SUPER EDGE-MAGIC DEFICIENCIES OF FORESTS OF ALPHA FAMILIES OF TREES

By

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Abstract

Let G be a graph which is admitting an edge-magic total valuation. The edge magic deficiency [38] of this graph G is the positive numbers α required to make it a edge magic total graph. The general notation for edge-magic deficiency is $\mu(G)$. If there does not exist such α then edge magic deficiency will be ∞. The super edge magic deficiency of graph G [17], follows the same definition and it is denoted by $\mu_s(G)$.
This dissertation studies about the super edge-magic deficiencies of forests of alpha families of trees. Firstly, We show that alpha trees with caterpillars as its components have super edge-magic deficiency is $\mu_s = 0$. We also prove that if the alpha tree has w-trees and alpha trees as its components then its super edge-magic deficiency is $\mu_s = 0$. At the end, we prove that forest of alpha trees with components as banana tree and stars preserve super edge-magic deficiency $\mu_s = 1$.

Chapter 1

Introduction

1.1 Graph

Let $G\prime$ be a graph which has two sets, one is called vertex set and other is called edge set. It is denoted by $G(V,E)$. The cardinality of vertex set is called order and cardinality of edge set is called size of graph. We will consider only finite and simple graphs.

1.2 Subgraph

A subgraph H of G is a trivial or non-trivial subpart of graph G. Obvious $V(H) \subseteq V(G)$ and $E(H) \subset E(G)$. If $V(H) = V(G)$ and $E(H) \subset E(G)$ then H is called spanning subgraph of G. If $V(H) = V(G)$ and $E(H) = E(G)$ then H is called trivial subgraph.

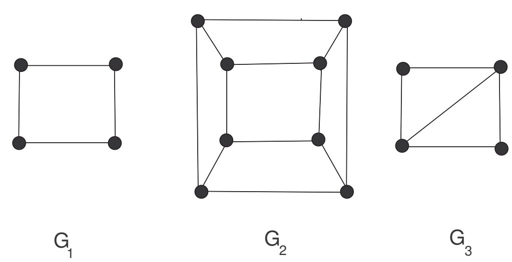


Figure 1.1: $G_1 \subseteq G_2$, $G_1 \subseteq G_3$, $G_3 \not\subseteq G_2$.

1.3 Walk

A walk in a graph a is a finite alternating sequence of vertex and edges. If no vertex is repeats in a walk ”W” the walk is called trail. If no edges no vertex repeats in a path.

1.4 Connected Graph

A graph is connected if we can travel from one vertex to other vertex without jumping. If we have to jumped then graph is called disconnected graph.

1.5 Tree

A tree graph G is special class of graphs which are connected and has no close walk. There are many properties of trees. If a tree has n vertices then number of edges will be $n-1$. In a tree has at least two vertices having degree one which is called called leaves i.e a vertex having degree 1. Every edge is a bridge.

1.6 Distance

Length of a shortest route between two vertices a and b is called the distance between them and it is denoted by $d(a,b)$.

1.6.1 Line Graph

Line graph of graph G is denoted by $L(G)$ such that its order is the size of graph G and $E(L(G)) = \{v_iv_j; e_j\}$ and e_j are adjacent in G v_i is corresponding to edge e_i. v_j is corresponding to edge e_j.

Planar Graph

If we have a graph which can be embedded in plane such that no two edges cross each other i.e. crossing number in zero, then we will have a planar graph.

1.7 Graph Operations

1.7.1 Union of Graph

If a graph G has two component say G_i and G_j, the union of these two graphs is represented by $G = G_i \cup G_j$, is placing of graphs G_i and G_j such that both remain independent components of graph G.

1.7.2 Sum of Graph

If a graph G has two component say G_i and G_j, the sum of these two graphs is represented by $G = G_i + G_j$, is placing of graphs G_i and G_j such that every vertex of G_i is adjacent with every vertex of G_j and both graphs preserves their properties.

1.7.3 Corona of Graph

The corona of two graphs is represented by $G_1 \odot G_2$, by taking one copy of G_1-graph and take V_1-time copies of G_2, then join $i-th$ vertex with $i-th$ copy of G_2.

1.7.4 Isomorphic Graphs

The isomorphism between two graphs G_1 and G_2 are isomorphic is a bijection between vertices and edges of both graphs. Isomorphic graphs have same order, size, degree sequences, number of cycles and paths etc. Its notion is $G_1 \cong G_2$.

1.8 Alpha Families of Tree

let $G \cong T_1 \cup T_2 \cup \ldots \cup T_k$ be a forest of $k \geq 2$ disjoint trees. An alpha tree denoted by $T_\alpha[T_1, T_2, \ldots T_k]$ is obtained by joining a single vertex say a, with a leaf of every tree $T_i : 1 \leq i \leq k$.The most famous families of Alpha trees are: Banana Trees, W-trees, Caterpillars, and Subdivided Stars etc.

1.9 Caterpillar

Let we have a tree and if we delete all the leaves of that tree and after this operation we get a path, then this tree is called caterpillar, $CT(n_1, n_2, \ldots n_k)$ or $CT(n,k)$.

