

General Linear Methods with Projection



By

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CIIT/FA16-RMT-031/LHR

MS Thesis

In

Mathematics

COMSATS University Islamabad

Lahore Campus

Spring, 2018



COMSATS University Islamabad, Lahore Campus

General Linear Methods with Projection

A Thesis Presented to

COMSATS University Islamabad, Lahore Campus

In partial fulfillment

of the requirement for the degree of

MS (Mathematics)

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Spring, 2018

General Linear Methods with Projection

A Post Graduate Thesis submitted to the Department of Mathematics as partial fulfillment of the requirement for the award of Degree of MS Mathematics.

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Declaration

I Lubna Mustafa with registration number **CIIT/FA16-RMT-031/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken my material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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Certificate

It is certified that Lubna Mustafa (CIIT/FA16-RMT-031/LHR) has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus, and the work fulfills the requirement for award of MS degree.

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DEDICATION

To my Parents and Teachers

ACKNOWLEDGEMENTS

I would like to thank all the people who have been helpful in any way in the completion of this work. I thank my supervisor Dr. **Yousaf Habib** for his guidance. I also pay my gratitude to **Dr. Sarfraz Ahmad**, head of the Mathematics Department at COMSATS University Islamabad, Lahore Campus. I am grateful to my parents and my siblings for their unfailing support and encouragement throughout my years of study.

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Abstract

The present work deals with the structure-preserving numerical methods for the solution of ordinary differential equations representing dynamic systems from the Hamiltonian perspective of classical mechanics. These differential systems involve quantities that should remain constant over the course of enquiry. Whereas the usual numerical methods do not account for the preservation of these invariants, structure-preserving methods such as the symplectic Runge-Kutta methods and the G -symplectic general linear methods faithfully maintain these characteristic properties of a Hamiltonian system during the numerical discretization of the problem. Physically significant invariant properties of a Hamiltonian system include the conservation of total energy and the symplecticity of flow.

In Chapter 1, basic theory of ordinary differential equations is introduced. The exposition combines definitions with the construction and implementation of numerical methods for ordinary differential equations and is central to all the subsequent investigations carried out in this work. In Chapter 2, long - term integrators for a Hamiltonian system are treated in detail. The general linear methods subsume the Runge-Kutta methods as well as the linear multi-step methods introduced previously. In Chapter 3, the important notion of projection is also introduced here. The basic idea of the standard projection technique is generalized to develop the heterogeneous class of numerical methods collectively known as the general linear methods. Projection technique is applied to the general linear methods and the similarity of the symplectic Runge-Kutta methods to the G -symplectic general linear methods is discussed.