^2\delta\Big)^2\Big(4-9m+3(-1+3m)\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}\Big)-\Big(m^2+\pi t^2\delta\Big)^2\\
&\times \quad \Big(3m^2\epsilon\text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^2\delta}{m^2}\Big)-2KM^2t^2\\
&\times \quad \text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{3\pi\delta\epsilon}{KM^2}\Big)\Big)\Big)+12\pi Rt^2\Big(m^3\\
&+ \quad m\pi t^2\delta\Big)^2\epsilon\Big(6-\frac{\sqrt{3}\sqrt{K}}{\sqrt{\frac{m^2\epsilon}{M^2t^2}}}\Big)C_3\Big), \qquad (3.5.9)
\end{aligned}
$$

We plot EoS parameter for SMHDE model with respect to redshift parameter as shown in Figure 3.13 for power-law scale factor. For $m=3$ and 4, the trajectories represent the transition from quintessence to phantom phase while $m=2$ indicates the phantom era throughout for z. The plot of this parameter with its evolution parameter is given in Figure 3.14 which shows the freezing region of the evolving universe. However, for higher values of m, we may get thawing region ($\omega'_{EA}>0$). Figure 3.15 gives the graph of squared speed of sound versus redshift. The trajectory for $m=2$ shows the stability of the model as redshift parameter decreases while other

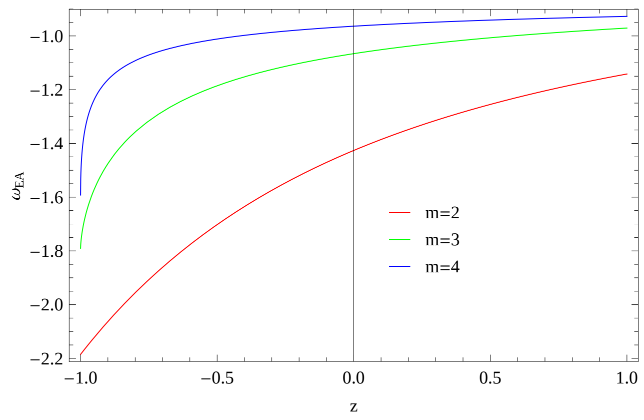

Figure 3.13: Plot of ω_{EA} versus z taking power-law scale factor for SMHDE model.

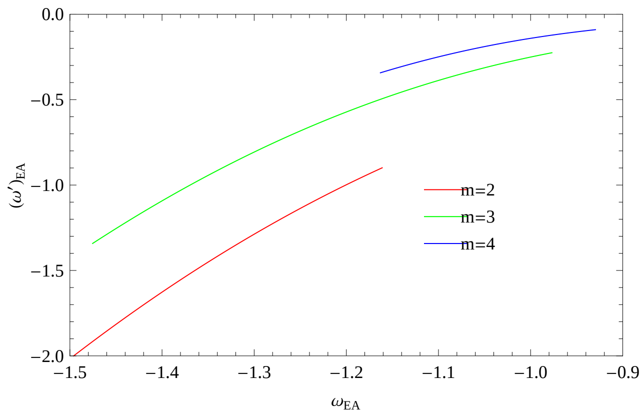

Figure 3.14: Plot of $\omega'_{EA} - \omega_{EA}$ taking power-law scale factor for SMHDE model.

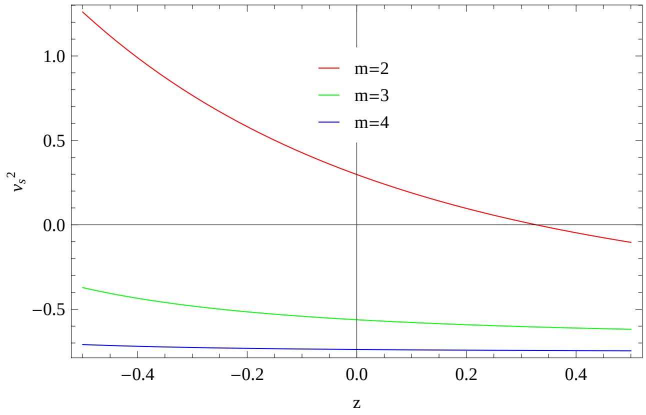

Figure 3.15: Plot of v_s^2 versus z taking power-law scale factor for SMHDE model.