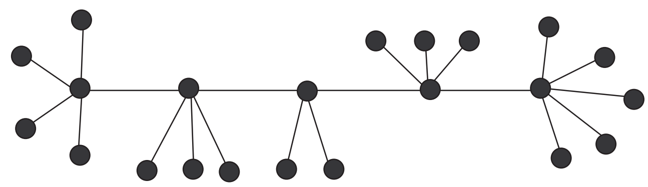


Figure 1.2: A caterpillar.

1.10 Star Graph

For $n \geq 3$, a star graph S_n, has a center and this center vertex is adjacent with all other $n-1$ vertices.

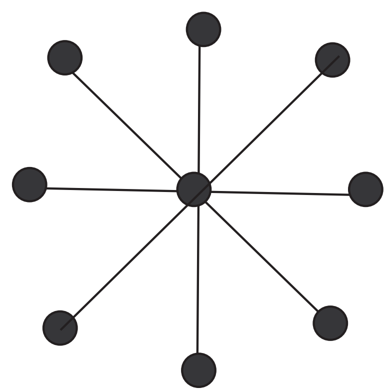


Figure 1.3: Star graph S_8.

1.11 Banana Tree

Let $S_p \cong K_{1,p}$ be a star graph and $G \cong nK_{1,p}$ where $p \geq 2$ and $n \geq 3$. A banana tree $BT(p,n)$ is obtained by joining a leaf of every star $K_{1,p}$ in G with a single vertex say a.

1.12 W-Tree

Let we have m copies of w-graphs $w(n)$. A w-tree $WT(n,m)$ is obtained by joining a vertex say α with a leave from every w-graph.from graph G, W-tree is denoted by $WT(p,m)$, where $p \geq 2$.

1.13 Cycle

A close walk in which number of vertices is equal to the edges and no edge and vertex repetition is not allowed then such walk is called cycle.

1.14 Complement of Graph

Let G be a graph,contains vertices and edges then complement of G has same number of vertices and having edges between those vertices which are in G. Let be G a graph, if the union of two or more graph not connected then its complement is connected.

Chapter 2

Basic Concepts

A total labelings of graph G is a bijective mapping from the fundamentals of a graph to the set of positive integers staring from 1 up to the sum of order and size of that graph. The vertices and edges are the fundamentals of graph. In a planar graph we also consider the face as the element of graph. An order and size of graph is the cardinality of vertex and edge set respectively. If we apply the restrictions on the range of this map then we have new labelings. Furthermore, after assigning the numbers to the graph elements of G, if all the weights of vertices, or edges or subgraph H of graph G is a constant number then we will say graph G is a vertex or edge or H magic. There are many graphs in the literature which preserve such properties. However, there are many unsolved conjectures and questions in the modern studies on such topics. For a detail study of this subject and underlying problems in this field, readers can consult a complete survey by Gallian [20]

2.0.1 Edge magic Total Labeling

Edge magic Total Labeling of a graph $G(V,E)$ was given by Kotzig and Rosa as a valuation say μ which is assigning of numbers to vertices and edges such that constant say k is being for any edge uv of graph G ,$\mu(u)+\mu(v)+\mu(uv)$. This edge magic labeling was named by Ringel and Llado in [50] in 1996. This labeling as edge magic total labeling was renamed by Wallis [49]. A graph is called Edge magic Total if it possesses an edge magic total labeling.

Kotzig and Rosa represented that, subsequent graphs have Edge magic Total Labeling of completeb partite graphs;$K_{a,b}$ C_s for all $s \geq 3$, tree of d paths of P_2 iff d is odd. Proof is given by Kotzig and Rosa ,an edge magic total labeling $iff = 1,2,3,4,5,or6$ (see [35],[13],and [16]) is admitted by K_n. Wallis et al. [49] investigated the path, crown and complete bipartite graph are edge magic.

Enomoto et al [15] gave proof of all complete bipartite graph have edge magic total. They also represented that, Edge magic Total Labeling does not exist in wheel graph W_n for different values of n and for some values of n it exist.

Slamin et al [28] gave proof that edge magic total labeling exist in all fan graph. That trees are edge magic total was conjectured by Ringel et al [50]. Phillips et al PRW and slamin [28] for $n \equiv 0,1(\text{mod}4)$ and $n \equiv 6(\text{mod}8)$ respectively gave the proof of this conjectured .

2.0.2 Super Edge Magic Total Labeling

A valuation say μ from $V \cup E$ to $\{1,2,3,...|V \cup E|\}$ is called super edge magic-total labeling if minimum numbers are assign to vertices and weight of all edges is a constant say k is being for any edge uv of graph G ,$\mu(u)+\mu(v)+\mu(uv)$. is called super edge magic total labeling if the weight of every edge under labeling λ is a constant number, where the weight of any edge is a sum of labels of end vertices and label of edge. There are many graphs which are supeer-edgei-magic-total. Here are some examples; Figueroa-Centroea et al. [16] showed that disjoint union of cycle and cycle with paths have super edge magic total labeling for different orders. Swaminathan et al. [63] and Baskoro et al. in [56] showed that banana trees and their forests preserve Super Edge Magic-Total behavior. Super Edge magic total labeling of banana trees were given by Hussain et al [31]. The super edge magic total labeling for different families of alpha families of trees are: subdivision of banana tree and forest of banana trees [1], w-tree, extended w-tree and disjoint union of extended w-tree, $K_{1,4}$ and w-trees [29], [36].

