Dynamical Study of Dengue Model with Caputo Time Fractional Derivative

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CIIT/FA17-RMT-037/LHR

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Declaration

I Maryam Anwar hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC

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Certificate

It is certified that has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus and the work fulfills the requirement for award of MS degree

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DEDICATION TO

I would like to dedicate this work to my parents (late).

Without the efforts of my parents in law, I dont think

so that I could make at such a level where is now i am standing.

I also insist whom ever will study my research work do pray for my parents.

I also dedicate my work to my family and all my teachers

who was being with me through thick and thin.

IN THE NAME OF

ALLAH

THE MOST GRACIOUSAND COMPASSIONATE

THE MOST MERCIFUL AND BENEIFICIENT

WHOSE HELP AND GUIDANCE ALWAYS

SOLICIT AT EVERY STEP AND MOMENT.

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Praise to be ALLAH, the Cherisher and Lord
of the World, Most gracious and Most Merciful

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ABSTRACT

We discussed a viral disease that is named as Dengue which is caused by biting a female mosquito (Aedes). This disease is mild in the beginning and becomes severe with time. Dengue is a fever type epidermic disease spreading in over 100 countries nationwide. Now it is affecting about half of world population. Analysis is being done about its spread. A number of models are presented to control the spread of dengue. In the mid of 2005, dengue fever started spreading as the severe viral disease that transmit by means of a vector, periling 2.5 billion persons [14]. For the control of dengue, a number of mathematical models are being introduced [31]. In order to control female mosquito production, sterile male mosquito is being introduced instead of insecticides [32]. Apart from its control, a number of drugs have also been introduced for its treatment. Vaccines are also available for dengue.

Fractional calculus is the study of an extended form of integrals and derivatives in fractional orders. The most vital aspect of fractional derivatives is that, these models hold memory. In this thesis we have investigated two fractional order dengue models. First a differential equation of non integer order dengue hemorrhagic fever for the susceptible-infected-recuperated (SIR) model is analysed. We find the disease free model of equilibrium and endemic model of equilibrium of the specific model analytically. Further we compute the R_0 and show that R_0 is less than unity, then disease-free equilibrium will be stable and for $R_0 > 1$ the endemic equilibrium will be stable. Furthermore, fractional order dengue model will be solved by trapezoidal method and simulations are presented. Secondly we have considered a dengue model with temperature effect where the man population split in three sections (susceptible, infected humans and resistant people), whereas the population of the female mosquito can be categorized in three parts (susceptible, infected but not infectious and infectious) and pre-adult the population of female mosquito is distributed in two parts (infected mosquitos and the susceptible mosquitos). R_0 be the number of basic reproduction and is derived using the method of next generation. We solved

the fractional order dengue model by L1 method and simulations that presented to different values of fractional order. Simulation results refers that the most risk for the transmission of dengue, occurs at 28 °C temperature.

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Preliminaries and Introduction

1 Preliminaries and Introduction

The use of fractional calculus (FC) just developed over the most recent two decades, because of the advancement in the area of science and engineering that revealed remarkable associations with the ideas of FC. Fractional differential conditions have appealed numerous specialists because of their vital applications in science, material science and the study of disease transmission. The realistic modeling of an infectious disease model does not consider current situation, but rather additionally depend on the historical backdrop of the past time. Fractional derivative is defined as an integration which is nonlocal operator so fractional order model describe the dynamics of biological systems better than integer order models. The book authored by Kilbas et al. [23] presents the theory of fractional differential equations.

Dengue fever is caused by the virus transported by mosquito. This infection is happened in the humid and stifling area of the globe. The most widely recognized districts where dengue fever contain Southeast Asia and the islands of western Pacific. In Latin America and the lands of Caribbean, a rapid increase in the disease is observed recently. Dengue virus is the positive stranded RNA single virus from the family of Flaviviridae. There are five serotypes of the virus that can cause full spectrum of the disease. Each year, the amount of patients that are hospitalized is about half of a million, most of them recover in a few days but some take severe form; fever of hemorrhagic dengue. Hemorrhagic fever can cause intense bleeding, drop in blood pressure suddenly and ultimately goes to death. Some symptoms of dengue may include: Headache, Muscle contraction, bone and joint pain, Vomiting, Nausea, pain in the eyes muscles, swollen glands and Rashes. Dengue causes the platelets to reduce. Normal count of platelets is 1.5 lac to 4.5 lacs per micro liter of blood but in viral fever it reduces to 90,000 to 1 lac. But in dengue, it reduces to 20,000 or even lower. Note that, the dengue fever is caused by a virus, which transmits via bites of the female mosquitoes, mainly Aedes albopictus and Aedes aegypti ones. When these mosquitoes bite an infected human, a specific possibility they become infected, and after that they endure infected until the end of their life. There is also a certain probability for a susceptible human to become infected when an infected mosquito bits him/her. However, contrary to the mosquitoes, the infected humans recover from the infection after a certain time period. Also, the recovered individuals are unable to transmit the virus back to the mosquitoes, and hence, they keep their immunity against further infections.

A deeper understanding of mathematical model is essential and reliable of the mechanism of spreading the viral disease and how to control the transmission of disease. In keep the dengue infection under control some better number of mathematical (compartmental) models has been suggested and examined (see [12]-[19], [31]-[36]. Abdelrazec et al. [4] developed a regulatory model for the dynamic study of transmission of dengue fever by means of a nonlinear rate of recovery to analyze the spread and control of the disease. A mathematical model for dengue is constructed to determine the chain of two epidemic diseases with different human populations. The reproductive number R_0 as a inception of bring down of the epidemics is explained by means of the stability analysis. Their model shown the only environmental control because the vector control is not enough; it may interrupt the spread of epidemics. The consequence of a vaccine may control simultaneously against the some serotypes residues a expected in the future. The critical Study centered on the analytical view of the compartmental integer-order epidemic model (that contains a system of ODE's) central tool for the research of the model. However, fractional order systems are in broad-spectrum memory less [4],[31],[36].

Fractional calculus is the study of an extended form of integrals and derivatives in fractional orders. The most vital aspect of fractional derivatives is that, these models hold memory. Sardar et al. [32] investigated a compartmental dengue transmission model having the memory, the memory is incorporated in the model exhausting a rational differential operator. A threshold quantity R_0 is derived, having the similarity with elementary reproduction number and dewtermined that although the value of R_0 is less than one, the disease-free equilibrium would not be constantly stable, and the system reveals a Hopf-type bifurcation. The SIR model is totally depending upon the fractional order differential equation of the dengue fever and it is investigated in [20]. Our goal is to investigate the fractional order dynamical model of the dengue fever centered at the distribution of the human population into these catagories (susceptible, infected, and resistant humans), whereas the population of the female mosquitos is distributed in two parts (infected and susceptible) to understand the dynamic of dengue disease in more realistic way.

1.1 A brief history of fractional calculus

Leibniz turned into the first, who has generated the idea of fractional calculus in 1965. Fractional calculus as a major element is prominent among the mathematicians department for the final three centuries. As a quick period of previous few years, it has gone to peaked within the several carried out range of engineering and sciences, within the attention of purposeful order can give the extra rational analysis of real problem. Literature can provide a lot of definitions of fractional derivative.

1.2 Euler's Gamma function

In the study of exceptional functions a fundamental cornerstone is given by Euler's Gamma function. The reason in this lies in the way that this function can be experienced in almost all parts of the subject and moreover numerous extraordinary functions can be communicated in term of the Gamma function straight forwardly or by contour integration. Before we give a formal definition of Euler's Gamma function we need an extra definition, which will be utilized in the proofs for certain properties of the Gamma function.