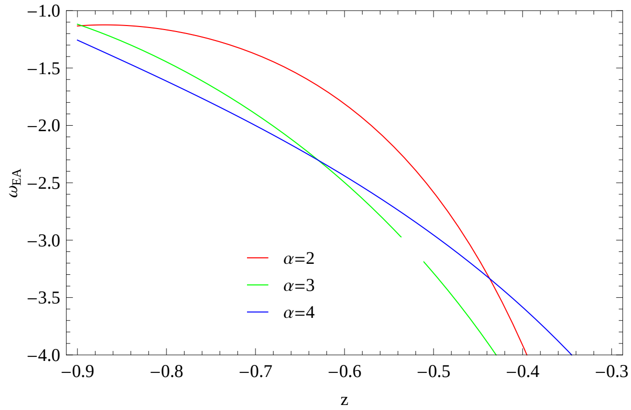

Figure 3.16: Plot of ω_{EA} versus z taking exponential scale factor for SMHDE model.

trajectories describe the unstable behavior of the model.

<u>Exponential form of scale factor:</u>

Following the same steps, we obtain the following expressions for energy density, pressure and parameters for exponential scale factor. These are

$$\rho_{EA} = \frac{KM^2 t^{-2+2\theta}\alpha^2\left(-1+\left(1+\frac{3\pi\delta\epsilon}{KM^2}\right)^{R/\delta}\right)\theta^2}{8\pi R}, \tag{3.5.10}$$

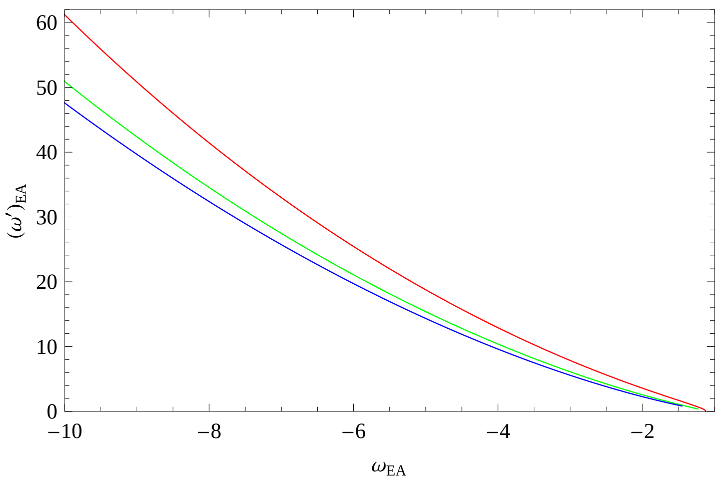

Figure 3.17: Plot of $\omega'_{EA} - \omega_{EA}$ taking exponential scale factor for SMHDE model.

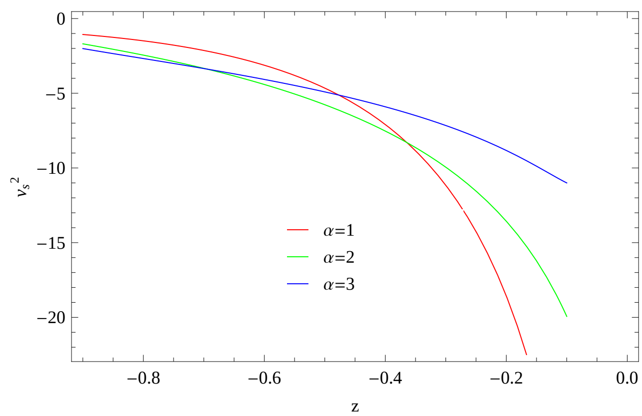

Figure 3.18: Plot of v_s^2 versus z taking exponential scale factor for SMHDE model.