2.0.3 Magic Labeling of Types(a,b,c)

In 1983, Lih [42] introduced the concept of a magic type method to label to the vertices, edges and faces of planar graphs. A magic labeling of type (a,b,c) of a planar graph G(V,E) is assigning of numbers from 1 to $v+e+f$ such that weight of every s-sided face is a constant numbers. There are many results on such kind of labelins. For instance: fans and ladders , grids [4] are magic graphs, Möbius ladders [5] and

$P_n \times P_3$ are also have (a,b,c) labelings. Convex polytopes, and some planar graphs of different faces [37] have magic labelings of type (1,1,1).

2.0.4 H-Magic Labeling

Gutierrez and Llado [26] gave the idea of H-magic labeling which was basically the general form of edge magic labeling. Let us consider a subgraph H_o of a graph $G_o(V_o, E_o)$ with the condition that each edge of G_o would be a member of at least one subgraph which is isomorphic to H. If every H_i is congruent to given graph H, then it is called H-covering. A bijective mapping $\gamma : V_o \cup E_o \to \{1,2,3,\ldots,v_o+e_o\}$ is an H-magic labeling of G_o if the weight of every subgraph H of graph G_o is a constant number.

Gutierrrez and Llado [26] were the persons who studied the star-magicness of complete graph K_n and complete bipartite graph K_m,n. The list of results is given ahead: a d_0-regular graph is not $K_{1,h}$ for any $1 < h < d_0; k_{n,n}$, P_n is P_h-super edge magic; for any $2 < h \leq n, K_n$ is not P_h supermagic; for any $2 \leq h < n$, C_n is P_h-magic. Llado and Moragas [43] analysed cycle-magic labelings of different families of graph. They proved that the wheels W_n are C_n are supermagic. They proved that the wheels W_n are C_3 are supermagic for odd $n \geq 5$. The windmill graphs C_r^k are C_r-supermagic for $r \geq 3$ and $k \geq 2$; and $G \times K_2$ graph is C_4-supermagic if G is C_4-free super edge magic graph of odd size. Moreover, for odd value of n,$C_n \times K_2$ which are prisms and $K_{1,n}$ which are books, both are C_4-magic.
Maryati et al [40] showed P_h-supermagic labelings of subdivisions of shurbs, star and banana trees. H,Maryati et al. [41] gave the idea that the disjoint union of k isomorphic copies of a connected graph H is H-supermagic graph, for any graph H, if and only if $|V(H)|+|E(H)|$ is even or k is odd. Maryati discussed those graphs whose disjoint union possess supermaginess. Naurah et al. found the C_4-(super)magic labelings for ladder graph, book graph and grids graph $P_m \times P_n$. They also proved that graph like fan graph, trianular ladder graph and the graph $K_{1,n}+K_1$ admits C_3-supermagic labelings.

Jeyanthi et al. proved that $P_m \times P_n$ are C_4-supermagic for $n > 4$. They also found the C_3-supermagic labeling of antiprism graph.

Roswitha et al. gave another result about Jahangir graph, that the generalized Jahangir graph $J_{k,s}$ is C_{s+2}-supermagic; $K_{2,n}$ admits C_4-supermagic; and W_n for even $n \geq 4$ possess C_3-supermagic. They also found that special families of tree graphs have star-magic valuaitons.

It has been demonstrated by Selvagopal and Jeyanthi that snake graphs prism graph and grid graphs are cycle-magic graphs. Jeyanthi et al. showed the result on magicness of families of fan graphs and generalized antiprisms.

For more details, one can see survey on graph labeling written by Gallian [20].

Chapter 3

Super Edge-Magic Deficiencies of Forests of Alpha Families of Trees

Let G be a graph which is admitting an edge-magic total valuation. The edge magic deficiency [38] of this graph G is the positive numbers α required to make it a edge magic total graph. The general notation for edge magic deficiency is $\mu(G)$. If there does not exist such α then edge magic deficiency will be ∞. The super edge magic deficiency of graph G [17], follows the same definition and it is denoted by $\mu_s(G)$.

This dissertation studies about the super edge-magic deficiencies of forests of alpha families of trees. Firstly, We show that alpha trees with caterpillars as its components have super edge-magic deficiency is $\mu_s = 0$. We also prove that if the alpha tree has w-trees and alpha trees as its components then its super edge-magic deficiency is $\mu_s = 0$. At the end, we prove that forest of alpha trees with components as banana tree and stars preserve super edge-magic deficiency $\mu_s = 1$.