Definition 1. For $z \in C \setminus 0,-1,-2,-3,...$ Eulers Gamma function $\Gamma(z)$ is defined as

$$\Gamma(\xi) = \begin{cases} \int_0^\infty t^{\xi-1}e^{-\tau}d\tau, & if, \quad Re(\xi > 0) \\ \Gamma(\xi+1/\xi, & if, \quad Re(\xi \leq 0), \xi \neq 0,-1,-2,-3,... \end{cases} \tag{1}$$

1.3 Cauchy formula

Goreno and Manardi defined the Cauchy formula by

$$J^n f(t) = \frac{1}{(n-1)!}\int_0^t (t-\tau)^{n-1}f(\tau)d\tau, t>0, n \in \mathbb{N}. \tag{2}$$

where $\mathbb{N}$ be the set of natural numbers. Extending from the natural numbers, the index for any positive real valued outcomes in fractional order Riemann-Liouville Integral, if $q > 0$.

1.4 Riemann-Liouville integral

1.4.1 Definition

Let $f : [0,T] \to R$ and $q \in R_+$. The operator I^q defined on $L_1[0,T]$ by,

$$I^q f(\xi) = \frac{1}{\Gamma(q)} \int_0^{\xi} (\xi - \tau)^{q-1} f(\tau) d\tau, \tag{3}$$

for $0 \leq x \leq T$, is called the fractional integral operator of Riemann-Liouville with order q. For $q = 0$, take I^0:=I, be the trivial operator.

1.4.2 Example:

Let $f(x) = x^c$, for some $c > -1$ and $q > 0$. Then,

$$I^q f(x) \quad = \quad \frac{\Gamma(c+1)}{\Gamma(q+c+1)} x^{q+c}.$$

1.5 Riemann-Liouville fractional derivative

1.5.1 Definition

Let $f : [0,T] \to R$, $q \in R_+$ and $n = [q]$. The operator D^q, defined by

$$D^q f(x) = D^n I^{n-q} f(x) = \frac{1}{\Gamma(n-q)} (\frac{d}{dx})^n \int_0^x (x-t)^{n-q-1} f(t) dt. \tag{4}$$

For $0 \leq x \leq T$, is called the Riemann-Liouville differential operator of order q. for $q = 0$, we set $D^0 := I$, the identity operator.

1.5.2 Example

Let $f(x) = \exp(\lambda a)$, for some $\lambda > 0$, and let $q > 0$, q does not belong to N. Then

$$D^q f(x) = \frac{\exp(\lambda a)}{\Gamma(1-q)} ((x)^{-q})_1 F_1(1; 1-q; \lambda(x)),$$

where ${}_1F_1$ used for the hypergeometric function of Kummer's Confluent.

1.6 Caputo fractional derivative

In this research, the main reason of using the Caputo derivative is the dominance in applied problems. There is similarity in each, the physical position for fractional order equation with Caputo fractional derivative and also the whole number of system, avoidance of solvable issues, and this is often called the main advantage of Caputos approach. The Caputo fractional derivative is outline as follows:

$$\begin{aligned} D_c^q f(\xi) &= I^{N-q}[f^{(N)}(\xi)] \\ &= \frac{1}{\Gamma(N-q)}\int_0^s (s-y)^{N-q-1} f^{(N)}(y)dy, \end{aligned} \tag{5}$$

where $N = [q]$.

For the Caputo fractional derivative, the Laplace transform is given by;

$$\mathscr{L}[D_c^q f(s)] = \lambda^q F(t) - \sum_{k=0}^{N-1} f^{(k)}(0)\lambda^{q-k-1}. \tag{6}$$

The Mittag-Leffler function is given by the power series:

$$E_{q,\beta^{(Z)}} = \sum_{k=0}^{\infty} \frac{Z^k}{\Gamma(qk+\beta)} \tag{7}$$

The Laplace transform of the function is:

$$\mathscr{L}[t^{\beta-1}E_{q,\beta}(\pm q t^q)] = \frac{S^{q-\beta}}{S^q \mp q} \tag{8}$$

Let $q, \beta > 0$ and Z $\in$ C, and the Mittag-Leffler functions satisfy the equality given by Theorem 5.2 in [15]

$$E_{q,\beta^{(Z)}} = ZE_{q,q+\beta^{(Z)}} + \frac{1}{\Gamma(\beta)}. \tag{9}$$

1.7 Numerical method for fractional differential equations

In this, we build a numerical plan to estimated the arrangement of the fractional differential condition.

$$\begin{cases} D_c^q y(t) = f(y(t)),\ 0 < t < T < \infty, \\ y(0) = y_0, \end{cases} \tag{10}$$

To do so, first we transform the FDE (10) into an equivalent integral equation and f is a real–valued continuous vector function, which satisfies

$$\|f(y_1(t)) - f(y_2(t))\| \leq L\|y_1(t) - y_2(t)\|, L > 0. \tag{11}$$

Applying the integral operator, we derive

$$y(t) = y_0 + I^q f(y(t)), 0 < t < T < \infty, \tag{12}$$

where I^q denotes the integral operator. To propose the numerical scheme, we choose an equidistant grid with the time step size $\tau = \frac{T}{N}$ on $[0,T]$; that is $0 = t_0 < t_1 < ... < t_N$ and $t_{n+1} - t_n = \tau$, $n = 0,1,2,...,N-1$. Also, we denote x_n as the approximate solution of $x(t_n)$, where $t_n = n\tau$, $n = 0,1,2,...,N$.

1.7.1 The L1 method

According to the definition of integral operator, we consider the following integral

$$I^q f(x(t_{n+1})) = \frac{1}{\Gamma(q)} \int_0^{t_{n+1}} (t_{n+1} - \xi)^{q-1} f(x(\xi))d\xi, n = 0,1,\ldots,N-1. \tag{13}$$

The L1 method for the fractional differential equation (14) is given by:

$$x_{n+1} = x_0 + \sum_{k=0}^{n-1} b_{n-k-1}(f(x_{k+1}) - f(x_k), \tag{14}$$

where

$$b_k = \frac{\tau^{-q}}{\Gamma(2-q)}[(k+1)^{1-q} - k^{1-q}].$$

1.7.2 The trapezoidal method

For trapezoidal method, on each sub–interval $[t_k, t_{k+1}]$, $f(x(\xi))$ is approximated by the following piecewise linear interpolation polynomial

$$f(x(\xi))|_{[t_k,t_{k+1}]} \approx \frac{t_{k+1} - \xi}{t_{k+1} - t_k} f(x_k) + \frac{\xi - t_k}{t_{k+1} - t_k} f(x_{k+1}), 0 \leq k \leq n. \tag{15}$$

Inserting the above approximation into Eq. (13), we obtain

$$ {}^{C}I^{q}f(x(t_{n+1})) \approx \tau^{q}\sum_{k=0}^{n+1} a_{n+1,k}f(x_k),\, n=0,1,\ldots,N-1, \tag{16} $$

where $a_{n+1,k}$ is defined by

$$ \begin{cases} a_{n+1,0} = \frac{1}{\Gamma(q+2)}\left(n^{q+1}-(n-q)(n+1)^{q}\right), \\ a_{n+1,k} = \frac{1}{\Gamma(q+2)}\left((n-k+2)^{q+1}-2(n-k+1)^{q+1}+(n-k)^{q+1}\right),\, k=1,2,...,n, \\ a_{n+1,n+1} = \frac{1}{\Gamma(q+2)}. \end{cases} \tag{17} $$

Thus, we can provide the following fractional trapezoidal method for Eq.(12) in the Caputo sense

$$ x_{n+1} = x_0 + \tau^{q}\sum_{k=0}^{n+1} a_{n+1,k}f(x_k),\, n=0,1,\ldots,N-1. \tag{18} $$

1.8 The SIR model

The SIR model is the least complex fractionary model, and large numbers of models are originated from this fundamental structure. The model consists of three compartments S for the number susceptible, I for the number of infectious, and R for the number recovered (or immune). This model is sensibly prescient for irresistible infections that are transferred from man to man, and here recuperation gives enduring opposition, for example, dengue, measles and rubella.