$$
\begin{aligned}
p_{EA} &= \frac{1}{24}t^{-4+\theta}\alpha\theta\Bigg(-\frac{3KM^2t^{2+\theta}\alpha\Big(-1+\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}\Big)\theta}{\pi R}-\epsilon(-1+\theta)\\
&\times \Bigg(t^2\Bigg(K^{3/2}M^2\Bigg(-4+3\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}\\
&+ \text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{3\pi\delta\epsilon}{KM^2}\Big)\Bigg)+36\pi R\epsilon C_3\Bigg)\\
&\times (3\sqrt{K}\pi R\epsilon)^{-1}+t^{2\theta}\alpha^2\theta^2\Bigg(-8-3\Big(1+\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\Big)^{R/\delta}\Big(\pi t^2(-2R\\
&+ 3\delta)+3t^{2\theta}\alpha^2\theta^2\Big)(\pi t^2\delta+t^{2\theta}\alpha^2\theta^2)^{-1}\\
&+ \text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\Big)(\pi R)^{-1}-4\sqrt{3}\epsilon\\
&\times \frac{C_3}{M^2\Big(\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}\Big)^{3/2}}\Bigg)\Bigg)\Bigg), \qquad (3.5.11)\\
\omega_{EA} &= \frac{1}{3KM^2\alpha\Big(-1+\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}\Big)\theta}\pi Rt^{-2-\theta}\Bigg(-3KM^2t^{2+\theta}\alpha\Bigg(-1\\
&+ \Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}\Bigg)\theta(\pi R)^{-1}-\epsilon(-1+\theta)\Bigg(t^2\Bigg(K^{3/2}M^2\Bigg(-4\\
&+ 3\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}+\text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{3\pi\delta\epsilon}{KM^2}\Big)\Bigg)\\
&+ 36\pi R\epsilon C_3\Bigg)(3\sqrt{K}\pi R\epsilon)^{-1}+t^{2\theta}\alpha^2\theta^2\Bigg(-8-3\Big(1+\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\Big)^{R/\delta}\Big(\pi t^2\\
&\times (-2R+3\delta)+3t^{2\theta}\alpha^2\theta^2\Big)(\pi t^2\delta+t^{2\theta}\alpha^2\theta^2)^{-1}\\
&+ \text{Hypergeometric2F1}\Big(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\Big)(\pi R)^{-1}-4\sqrt{3}\epsilon\\
&\times \frac{C_3}{M^2\Big(\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}\Big)^{3/2}}\Bigg)\Bigg)\Bigg), \qquad (3.5.12)\\
\omega'_{EA} &= \frac{1}{9KM^2\alpha^2\Big(-1+\Big(1+\frac{3\pi\delta\epsilon}{KM^2}\Big)^{R/\delta}\Big)\theta^2}\pi Rt^{-2(1+\theta)}(-1+\theta)
\end{aligned}
$$

$$
\begin{aligned}
&\times \quad \Bigg(\frac{1}{\pi R\left(\pi t^2\delta + t^{2\theta}\alpha^2\theta^2\right)^2}\theta\Bigg(KM^2t^2\Bigg(-4+3\left(1+\frac{3\pi\delta\epsilon}{KM^2}\right)^{R/\delta}\Bigg)\\
&\times \quad \left(\pi t^2\delta + t^{2\theta}\alpha^2\theta^2\right)^2 + 3t^{2\theta}\epsilon\theta\Bigg(8(-2+\theta)\left(\pi t^2\alpha\delta + t^{2\theta}\alpha^3\theta^2\right)^2 - 3\alpha^2\\
&\times \quad \left(1+\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\right)^{R/\delta}\Bigg(t^{4\theta}\alpha^4\theta^4(-5+2\theta) + 2\pi t^{2+2\theta}\alpha^2\theta^2(R(3-2\theta)\\
&+ \quad \delta(-5+2\theta)) + \pi^2t^4\left(2R\delta(5-4\theta) + 4R^2(-1+\theta) + \delta^2(-5+2\theta)\right)\Bigg)\Bigg)\\
&+ \quad \left(\pi t^2\delta + t^{2\theta}\alpha^2\theta^2\right)^2\Bigg(KM^2t^2\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{3\pi\delta\epsilon}{KM^2}\right)\\
&- \quad 3t^{2\theta}\alpha^2\epsilon\theta(-1+2\theta)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\right)\Bigg)\Bigg)\\
&+ \quad 12t^2\epsilon\left(\frac{3\theta}{\sqrt{K}} + \frac{\sqrt{3}(1-2\theta)}{\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}}\right)C_3\Bigg),
\end{aligned}
\tag{3.5.13}
$$