3.1 K_2-Supermagic Labeling of Star Like Graphs

Theorem 3.1.1. *The graph $G \cong T_\alpha[CT(p,m), CT(p,m), CT(p,m)]$ admits a super edge-magic deficiency $\mu_s = 0$ for odd $m \geq 2$ and odd $p \geq 3$.*

Proof. Now, we define the labeling for vertices of graph G as follows:
For odd g_0,

$$\rho(\alpha_h^g) = h_0 + \frac{g_0-1}{2}(z_0+1)$$

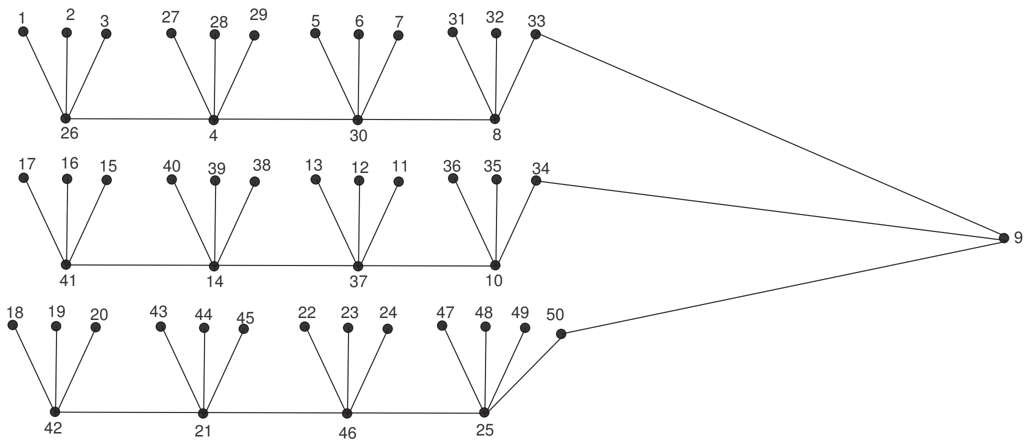
$$\rho(\beta_h^g) = (z_0+1)x_0 + (y_0-1) - h_0 - \frac{g_0-1}{2}(z_0+1)$$

$$\rho(\gamma_h^g) = (z_0+1)x_0 + (y_0-1) + h_0 + 1 + \frac{g_0-1}{2}(z_0+1)$$

For even g_0,

$$\rho(\alpha_h^g) = \frac{\alpha}{2} + h_0 + 1 + \frac{g_0-2}{2}(z_0+1)$$

$$\rho(\beta_h^g) = \frac{\alpha}{2} + (z_0+1)x_0 - h_0 - \frac{g_0-2}{2}(z_0+1)$$



$$\rho(\gamma_h^g) = \frac{\alpha}{2} + (z+1)x + h + 1 + \frac{g-2}{2}(z+1)$$

For odd g_0,

$$\rho(\alpha_0^g) = \frac{\alpha}{2} + 1 + \frac{g_0-1}{2}(z_0+1)$$

$$\rho(\beta_0^g) = \frac{\alpha}{2} + (z_0+1)x - \frac{g_0-1}{2}(z_0+1)$$

$$\rho(\gamma_o^g) = \frac{\alpha}{2} + (z_0+1)x + 1 + \frac{g_0-1}{2}(z_0+1)$$

for even g_0,

$$\rho(\alpha_0^g) = \frac{z_0+1}{2} g_0$$

$$\rho(\beta_0^g) = (z_0+1)x_0 - (z_0-1) - \frac{g_0-2}{2}(z_0+1)$$

$$\rho(\gamma_0^g) = (z+1)(x+1) + 1 + \frac{g-2}{2}(z+1)$$

$$\rho(l) = \frac{\alpha}{2} - (z_0+1)x$$

The set of edge weights of all edges form Arithmatic sequence starting from.

Theorem 3.1.2. *The graph $G \cong T_\alpha[CT(p,m),CT(p,m),CT(p,m)]$ admits a super edge-magic deficiency $\mu_s = 0$ for even $m \geq 2$ and even $p \geq 2$.*

Proof. Now, we define the labeling for vertices of graph G as follows:

For even j_0,

$$\rho(\alpha_h^g) = h_0 + 1 + \frac{g_0 - 2}{2}(z_0 + 1)$$

$$\rho(\beta_h^g) = \alpha - 2x_0(z_0+1) - (h_0+1) - \frac{g_0-2}{2}(z_0+1)$$

$$\rho(\gamma_h^g) = \alpha - 2x_0(z_0+1) + h_0 + \frac{g_0-2}{2}(z_0+1)$$

For odd g_0

$$\rho(\beta_h^g) = \frac{\alpha}{2} - z_0 + h_0 + 1 + \frac{g_0-1}{2}(z_0+1)$$

$$\rho(\beta_h^g) = \frac{\alpha}{2} + x_0(z_0+1) - h_0 + 1 - \frac{g_0-1}{2}(z_0+1)$$

$$\rho(\gamma_h^g) = \frac{\alpha}{2} + x_0(z_0+1) + h_0 + \frac{g_0-1}{2}(z_0+1)$$

For Even g_0,

$$\rho(\alpha_0^g) = \frac{\alpha}{2} + y_0 - 1 + \frac{g_0-2}{2}(z_0+1)$$

$$\rho(\beta_0^g) = \frac{\alpha}{2} + z_0(z_0-1) + x_0 - \frac{g_0-2}{2}(z_0+1)$$

$$\rho(\gamma_0^g) = \frac{\alpha}{2} + (z_0+1)(x_0+1) + \frac{g_0-2}{2}(z_0+1)$$