1.9 Basic reproduction number (R_0)

Mathematical modelling is very helpful in planning disease control schemes by concentrating on the significant parts of the disease, to determine initial quantities for the survival of disease, assessing the impact of specific mechanism polices. A very significant initial quantity is the reproduction number, often named as basic reproductive number or basic reproductive ratio [21] that is generally represented by R_0. The epidemiological definition of R_0 is the number of other suspects originated from one of the infected individuals transmitted to a population of disposed individuals, and the infected person may acquire the disease, and disposed people may be healthy but they may get the disease.

Chapter 2

SIR Fractional order epidemic model of dengue fever

2 SIR Fractional order epidemic model of dengue fever

Susceptible-Infected-Recovered model (SIR) has been applied for the better interpretation of the infectious sickness and its dynamical behavior. We develop a vector-host dengue transmission SIR model of fractional order. In this chapter, we will analyze the fractional order dengue model proposed by Bailey [10] which include the aquatic population in addition to adult population of the female mosquitoes. The population which is getting used inside the proposed fractional model is based on three human epidemiological states:

$H_1(t)$ susceptible(individuals who can contract the virus)

$H_2(t)$ infected(individuals)

$H_3(t)$ recovered/resistant(individuals who have required immunity)

In the assumption by research that the total human population $N_h = H$ is constant

So $H = H_1 + H_2 + H_3$.

During the study of female mosquito model, one have to study only the infected and susceptible mosquito, recovery in mosquito is not included beacause of short time of life after infected.

$A_v(t)$ aquatic phase(which includes pupa,stages,larva and the egg)

$M_1(t)$ susceptible(the mosquitoes who are ready to virus contracting)

$M_2(t)$ infected(the mosquitoes who has virus transmit capability to human)

Here, $M = M_1 + M_2$, Similarly the formulation of model is based on following assumptions:

- Immigration of infected individuals is human population is not possible.
- Mounted the constant of transmission of the illness.
- Both mosquito and human are born susceptible.

- There is not memory possession in mosquito and human regarding death and birth.
- Vector-host dengue transmission and host-vector undergoes a common dynamic so between fractional order $0 < q < 1$ q for both is assumed to be the same.

On the basis of mentioned assumptions, the characterization of the animation of a single mosquito infected by non-integral differential equation is as [32]:

$$\begin{aligned}
D_c^q A_v =& P\xi(1 - A_v/c)M - (P + \eta)A_v \\
D_c^q M_1 =& A_v - \frac{b^q a_m}{H} M_1 H_2 - d_m M_1 \\
D_c^q M_2 =& \frac{b^q a_m}{H} M_1 H_2 - d_m M_2 \\
D_c^q H_1 =& d_n(H - H_1) - \frac{b^q a_h}{H} H_1 M_2 \\
D_c^q H_2 =& \frac{b^q a_n}{H} H_1 M_2 - (\gamma + d_n) H_2 \\
D_c^q H_3 =& \gamma H_2 - d_n H_3.
\end{aligned} \tag{19}$$

Parameter are considered from different studies. A couple of parameter has been used inside the model construction at some point of the whole studies process. The Caputo used inside the model is nice illustration of fractional derivative. Diverse assets has been used for the dedication of the model parameters. Due to the lack of valid records from information within the Ministry of health Malaysia (KKM) data for pre-adult mosquito is collected from [27] and data for transmission probability rate for human and the vector is taken [17]. The population of the tropical America and Southeast Asia is similar and both location are effected by all four dengue viruses. Therefore the reliability of the parameters estimation is acceptable. Some parameters associated with this study has also been taken Singapore. As weather in Malaysia and Singapore of each place is similar data available for any of the parameter is suitable.

Table 1: Dengue model's parameters values.

Parameters	Biological meaning	Range of values	References
P	proportion of eggs	0-1	[27]
ξ	Oviposition rate	0-11.2 per day	[27]
ρ	Transition rate from aquatic to adult	0-0.19 per day	[27]
η	Average aquatic morality rate	0.01-0.47 per day	[27]
$\frac{1}{d_m}$	Average lifespan of adult mosquito	11-56 days	[35]
$\frac{1}{d_n}$	Average lifespan of human	73-75 years	[24]
b	The biting rate	0-1 per day	[3]
a	Transmission probability from human to vector	0.375	[17]
d	Transmission probability from vector to human	0.375	[17]
γ	Recovery rate in the host population	0.328833 per day	[35]
c	Mosquito carrying capacity	–	–

2.1 Stability analysis for the generalized model

We characterized that, Ω be a closed and bounded set, that has positive impact and attacking for the system (19).

Theorem 2.1. *The closed set*

$$\Omega = \{(A_v, M_1, M_2, H_1, H_2, H_3) \in \mathbb{R}^5_+ : 0 \leq H_1 + H_2 + H_3 = K,$$
$$0 \leq M_1 + M_2 \leq V_1 \text{ and } V_1 \geq \frac{\rho A_v}{d_m}, 0 \leq A_v \leq V_2 \text{ and } V_2 \geq P\xi M\},$$

*is positively impact with respect to model (*19*).*

Proof. Adding the last three equations of and adding equation $D_c^q M_1 + D_c^q M_2$,we get two fractional order differential equations with respect to the total human and vector population as follows:

$$D_c^q H(t) = 0. \tag{20}$$

and

$$D_c^q M(t) = \rho A_v - d_m M. \tag{21}$$

Solution to Eq.(20) is H(t)=K, since the Caputo derivative of a constant is zero and H(0)=K. Now we solve Eq.(21) by using the Laplace transform method [17], we have the following solution:

$D_c^q M(t) = \rho A_v - d_m M.$

Applying the Laplace transform (2), we have

$$\lambda^q \mathscr{L}(M(t)) - \lambda^{q-1}(M(0)) = \rho A_v - d_m \mathscr{L}(M(t)), \tag{22}$$

that can be written as below using the Laplace transform,

$$\mathscr{L}(M(t))(\lambda^q + d_m) = \rho A_v + \lambda^{q-1} M(0) \lambda^q + d_m$$

$$\begin{aligned} \mathscr{L}(M(t)) =& \frac{\rho A_v}{\lambda^q + d_m} + \frac{\lambda^{q-1} M(0)}{\lambda^q + d_m} \\ \leq& \frac{\rho A_v}{d_m} + t^{q-1} E_{q,q}(-d_m t^q M(0) \\ \leq& V_1 + C_1 t^{q-1} E_{q,q}(-d_m t^q), \end{aligned} \tag{23}$$

where $V_1 \geq \frac{\rho A_v}{dm}$, C_1 is an arbitrary constant, $0 < q \leq 1$ and $E_{a,b(Z)}$ is the parameter Mittag-Leffler function with parameter a and b [15], [29], Since Mittag-Leffler function is an entire function, thus $E_{q,q}(-d_m t^q$ is bounded for all $t > 0$. Therefore,as $t \to \infty$ we have $M(t) \leq V_1$. For the fractional order differential equation of aquatic phase of the vector, we have:

$$D_c^q A_v(t) = pM\xi - \frac{pM\xi}{c} A_v - (\rho + n) A_v \tag{24}$$

$$\leq pM\xi - (\rho + n) A_v. \tag{25}$$

Applying Laplace transform, we have

$$\lambda^q \mathscr{L}(A_v) - \lambda^{q-1}(A_v(0)) \leq pM\xi - (\rho + n) \mathscr{L}(A_v(t)), \tag{26}$$

that can be written as below using the Laplace transform,

$$\mathscr{L}(A_v(t))(\lambda^q+(\rho+n)) \leq pM\xi+\lambda^{q-1}A_v(0)$$

$$\begin{aligned}
\mathscr{L}(A_v(t)) &\leq \frac{pM\xi}{\lambda^q+(\rho+n)}+\frac{\lambda^{q-1}A_v(0)}{\lambda^q+(\rho+n)} \\
&\leq \frac{pM\xi}{\rho+n}+t^{q-1}E_{q,q}(-(\rho+n)t^q)A_v(0) \\
&\leq V_2+C_2t^{q-1}E_{q,q}(-(\rho+n)t^q)
\end{aligned} \tag{27}$$

where $V_2 \geq \frac{p\xi M}{\rho+n}$ and C_2 are the arbitrary constant. Hence, it follows the same argument as in the vector model. Thus, closed set Ω is positively invariant and attracting to the system (19).