$$
\begin{aligned}
v_s^2 \quad &= \quad \frac{1}{18KM^2\alpha\left(-1+\left(1+\frac{3\pi\delta\epsilon}{KM^2}\right)^{R/\delta}\right)\theta}\pi Rt^{-2-\theta}\Bigg(\frac{1}{\pi R\left(\pi t^2\delta + t^{2\theta}\alpha^2\theta^2\right)^2}\\
&\times \quad \Bigg(KM^2t^2\left(\pi t^2\delta + t^{2\theta}\alpha^2\theta^2\right)^2\Bigg(-8+4\theta+18t^\theta\alpha\theta - 3\left(1+\frac{3\pi\delta\epsilon}{KM^2}\right)^{R/\delta}\\
&\times \quad \left(-2+\theta+6t^\theta\alpha\theta\right)\Bigg)\Bigg) + 3t^{2\theta}\epsilon\Bigg(8(-4+3\theta)\left(\pi t^2\alpha\delta\theta + t^{2\theta}\alpha^3\theta^3\right)^2 - 3\alpha^2\\
&\times \quad \left(1+\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\right)^{R/\delta}\theta^2\Bigg(t^{4\theta}\alpha^4\theta^4(-11+8\theta) + 2\pi t^{2+2\theta}\alpha^2\theta^2(R(5-4\theta)\\
&+ \quad \delta(-11+8\theta)) + \pi^2t^4\left(2R\delta(7-6\theta) + 4R^2(-1+\theta) + \delta^2(-11+8\theta)\right)\Bigg)\Bigg)\\
&- \quad \left(\pi t^2\delta + t^{2\theta}\alpha^2\theta^2\right)^2\Bigg(KM^2t^2(-2+\theta)\\
&\times \quad \text{Hypergeometric2F1}\left[-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{3\pi\delta\epsilon}{KM^2}\right] + 3t^{2\theta}\alpha^2\epsilon\theta^2\\
&\times \quad \text{Hypergeometric2F1}\left[-\frac{3}{2},-\frac{R}{\delta},-\frac{1}{2},-\frac{\pi t^{2-2\theta}\delta}{\alpha^2\theta^2}\right]\Bigg)\Bigg) + 12t^2\epsilon C_3\\
&\times \quad \frac{\left(6-3\theta-\frac{\sqrt{3}\sqrt{K}}{\sqrt{\frac{t^{-2+2\theta}\alpha^2\epsilon\theta^2}{M^2}}}\right)}{\sqrt{K}}\Bigg).
\end{aligned}
\tag{3.5.14}
$$

For exponential scale factor for SMHDE model, the plot of EoS parameter in Figure 3.16 represents the phantom behavior initially but converges to cosmological constant behavior for $\alpha = 2, 3$ as z decreases. For $\alpha = 4$, the EoS parameter gives the phantom behavior. Figure 3.17 represents the graph of $\omega'_{EA} - \omega_{EA}$ plane which shows the positive behavior of ω'_{EA} versus negative ω_{EA} expressing thawing region of the universe. The squared speed of sound graph gives unstable behavior of the SMHDE model in the framework of Einstein-Aether theory of gravity.