For odd g_0,

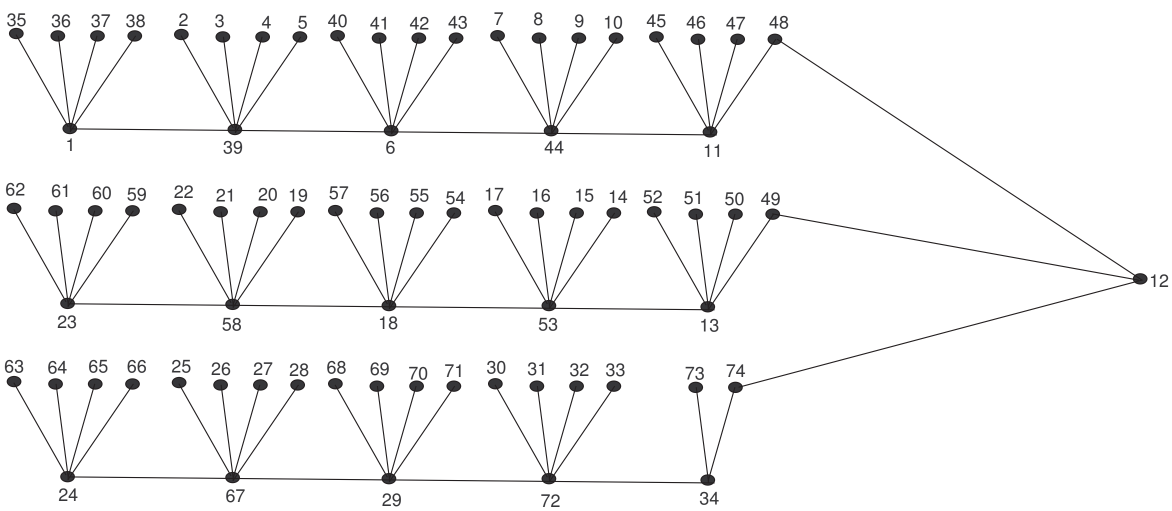
$$\rho(\alpha_0^g) = 1 + \frac{g_0-1}{2}(z_0+1)$$

$$\rho(\beta_0^g) = \alpha - 2x_0(z_0+1) - 1 - \frac{g_0-1}{2}(z_0+1)$$

$$\rho(\gamma_0^g) = \alpha - 2x_0(z_0+1) + \frac{g_0-1}{2}(z_0+1)$$

$$\rho(l) = \frac{\alpha}{2} - (z_0+1)x_0$$

The set of edge weights of all edges form Arithmatic sequence starting from $\frac{v}{2} - p + 3, \frac{v}{2} - p + 4, \ldots$ and magic constant is



Theorem 3.1.3. *The graph $G \cong T_{\alpha}[WT(p_1,m), CT(p_2,m), CT(p_3,m)]$ admits a super edge-magic total labeling for even $m \geq 2$, and p_1, p_2, $p_3 \geq 3$.*

Proof.

Now, we define the labeling for vertices of graph G as follows:

For odd g_0,

$$\rho(\alpha_{h_0}^{g}) = h_0$$

$$\rho(l) = r_0 + z_0 + (x_0 + 1)$$

$$\rho(\alpha_0^{g}) = r_0 + (z_0 + 1) + (g_0 - 1)$$

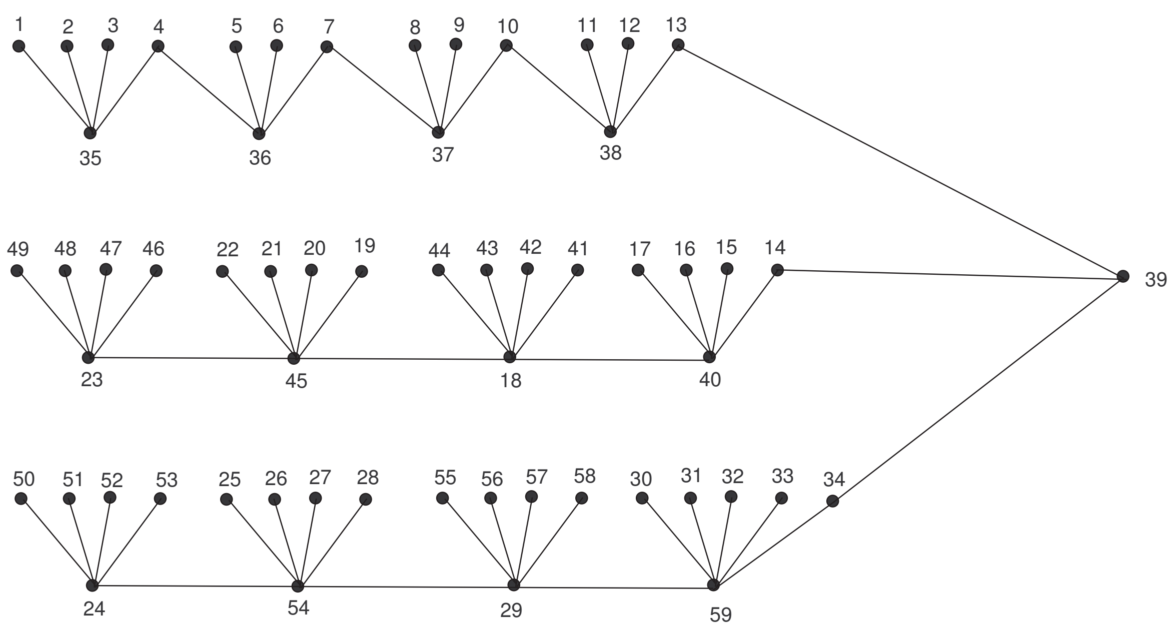
$$\rho(\gamma_o^{j}) = \begin{cases} \alpha - \frac{(z_0+1)(x_0+1)}{2} + z_0 - \frac{g_0-1}{2}(z_0+1), \ if \ \ g_0 - odd \\ \\ r_0 - \frac{(z_0+1)(x_0+1)}{2} + z_0 - 1 - \frac{g_0-2}{2}(p_0+1) \ if \ \ g_0 - even \end{cases}$$