With the relation of $N_h = H = H_1+H_2+H_r$, we have $H_r = H-H_1-H_2$. Thus system (19) can be reduced as follows:

$$\begin{aligned}
D_c^q A_v =& p\xi(1-\frac{A_v}{C})M-(\rho+n)A_v \\
D_c^q M_1 =& \rho A_v-\frac{b^q a_m}{H}M_1H_2-d_mM_1 \\
D_c^q M_2 =& \frac{b^q a_m}{H}M_1H_2-d_mM_2 \\
D_c^q H_1 =& d_h(H-H_1)-\frac{b^q a_m}{H}M_1H_2 \\
D_c^q H_2 =& \frac{b^q a_m}{H}M_1H_2-(\gamma+d_h)H_2
\end{aligned} \tag{28}$$

For the system (28), the region $\Omega_1 = (A_v, M_1, M_2, H_1, H_2) \in \mathbb{R}_+^4 | 0 \leq (H_1+H_2) \leq K; 0 \leq (M_1s+M_2) \leq$ is positively invariant and attracting. .

5.1. Equilibrium points □

Theorem 2.2. *Consider R_m to be the basic offspring of the mosquito population*

$$R_m = \frac{p\xi\rho}{d_m(\rho+n)}$$

. The set of equations (28) may have maximum two disease-free points of equilibrium:

- *if* $R_m \leq 0$*,we have the disease-free equilibrium (DFE), knows as trivial equilibrium,* $E_0 = (0,0,0,H,0)$*;*

- *if* $R_m > 0$*,there is a biologically realistic disease-free equilibrium (BRDFE),* $E_2 = (\overline{A_v}, \overline{M_1}, 0, H, 0)$.

Proof. To evaluate the equilibrium points, we let $D_c^q A_v = 0$, $D_c^q M_1 = 0$, $D_c^q M_2 = 0$, $D_c^q H_1 = 0$, $D_c^q H_2 = 0$

By calculation, we obtain four equilibrium points for system (28). Two of them are known as disease-free equilibrium (DFE).

The first DFE is called trivial equilibrium, since $A_v = 0$, so the mosquitoes do not exist, hence no disease occurs:

$$E_0 = (0,0,0,H,0)$$

The second dfe is higher known as as the biologically sensible ailment-unfastened equilibrium (BRDFE), is the case whilst human and vector interact, however simplest one outbreak of the ailment. Hence, the ailment will die out over time without ant movement taken on the mosquito population. This equilibrium is greater practical than the preceding one as it is associated with the herbal phenomena

$$E_1 = (\overline{A_v}, \overline{M_1}, 0, H, 0).$$

where $\overline{A_v}$ and $\overline{M_1}$ are given by
$\overline{A_v} = c(1 - \frac{1}{R_m})$ and $\overline{M_1} = \frac{\rho \overline{A_v}}{d_m}$
Thus we obtain the following:

$$\begin{aligned} E_1 &= (\frac{c(p\xi\rho - nd_m - d_m\rho)}{p\xi\rho}, \frac{c(p\xi\rho - nd_m - d_m\rho)}{p\xi d_m}, 0, H, 0) \\ E_1 &= (c(1 - \frac{1}{R_m}), \frac{c\rho}{dm}(\frac{R_m - 1}{R_m}), 0, H, 0) \end{aligned}$$

where $R_m = \frac{p\xi\rho}{d_m(\rho+n}$. This is biologically interesting only ir $R_m > 1$ as this implies a positive population of mosquitoes. □

Theorem 2.3. *If $R_m > 0$ and $R_0 > 1$, the system (28) has an endemic equilibrium as following:*

$$E_2 = (Z_m^*, M_1^*, M_2^*, H_1^*, H_2^*).$$

Proof. The second one DFE is higher called as the biologically sensible disorder-free equilibrium (BRDFE), is the case at the same time as human and vector interact, however best one outbreak of the ailment. Consequently, the disease will die out over time without ant motion taken on the mosquito population. This equilibrium is greater practical than the preceding one as it is related to the herbal phenomena.

$$\begin{aligned}
E_2 &= (Z_m^*, M_1^*, M_2^*, H_1^*, H_2^*), \\
\textit{where} & \\
Z_m^* &= \frac{c(p\xi\rho - nd_m - d_m p)}{p\xi\rho}, \\
M_1^* &= \frac{(\gamma+d_h)[-d_m(nca_h + pca_h - d_h H\phi\xi b^{-q}) + cd_h\phi\xi p]}{\phi\xi a_h(d_h b^q a_m + d_m(\gamma+d_h))}, \\
M_2^* &= \frac{d_h H(R_o^2-1)}{b^q a_h}, \\
H_1^* &= \frac{H^2 d_m\phi\xi(d_h b^q a_m + d_m(\gamma+d_h)}{b^{2q}a_m[-d_m(nca_h + pca_h - d_h H\phi\xi b^{-q}) + cd_h\phi\xi p]},
\end{aligned}$$

$$H_2^* = \frac{d_h d_m H^2(R_o^2-1)[\phi\xi(d_m(\gamma+d_h) + d_h b^q a_m)]}{(\gamma+d_h)a_m b^q[-d_h H\phi\xi - ca_n d_m b^q(n+\rho) + cd_h d_m\phi\xi p b^q]}. \tag{29}$$

The fourth equilibrium will not be examined because of same factors which are negative, which suggest that it does reffer to the Ω set. □

2.2 The basic reproduction number

Usually, the basic reproduction number R_0 of the widespread disease is known as the predicted domestic conditions caused by using. In terms of vector-endured disease, it may be explained as the quantity of people which might be affected from a single human at the beginning of infection from a mosquito. Hypothetically, when $R_0 < 1$ the transmissions chains are not significant and are not able to impotent a huge epidemic. Although if $R_0 > 1$, each generation will be raised by the number of infected humans and disease will spread out in the population. Using the next generation matrix approach, we reach the given below R_0 of system (28) classifying at the BRDFE. In this situation R_0 is equal to the spectral radius of $K = FV^{-1}$ of BRDFE, where F is a non-negative matrix of the infection items, and V is the singular M-Matrix of the transmission terms.

$$F = \begin{pmatrix} 0 & \frac{b^q a_m}{H}\overline{M_1} \\ b^q a_h & 0 \end{pmatrix}$$

and

$$V^{-1} = \begin{pmatrix} \frac{-1}{d_m} & 0 \\ 0 & \frac{-1}{\gamma_i + d_h} \end{pmatrix}$$

therefore,

$$FV^{-1} = \begin{pmatrix} 0 & \frac{-b^q a_m \overline{M_1}}{H(\gamma_i + d_h)} \\ \frac{-b^q a_h}{d_m} & 0 \end{pmatrix}$$

The eigenvalues of FV^{-1} is: $\pm\sqrt{\frac{b^{2q} a_m a_h \overline{M_1}}{H(\gamma_i + d_h) d_m}}$
Therefore, the dominant eigenvalue of FV^{-1} is

$$R_0 = \sqrt{\frac{b^{2q} a_m a_h \overline{M_1}}{H(\gamma_i + d_h) d_m}} \tag{30}$$

Where $\overline{M_1} = \frac{\rho \overline{A_v}}{d_m}$ Note that R_0 computed for fractional system (19) is of dimension $[time]^{-q}]$, although this is not a dimensionless quantity as usually described for the basic reproduction number in the integer order setting. The threshold quantity R_0 described in [17] is a memory dependent threshold quantity as $R_0 \propto b^q$. Accordingly, as the value of q is obtaining smaller, the value of b^q is declining, hence, less effective the average bite rate per mosquito per month [21].