Summary

In this work, we have discussed about the Einstein-Aether gravity. For this theory, we have talked about the modified Friedmann equations and hence from these equations we obtain the effective density and pressure for Einstein-Aether theory. We have analyzed the reconstructed Einstein-Aether models by using some holographic dark energy models. In the presence of free function $F(K)$, we have treated the affective density and pressure as DE. From the modified HDE models such as Tsallis HDE, R$\acute{e}$nyi HDE model and Sharma-Mittal HDE models, we have formed the unknown function $F(K)$ for Einstein-Aether theory by considering the power-law form and exponential form of scale factor. We have discussed some cosmological parameters like, EoS parameter with its evolutionary parameter and squared speed of sound to check the stability of reconstructed models for this theory.

The remaining results have been summarized as follows:

EoS parameter for power-law scale factor:

- THDE $\Rightarrow$ phantom behavior,
- RHDE $\Rightarrow$ quintessence phase,
- SMHDE $\Rightarrow$ transition from quintessence to phantom phase for $m = 3, 4$, phantom era for $m = 2$.

EoS parameter for exponential scale factor:

- THDE $\Rightarrow$ transition from quintessence to phantom era for $\alpha = 2$, phantom behavior for $\alpha = 3$, ΛCDM model for $\alpha = 4$,
- RHDE $\Rightarrow$ phantom phase for $\alpha = 2$, quintessence phase for $\alpha = 3, 4$

- SMHDE $\Rightarrow$ cosmological constant behavior for $\alpha = 2, 3$, phantom behavior for $m = 4$.

<u>$\omega' - \omega$ **plane for power-law scale factor:**</u>

- THDE $\Rightarrow$ freezing region for $m = 2, 3$, thawing region for $m = 4$,
- RHDE $\Rightarrow$ freezing region,
- SMHDE $\Rightarrow$ freezing region.

<u>$\omega' - \omega$ **plane for exponential scale factor:**</u>

- THDE $\Rightarrow$ freezing region to thawing region,
- RHDE $\Rightarrow$ freezing region,
- SMHDE $\Rightarrow$ thawing region.

<u>**Squared speed of sound for power-law scale factor:**</u>

- THDE $\Rightarrow$ unstable,
- RHDE $\Rightarrow$ unstable,
- SMHDE $\Rightarrow$ stable for $m = 2$, unstable for $m = 3, 4$.

<u>**Squared speed of sound for exponential scale factor:**</u>

- THDE $\Rightarrow$ unstable,
- RHDE $\Rightarrow$ stability for higher values and instability for lower values,
- SMHDE $\Rightarrow$ unstable.

Bibliography

[1] B. Gold. et al.: Astrophys. J. Suppl. 180, 265 (2009).

[2] M.R. Nolta, et al.: Astrophys. J. Suppl. 180, 296 (2009).

[3] N.A. Bachall. et al.: Science 284, 1481 (1999).

[4] S.J. Perlmutter. et al.: ApJ. 517, 565 (1999).

[5] S.J. Perlmutter. et al.: Nature 391, 51 (1998).

[6] A.G. Riess. et al.[Supernova Search Team Collaboration]: Astron. J. 116, 1009 (1998).

[7] A.G. Riess. et al.: Astrophys. J. 607, 665 (2004).

[8] C. Bennet. et al.: Phys. Rev. Lett. 85, 2236 (2000).

[9] D.N. Spergel. et al.: Astrophys. J. Suppl. Ser. 170, 377 (2007).

[10] L. Carl, Gardner: Nucl. Phys. B 707, 278 (2005).

[11] J. Ponce de Leon: Gen. Rel. Grav. 38, 61 (2006).

[12] J. V. Cunha, Phys. Rev. D 79, 047301 (2009).

[13] M. Roos, Introduction to Cosmology (John Wiley andSons, UK) (2003).

[14] S. Briddle. et al.: Science 299, 1532 (2003).

[15] D.N. Spergel. et al.: Astrophys. J. Suppl. 148, 175 (2003).

[16] K. Enqvist, S. Hannested and M. S. Sloth, JCAP 2, 004 (2005).