$$\rho(\gamma_h^g) = \begin{cases} \alpha - \frac{(x_0+1)(x_0+1)}{2} + z_0 - h_0 - \frac{g_0-2}{2}(z_0+1), \; if \;\; g_0 - odd \\ \\ r_0 - \frac{(z_0+1)(x_0+1)}{2} + 2z_0 - h_0 - \frac{g_0-1}{2}(z_0+1) \; if \;\; g_0 - even \end{cases}$$

$$\rho(\beta_0^g) = \begin{cases} \alpha - \frac{(z_0+1)(x_0+1)}{2} + z_0 + 1 - \frac{g_0-1}{2}(z_0+1), \; if \;\; g_0 - odd \\ \\ r_0 - \frac{(z_0+1)(x_0+1)}{2} + 3z_0 + \frac{g_0-2}{2}(z_0+1) \; if \;\; g_0 - even \end{cases}$$

$$\rho(\beta_h^g) = \begin{cases} \alpha - \frac{(z_0+1)(x_0+1)}{2} + z_0 + h_0 + 1 + \frac{g_0-2}{2}(z_0+1), \; if \;\; g_0 - odd \\ \\ r_0 - \frac{(z_0+1)(x_0+1)}{2} + 2z_0 + (h_0-1) + \frac{g_0-1}{2}(z_0+1) \; if \;\; g_0 - even \end{cases}$$

The set of edge weights of all edges form Arithmatic sequence starting from $1 + \alpha - \frac{(z_0+1)(x_0+1)}{2} + z_0, 2 + \alpha - \frac{(z_0+1)(x+1)}{2} + z_0, \ldots$ and magic constant is $v+e$.



Theorem 3.1.4. *The graph $G \cong T_\alpha[WT(p_1,m),CT(p_2,m),CT(p_3,m)]$ admits a super edge-magic total labeling for odd $m \geq 2$, and p_1, p_2, $p_3 \geq 3$.*

Proof.

Now, we define the labeling for vertices of graph G as follows:

For odd g_0,

$$\rho(\alpha_h) = h_0$$

$$\rho(l) = r + x + 2$$

$$\rho(\alpha_o^g) = r_0 + 2 + (g_0 - 1)$$

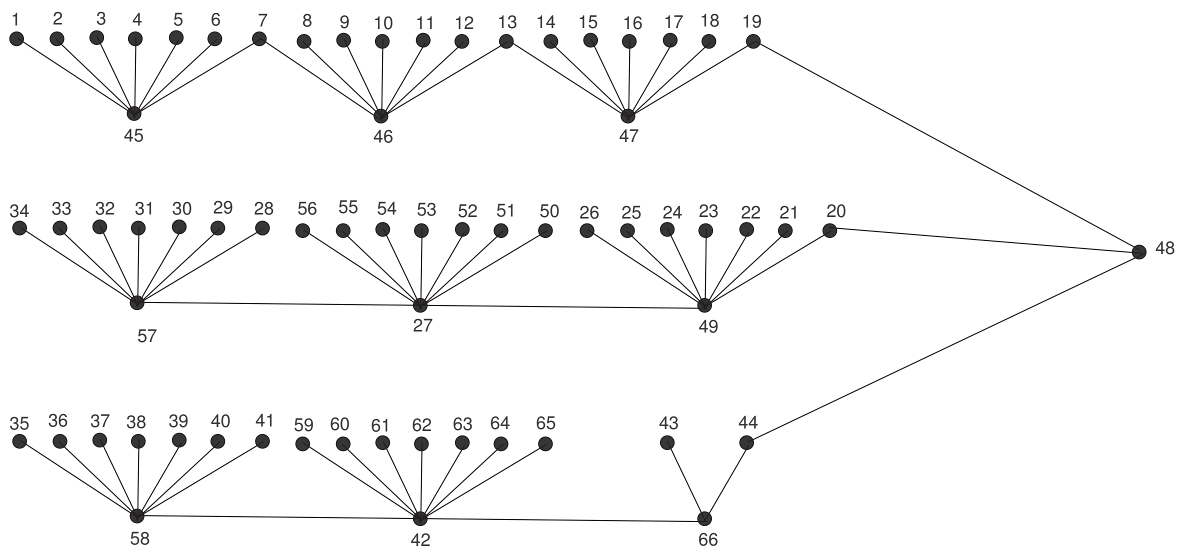
$$\rho(\gamma_h^g) = \begin{cases} \alpha - \frac{x_0}{2}(z_0+1) - (h_0-1) - \frac{g_0-1}{2}(z_0+1), \ if \ \ g_0 - odd \\ \\ r - \frac{x)}{2}(z+1) - h - \frac{g-2}{2}(z+1) \ if \ \ g - even \end{cases}$$