2.3 Stability analysis of the disease free equilibrium

Theorem 2.4. *The BRDFE, E_1 is locally asymptotically stable when all the eigenvalues, $\lambda_i, i = 1,2,3,4,5$ of the jacobin matrix $J(E_1)$ entertain the given condition as below: $|arg(\lambda_i)| > \frac{q\pi}{2}$.*

The jacobin matrix of the system can be calculated at the equilibrium point, E_1:

$$J(E_1) = \begin{pmatrix} -R_m(\rho+n) & 0 & 0 & 0 & 0 \\ \rho & -d_m & 0 & 0 & \frac{b^q a_m}{H}\overline{M_1} \\ 0 & 0 & -d_m & 0 & \frac{b^q a_m}{H}\overline{M_1} \\ 0 & 0 & -b^q a_h & -d_h & 0 \\ 0 & 0 & -b^q a_h & 0 & -(\lambda+d_h) \end{pmatrix}$$

The eigenvalues which have been calculated are $\lambda_1 = -R_m(\rho+n)$, $\lambda_2 = -d_m$, $\lambda_3 = -d_h$; the remaining two roots has been calculated with the quadratic equation $\lambda^2 + (d_m + \gamma_h + d_h)\lambda + d_m(\gamma_h + d_h)(1 - R_0) = 0$ So it is proved that E_1 is asymptotically stable if $R_0 < 1$ and is unstable if $R_0 > 1$ and the situation $R_m > 1$ is satisfied [17].

2.4 Stability analysis of the endemic equilibrium

Now, we study about the local stability of the positive endemic equilibrium (16). The jacobin matrix of the system calculated at the epidemics equilibrium, E_2 for the equations (28):

$$J(E_2) = \begin{pmatrix} \frac{-p\xi(M_1^*+M_2^*)}{c} - (\rho+n) & 0 & 0 & 0 & 0 \\ \rho & \frac{-b^q a_m}{H}H_2^* - d_m & 0 & 0 & \frac{b^q a_m}{H}M_1^* \\ 0 & \frac{b^q a_h}{H}H_2^* & -d_m & 0 & \frac{b^q a_m}{H}M_1^* \\ 0 & 0 & \frac{-b^q a_h}{H}H_1^* & -d_h - \frac{b^q a_h}{H}M_2^* & 0 \\ 0 & 0 & \frac{-b^q a_h}{H}H_1^* & \frac{b^q a_h}{H}M_2^* & -(\gamma_h+d_h) \end{pmatrix} \tag{31}$$

The characteristics equation of (2.4) is as follows:

$$(\lambda + d_m)(\lambda + (\gamma_h + d_h))(\lambda^3 + a_1\lambda^2 + a_2\lambda^3 + a_3) = 0 \tag{32}$$

Where

$$\begin{aligned}
a_1 &= \theta + b_1 + \frac{p}{c}(M_1^* + M_2^*) \\
a_2 &= \frac{b_1 p}{c}(M_1^* + M_2^*) + \theta b_1 + b_2 \\
a_3 &= \frac{a_3 p}{c}(M_1^* + M_2^*) + \theta b_2 \\
\theta &= \rho + n \\
b_1 &= d_h + \frac{b^q a_h}{H} M_2^* + \frac{b^q a_m}{H} H_2^* + d_m \\
b_2 &= d_h d_m + \frac{d_h b^q a_m}{H} H_2^* + \frac{b^{2q} a_m a_h}{H^2} M_2^* H_2^* + \frac{d_m b^q a_h}{H} M_2^*
\end{aligned}$$

If $p(x) = x^3 + a_1 x^2 + a_2 x + a_3$. Concider that D(p) is discriminant of p(x); so

$$D(p) = -1 \begin{pmatrix} 1 & q_1 & q_2 & q_3 & 0 \\ 0 & 1 & q_1 & q_2 & q_3 \\ 3 & 2q_1 & q_2 & 0 & 0 \\ 0 & 3 & q_1 & q_2 & 0 \\ 0 & 0 & 3 & 2q_1 & q_2 \end{pmatrix}$$

$D(p) = 18q_1q_2q_3 + (q_1q_2)^2 - 4q_3q_1^3 - 4q_2^3 - 27q_3^2$. Following [2] [30] we have the following proposition.

Proposition 2.5. *We can consider that E_2 lies in R_+^3.*

1. *If $R_0 > 1$, then E_2 is asymptotically stable locally.*

2. *If $D(p)$, be positive and Routh-Hurwitz may fulfils, i.e., $D(p) > 0$, $q_1 > 0$, $q_3 > 0$, and $q_1q_2 > q_3$, then E_2 is asymptotically stable locally.*

3. *If $D(p) < 0$, $q_1 > 0$, $q_(2) > 0$, $q_1q_2 = q_3$ and $q\varepsilon[0,1]$, then E_2 is locally asymptotically stable.*

4. *If $D(p) < 0$, $q_1 < 0$, $q_2 < 0$, and $q > \frac{2}{3}$, then E_2 is unstable.*

5. *For equilibrium point E_2, the necessary condition, to get asymptotically stable locally, be $q_3 > 0$.*

2.5 Simulation results

Many proposed strategies concerning numerical and analytical solutions may be used to solve such model. We applied Trapezoidal method to solve the fractional model. Parameters values is calculated by using the numerical simulation:

$$\xi = 7.5, b = 0.7, a_m = 0.375, a_h = 0.375, \eta = \frac{1}{4}, d_m = \frac{1}{11},$$
$$d_n = \frac{1}{74}, \gamma = \frac{1}{3}, \rho = 0.08, H = 31,200,000, c = 3H, k = 3, m = 6,$$

the first state can be

$$H_1(0) = H - 2511, H_2(0) = 2511, H_3(0) = 0, A_v(0) = kH, M_1(0) = mH, M_2(0) = 0.$$

The parameter values associated to this research belong to Malaysia database of 2016 [24]. We stimulate the model for period near to 52 weeks for real epidemic. The step size that is generally used is basically 10^{-3}. Figures 1-2 presents the dynamics of vector and host compartment for $q = 1$. Simulations are carried out using fractional order $q = 0.5, 0.7, 0.9$ (Figures 3-8).

2.6 Conclusion

In this chapter, the dengue transmission model of rational order is investigated. In the light of all the results regarding epidemiological and the theoretical findings are given below:

- The Threshold quantitative and the dynamical order behaviour of fractional system is quick same with the re-production numbers in which r is nearly driven as R_0 and it can be shown that if the R_0 is less than unity then the disease can be eradicated.