[17] X. Zhang, Int. J. Mod. Phys. D 14, 1597 (2005).

[18] D. Pavon and W. Zimdahl, hep-th/0511053.

[19] W. Fischler and L. Susskind, hep-th/9806039.

[20] S. Tsujikawa, Lect. Notes Phys. 800, 99 (2010).

[21] G. R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B 484, 112 (2000).

[22] S. Nojiri. et al.: Phys. Lett. B 659, 821 (2008).

[23] B. Li, J. D. Barrow, Phys. Rev. D 75, 084010 (2007).

[24] S. Nojiri, S. D. Odintsov, Phys. Rev. D 74, 086005 (2006).

[25] M.C.B. Abdalla, S. Nojiri, S.D. Odintsov, Class. Quantum Grav. 22, L35 (2005).

[26] E.V. Linder, Phys. Rev. D 81, 127301 (2010).

[27] K. K. Yerzhanov et al.: arXiv:1006.3879v1 [gr-qc](2010).

[28] S. Nojiri, S. D. Odintsov, Phys. Lett. B631, 1 (2005).

[29] I. Antoniadis, J. Rizos, K. Tamvakis, Nucl. Phys. B 415, 497 (1994).

[30] P. Horava, JHEP 0903, 020 (2009).

[31] C. Brans, H. Dicke, Phys. Rev. 124, 925 (1961).

[32] T. Jacobson, D. Mattingly, Phys. Rev. D 64, 024028 (2001).

[33] T. Jacobson, D. Mattingly, Phys. Rev. D 70, 024003 (2004).

[34] J.D. Barrow, Phys. Rev. D 85, 047503 (2012).

[35] T.G. Zlosnik, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 75, 044017 (2007).

[36] T.G. Zlosnik, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 77, 084010 (2008).

[37] D. Garfinkle, T. Jacobson, Phys. Rev. Lett. 107, 191102 (2011).

[38] E. V. Linder, R. J. Scherrer, Phys. Rev. D 80, 023008 (2009).

[39] J. D. Barrow, Phys. Rev. D 85, 047503 (2012).

[40] J. Zuntz, T.G. Zlosnik, F. Bourliot, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 81, 104015 (2010).

[41] B. Li, D. Fonseca Mota, J. D. Barrow, Phys. Rev. D 77, 024032 (2008).

[42] M. Gasperini, Class. Quant. Grav. 4, 485 (1987).

[43] M. Gasperini, Gen. Rel. Grav. 30, 1703 (1998).

[44] X. Meng, X. Du, Phys. Lett. B 710, 493 (2012).

[45] X. Meng, X. Du, Comm. Theor. Phys. 57, 227 (2012).

[46] A. Achcarro, J. O. Gong, S. Hardeman, G. A. Palma and S. P. Patil, Journal of Cosmology and Astroparticle Physics, 01, 030 (2011)

[47] R. R. Caldwell, E. V. Linder, Phys. Rev. Lett. 95, 141301 (2005).

[48] M. Gasperini, Gen. Rel. Grav. 30, 1703 (1998).

[49] T. Jacobson, D. Mattingly, Phys. Rev. D 64, 024028 (2001).

[50] T.G. Zlosnik, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 75, 044017 (2007).

[51] T.G. Zlosnik, P.G. Ferreira, G.D. Starkman, Phys. Rev. D 77, 084010 (2008).

[52] S. Nojiri, S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007).

[53] M. Tavayef, A. Sheykhi, K. Bamba, H. Moradpour, Physics Letters B 781, 195-200 (2018)

[54] H. Moradpour, S. A. Moosavi, I. P. Lobo, J. P. M. Graca, A. Jawad, I. G. Salako, Eur. Phys. J. C 829, 7883 (2018)

[55] A. S. Jahromi, S. A. Moosavi, H. Moradpour, J. P. M. Grac, I. P. Lobo, I. G. Salako, A. Jawad, Phys. Lett. B 780, 2124 (2018)