$$\rho(\beta_h^g) = \begin{cases} \alpha - \frac{x_0}{2}(z_0+1) + h_0 + \frac{g_0-1}{2}(z_0+1), \ if \ \ g_0 - odd \\ \\ r - \frac{x_0)}{2}(z_0+1) + (h_0+1) + \frac{g_0-2}{2}(z_0+1) \ if \ \ g_0 - even \end{cases}$$

$$\rho(\gamma_o^g) = \begin{cases} \alpha - \frac{x_0}{2}(z_0+1) + (z_0+1) + \frac{g_0-2}{2}(z_0+1), \ if \ \ g_0 - odd \\ \\ r_0 + 1 - \frac{x_0)}{2}(z_0+1) + \frac{g_0-1}{2}(z_0+1) \ if \ \ g_0 - even \end{cases}$$

$$\rho(\beta_o^j) = \begin{cases} \alpha - \frac{x_0}{2}(z_0+1) + (z_0+1) - \frac{g_0-2}{2}(z_0+1), \; if \;\; g_0 - odd \\ \\ r_0 - \frac{x_0)}{2}(z_0+1) - \frac{g_0-1}{2}(z_0+1) \; if \;\; g_0 - even \end{cases}$$

The set of edge weights of all edges form Arithmatic sequence starting from



Theorem 3.1.5. *The graph* $G \cong T_\alpha[CT(n,l), CT(n,l), CT(n,n,\ldots,n,n+1),$ $CT(n,n,\ldots,n,(\frac{l}{2}+1)(n+1)),\ldots,CT(n,n,\ldots,(\frac{l(j-3)}{2}+1)(n+1))]$ *admits a super edge-magic total labeling for odd* $n \geq 2$*,* $k \geq 4$ *and* $p \geq 3$*.*

Proof.

Here j= Number of Caterpillar, M_0=Length of caterpillar and $M_0 = \frac{l_0+1}{2}$, $A_0 = [1_0 + \frac{l+1}{2} - 1]k_0$

Theorem For first row

$$f(v_{2v_0}) = 1 + (m_0-1)(n_0+1)$$

For 2nd Row

$$f(v_{2m-1}) = [1+(M_0-1)(n_0+1)]\,j_0+1-(n_0+1)(m_0-1)$$

For each row except 2nd row

$$(v_{2m-1}) = (1+(n_0+1)(M_0+1))\,j_0+(1+(n_0+1)(m_0-1))+1$$

for first row For a odd length of catterpillar and its even vertices

$$f(v_{2m}) = A_0+m_0(n_0+1)$$

For 2nd Row

$$f(v_{2m}) = A_0+2M_0(n_0+1)+n_0-(n_0+1)(m_0-1)$$

for 3rd row

$$f(v_{2m}) = A_0+2M_0(n+1)+2n_0+m_0(n_0+1)$$

For 4th row

$$f(v_{2m} =)A_0+3M_0(n_0+1)+3n_0+m_0(n_0+1)$$

For 5th and own ward row

$$f(v_{2m}) = A_0+(j_0-1)M_0(n_0+1)+(j_0-1)n_0+m_0(n_0+1)+\lfloor\frac{(j_0-41)(j_0-3)}{2}[1+M_0(n_0+1)]\rfloor$$

Theorem 3.1.6. *The graph* $G \cong T_\alpha[CT(n,l),CT(n,l),CT(n,n,\ldots,n,n+1),$ $CT(n,n,\ldots,n,(\frac{l}{2}+1)(n+1)),\ldots,CT(n,n,\ldots,(\frac{l(j-3)}{2}+1)(n+1))]$ *admits a super edge-magic total labeling for even* $n \geq 2$, $k \geq 4$ *and* $p \geq 3$.