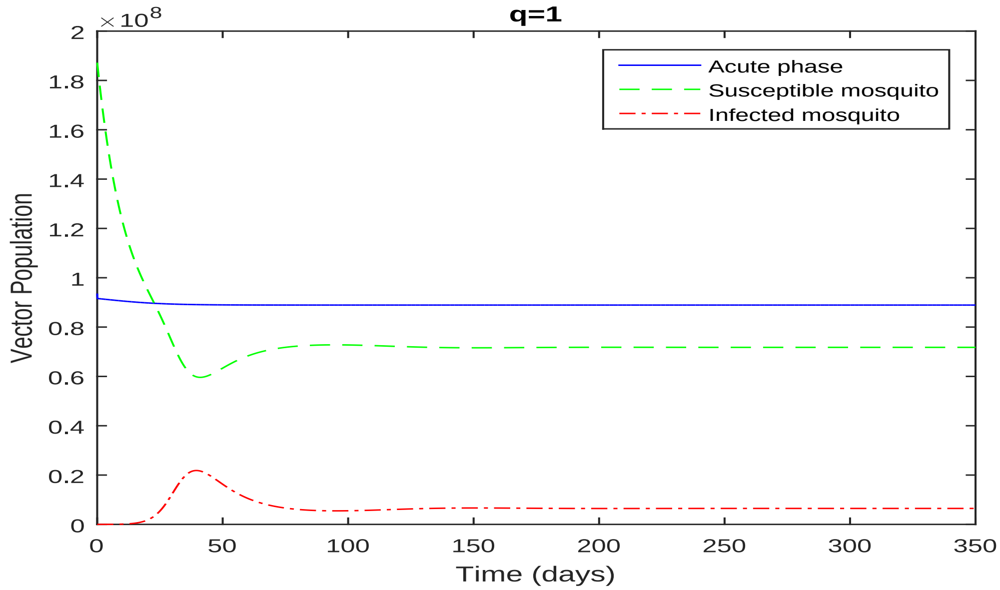

Figure 1: Simulation of the vector population with fractional order $q = 1$.

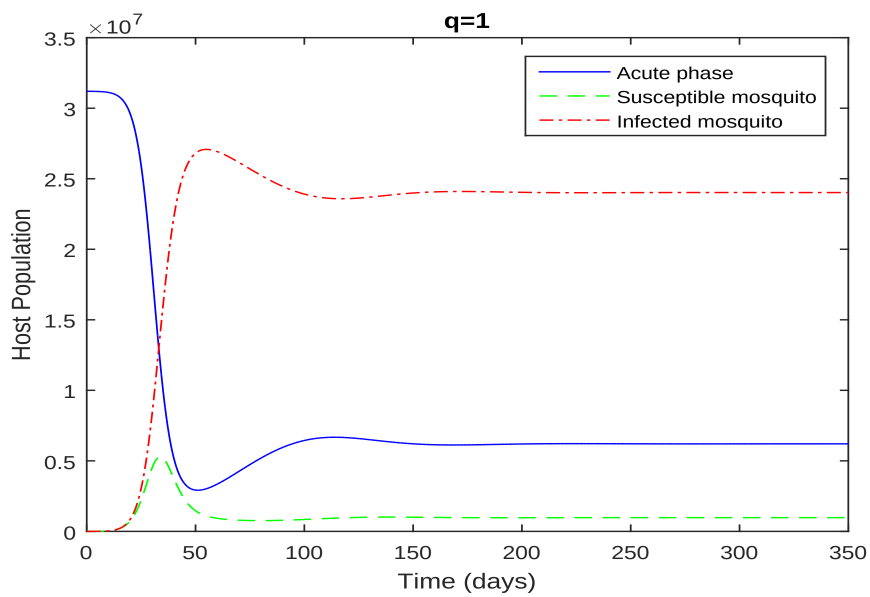

Figure 2: Simulation of the host population with fractional order $q = 1$.

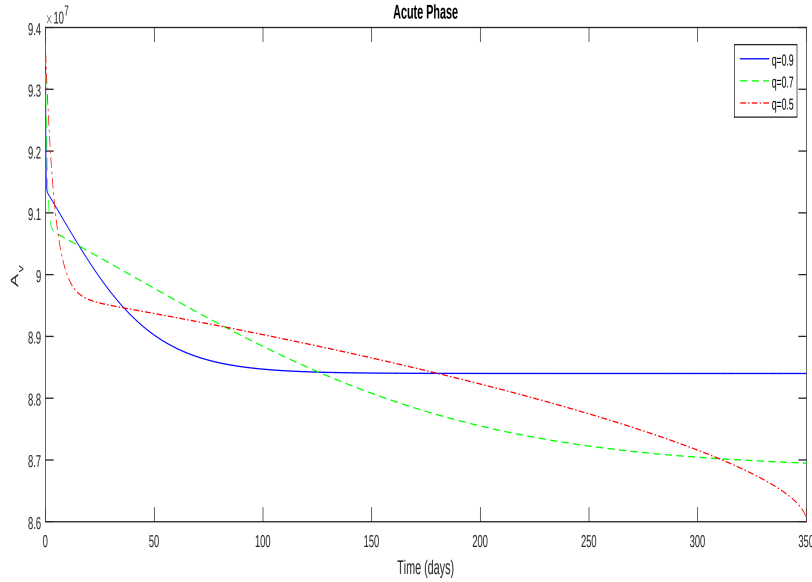

Figure 3: Simulation of the acute phase with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

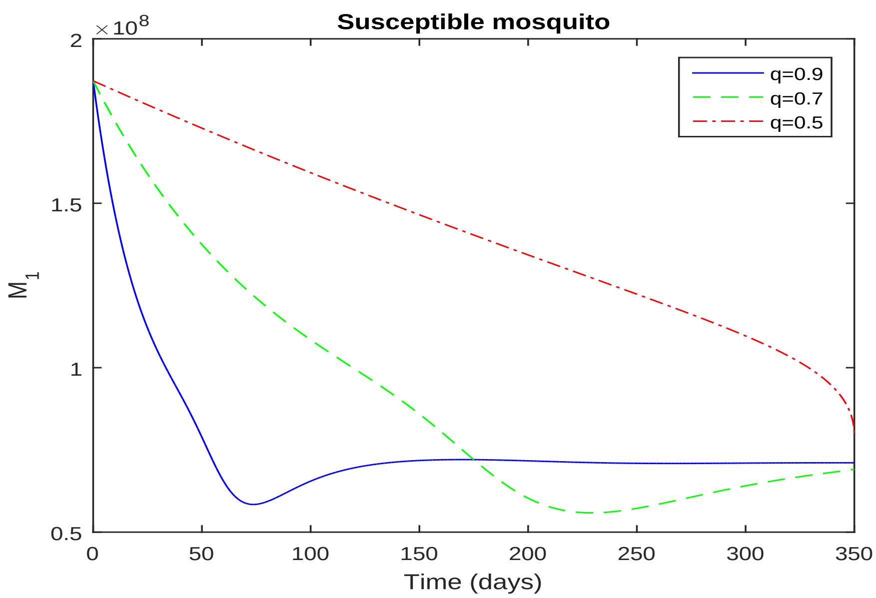

Figure 4: Simulation of the susceptible mosquito with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

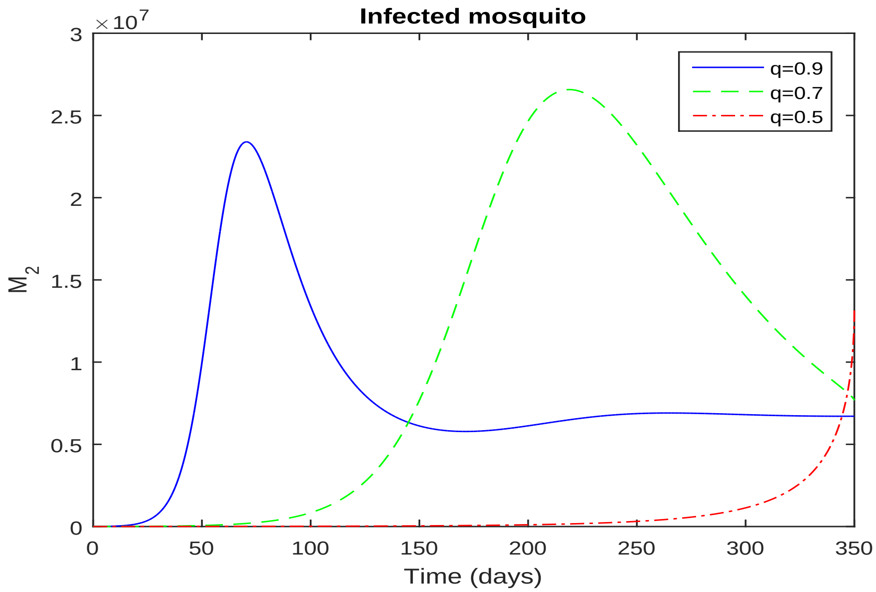

Figure 5: Simulation of the infected mosquito with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