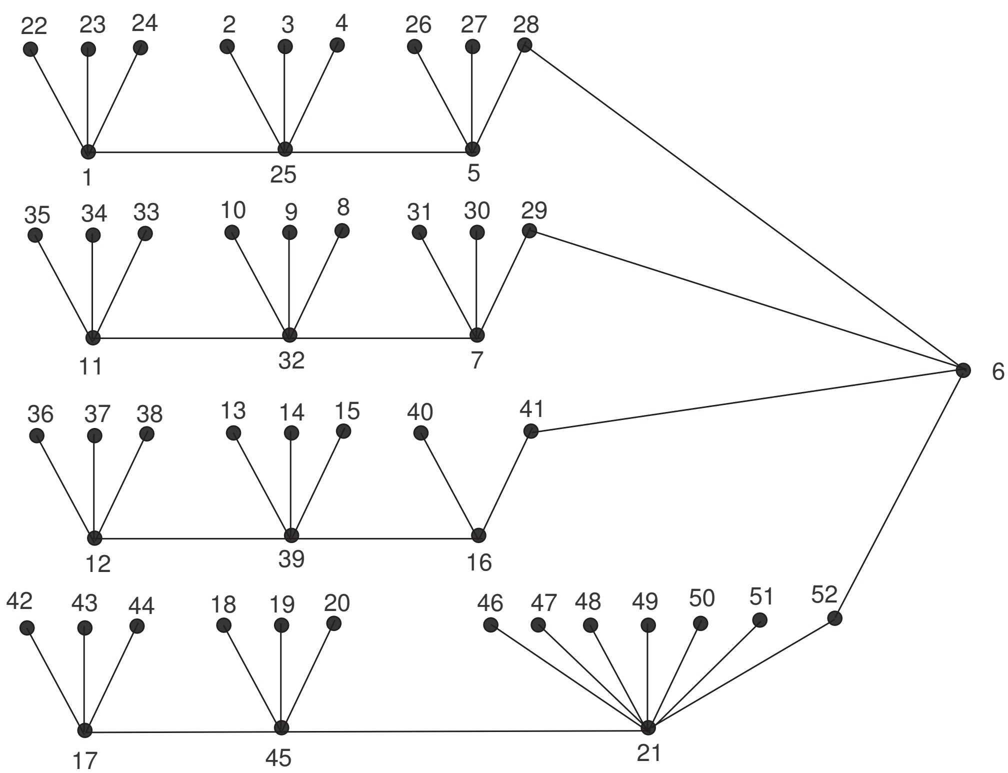


Figure 3.1: $Ewt(5,4,3:3,3)$

Theorem For Even For first Row

$$f(V_{2m}) = m_0(m_0+1)$$

For 2nd Row

$$f(v_{2m}) = [(n_0+1)m_0]j_0 - 1 - (m_0-1)(n_0+1)$$

for 3rd and own ward row

$$f(v_{2m}) = [(n_0+1)M_0]j_0 + 1 + m_0(n_0+1)$$

For first row

$$f(V_{2m-1}) = A_0 + 1 + (m_0-1)(n_0+1)$$

for 2nd row

$$f(v_{2m-1}) = A_0 + 2M_0(n_0+1) - (m_0-1)(n_0+1)$$

for 3rd row

$$f(v_{2m-1}) = A_0 + 2M_0(n_0+1) + 1 + (m_0-1)(n_0+1)$$

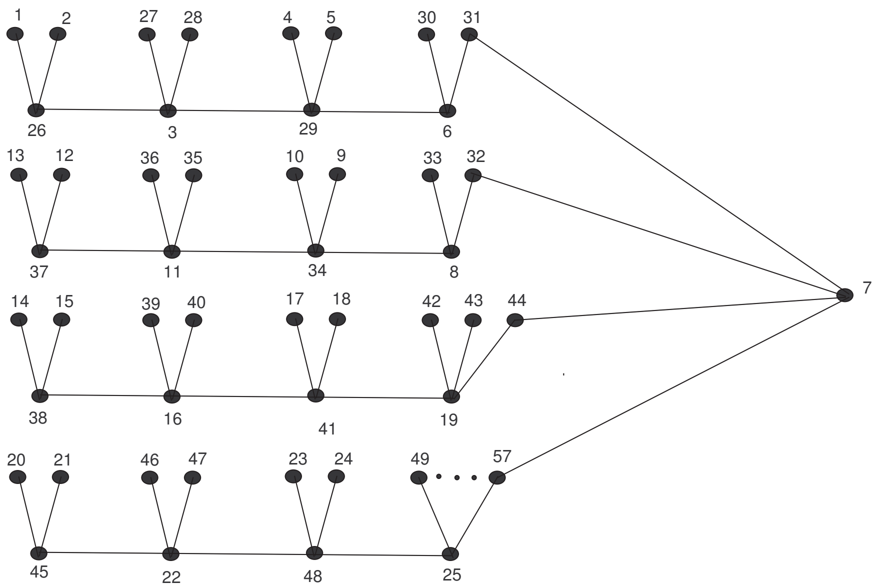
for 4th row

$$f(v_{2m-1}) = A_0 + 3M_0(n_0+1) + 1(m_0-1)(n_0+1) + 1$$

for 5th and own ward row

$$f(v_{2m-1}) = A_0 + (j_0-1)M_0(n_0+1) + 1 + (j_0-3) + (m_0-1)(n_0+1) + \lfloor \frac{M_0(j_0-4)(j_0-3)(n_0+1)}{2} \rfloor$$

The set of edge weights of all edges form Arithmatic sequence starting from $3+\frac{k}{2}(n+1)l, 4+\frac{k}{2}(n+1)l, \ldots,$ and magic constant is $v+3+\frac{k}{2}(n+1)l$



□

Our results from Theorem 4.2.1 to Theorem 4.3.2 have been accepted for publication in Utilitas Mathematica [53].