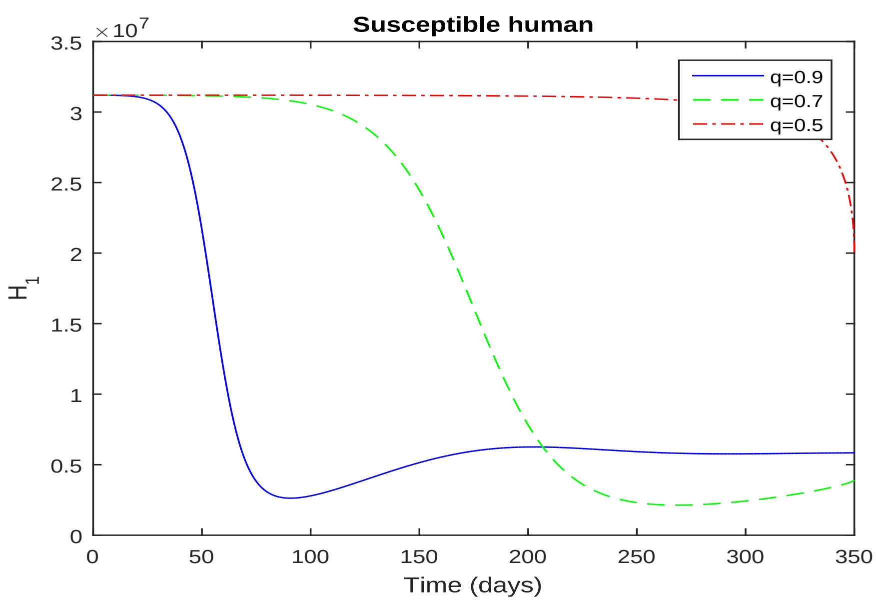

Figure 6: Simulation of the susceptible human with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

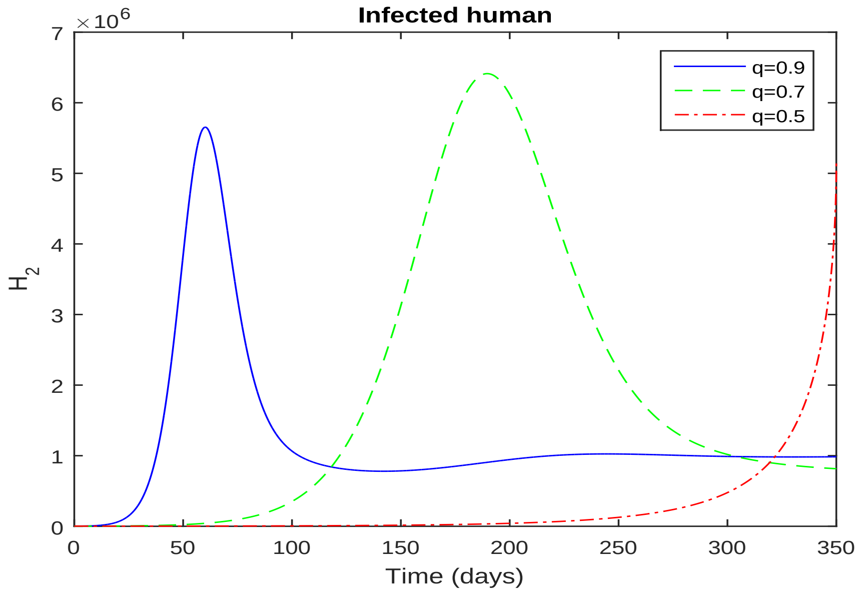

Figure 7: Simulation of the infected human with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

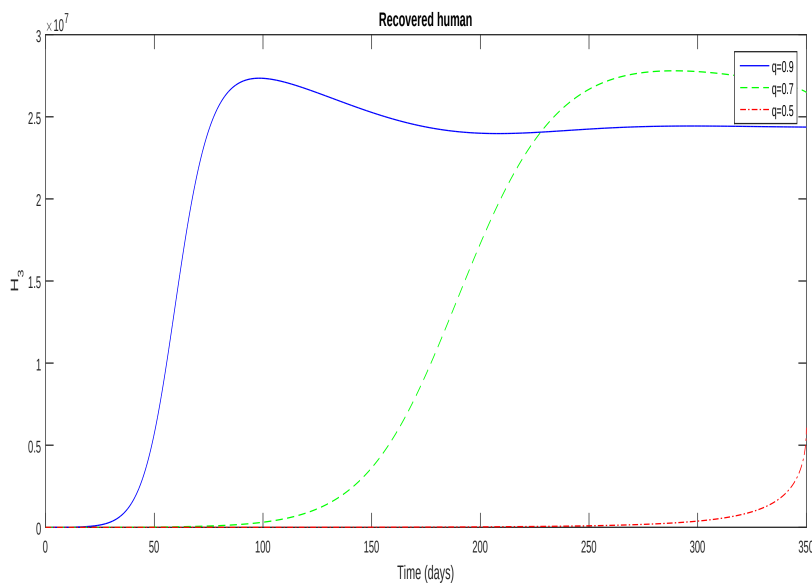

Figure 8: Simulation of the recovered human with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

- During the study it is found that local asymptotically endemic equilibrium can be stable when $R_0 > 1$ and the disease free equilibrium stability within model will be locally asymptotical whenever is $R_0 < 1$.

- It is found that in case of $R_0 > 1$ endemic equilibrium is globally asymptotical stable numerically.

Chapter 3

Fractional order dengue model with temperature effect

3 Fractional order dengue model with temperature effect

In this chapter, we will study a mathematical model for the rational order of dengue fever, the real data used in this mathematical model was collected from FongShan district, Kaohsiung, Taiwan. Kaohshiung is located in the south between $120^0 14' 30'' - 120^0 23' 30''$ east longitude and $22^0 30' 30'' - 22^0 45' 30''$ north latitude and is also the second most famous city-based city of Taiwan. Information data showed about that Kaohsiung was at high risk for dengue fever from 1998 to 2010 and total 2415 number of cases of dengue fever were reported. For our study weekly metro-logical data has been collected in Kaohsiung which was recorded by 11 supervising stations of the Taiwan related to surrounding conditions or the health of the Earth Protection government unit (EPA Taiwan, 2011), With the help of this metro-logical data weekly highest possible value, mean, and minimum temperatures in Kaohsiung from 2001 to 2010 were gotten and incorporated in our study.

3.1 Fractional order vector-host dengue model

Especially this study is based upon the vector-host transmission model from [1] for modeling transmission patterns of relationships, movement, or sound of dengue fever in Southern Area of Taiwan. Following ideas you think are true are made, population has been divided into host (human), vector (pre-adult), and vector (adult female mosquito population). There are two limits of A, aegypti pre-adult female mosquito population were defined, effective eggs and larvae, in able to be susceptible or influenced L_1 and infected T_1 populations. There has been three parameters of vector (adult) defined as: L_2, I_2, and T_2, these are numbers at time t of susceptible or influenced, infected but not being factious and infectious female mosquitoes respectively. Also, there are three parameters defined for host (human population): L_3, T_3, and C_3, these are the sizes at the time t of able to be susceptible or influenced, infected/infectious and recovered/immune human populations respectively . This way eight ordinary differential equations [1] define the system:

$$D_c^q L_1 = e_v^q (1 - p(\frac{T_2}{L_2 + I_2 + T_2})) - \alpha^q L_1, \tag{33}$$

$$D_c^q T_1 = e_v^q p(\frac{T_2}{L_2 + I_2 + T_2}) - \alpha^q T_1, \tag{34}$$

$$D_c^q L_2 = \alpha^q L_1 - b^q \frac{T_3}{M_h} L_2 - \delta^q L_2, \tag{35}$$

$$D_c^q I_2 = b^q \frac{T_3}{M_h} L_2 - \gamma^q I_2 - \delta^q I_2, \tag{36}$$

$$D_c^q T_2 = \gamma^q I_2 + \alpha^q T_1 - \delta^q T_2, \tag{37}$$

$$D_c^q L_3 = R_{hb}^q M_h - b^q \frac{L_3}{M_h} T_2 - R_{hd}^q L_3, \tag{38}$$

$$D_c^q T_3 = b^q \frac{L_3}{M_h} T_2 - \xi^q T_3 - R_{hd}^q T_3, \tag{39}$$

$$D_c^q C_3 = \xi^q T_3 - R_{hd}^q C_3. \tag{40}$$

The parameters of e_v, p, and q shows the deposition ratio of eggs (per day), upward infestation rate, and the immature mosquito infection rate (per day) in (33) and (34), respectively. As $L_2 + I_2 + T_2$ is the totalize of entire population (M_v) in that $T_2/(L_2 + I_2 + T_2)$ is the infestation probability. The $1/e_v$ shows the duration of medium deposition, and $1/q$ shows the immature mosquito medium conversion from hatched eggs into grown-up.

The parameter b be the tranmission biting rate/day. Female grown-up mosquito is contaminated by means of the dengue infection at some stage in a blood-meal as described by $b^q(T_3/M_h)L_2$. In the same way, the infected mosquitoes transfer the infection to healthy people as defined by $b^q(L_3/M_h)T_2$. This model think about that the basic transmission likelihood among people and mosquitoes are comparable in the both guideline [1]. The parameter ξ shows the man recovery rate/day. Hence, due to increase in human birth numbers $(R_{hb}^q, M_h), L_3$ also increases accordingly and decreases by the term $b^q(L_3/M_h)T_2$. Eventually due to increase in the human recovery number $(\xi^q T_3), C_3$ increases accordingly and diminishes by the human passing numbers $(R_{hd}^q C_3)$.

3.2 Initial conditions of modeling parameters

Table 2 presents the dengue model parameters their indicating data is taken from Civil Affairs Bureau of Kaohsiung [13] with human population size (M_h) of 341120.

Table 2: Given below the parameters of this model.

symbol	Meaning unit	Range of values	References
p	proportion of eggs	0.028	[22]
α	pre-adult mosquito mature rate (per day)	0.099^a	Estimated
b	the biting rate (per day)	0.33	[1]
e_v	Oviposition rate (per day)	6.218^a	Estimated
γ	Virus incubation rate in mosquito (per day)	0.0607^a	Estimated
δ	Adult mosquito death rate (per day)	0.0331^a	Estimated
ξ	Human recovery rate (per day)	1/7	[1]
R_{hd}	Human death rate (per day)	0.000016	[13]
R_{hb}	Human birth rate(per day)	0.00002	[13]
M_v	Total number of mosquitoes	341,120	Assumed
M_h	Total size of human population	341,120	[13]

The initial values of $L_3(0), T_3(0)$, and $C_3(0)$ were presumed to be 341,094, 26, and 0, respectively, cases in accordance with the number of confirmed cases in December, 2010 in FongShan district, the infectious population was estimated to be 26. Implying one bite per three days for one female mosquito, the transmissible biting rate was equal to 0.33 per day. The initial values of $L_1(0), T_1(0), L_2(0), I_2(0)$, and $T_2(0)$ were presumed to be 0, 0, 341120, 0, and 0, respectively. The mosquito population (M_v) was assumed to be constant at 341,120 female mosquitoes. Because of restricted mosquito population size in the modeling area, we presumed a 0.5-fold, one-fold, and two-fold difference between the number of female mosquitoes and the human population.

3.3 Equilibrium Points

We discuss two types of equilibrium points for the system such as disease-free equilibrium (DFE) E_0 and ”biologically realistic disease-free equilibrium” (BRDFE) E_1.

where $E = (L_1, T_1, L_2, I_2, T_2, L_3, T_3, C_3)$

Equilibrium points can be characterized as,

$$D_c^q L_1 = 0, D_c^q T_1 = 0, D_c^q L_2 = 0, D_c^q I_2 = 0, D_c^q T_2 = 0,$$

$$D_c^q L_3 = 0, D_c^q T_3 = 0, D_c^q C_3 = 0.$$

where, $\bullet E_0^q = (0,0,0,0,0,\frac{R_{hb}^q M_h}{R_{hd}^q})$
$\bullet E_1^q = (\frac{e_v^q}{\alpha^q},0,\frac{e_v^q}{\delta^q},0,0,\frac{R_{hb}^q M_h}{R_{hd}^q})$

3.4 (R_0)The basic reproduction number

Usually, the basic reproduction number R_0 of the widespread disease is known as the anticipated wide variety of secondary infections caused by a single, normal contamination in an entire able to be susceptible or influenced population. On the side of vector-borne disease, it can be able to be explained, as the number of humans which would be affected from a single human at the beginning of infection from a mosquito.

Hypothetically, when $R_0 < 1$ the transmissions chains are not beneficial and are impotent to achieve a huge epidemic. Although if $R_0 > 1$, each generation will be raised by the number of infected humans and infection will be taken advantage of with the usage of next generation matrix approach. In this situation R_0 is equal to the spectral radius of $K = FV^{-1}$, where F is a non-negative matrix of the infection items, and V is the singular M-Matrix of the transmission terms.

$$F = \begin{pmatrix} 0 & 0 & 0 & b^q \\ 0 & 0 & 0 & p\delta \\ b^q \frac{M_v}{M_h} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$V^{-1} = \begin{pmatrix} \frac{1}{\xi^q + R_{hd}^q} & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha^q} & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma^q + \delta^q} & 0 \\ 0 & \frac{1}{\alpha^q} & \frac{1}{\gamma^q} & \frac{1}{\delta^q} \end{pmatrix}$$

therefore,

$$FV^{-1} = \begin{pmatrix} 0 & \frac{b^q}{\delta^q} & \frac{b^q}{\gamma^q} & \frac{b^q}{\delta^q} \\ 0 & p & \frac{p\delta^q}{\gamma^q} & p \\ \frac{b^q M_v}{M_h}(\xi^q + R_{hd}^q) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The eigenvalues of FV^{-1} is: $\frac{p}{2} \pm \frac{1}{2}\sqrt{p^2 + \frac{4b^{2q}M_v}{M_h \gamma^q (\xi^q + R_{hd}^q)}}$

Therefore, the dominant eigenvalue of FV^{-1} is

$$R_0 = \frac{p}{2} + \frac{1}{2}\sqrt{p^2 + \frac{4b^{2q}M_v}{M_h \gamma^q (\xi^q + R_{hd}^q)}} \tag{41}$$

3.5 Simulation results

Using the L1 method given in Chapter 1, we solve the fractional dengue model numerically. Figures 9-11 have the action of the temperature-dependent vectorhost view for the man and mosquito populations where temperature is considered 18° C, 25° C and 28° C. The impact of fractional order ($q = 0.9, 0.7$ and $q = 0.5$.) is presented in Figures 12-19.

3.6 Conclusion

A fractional order dengue model is investigated that incorporate temperature dependent feature in entomological parameters. In the modeling process four entomological parameters will be used for temperature-dependent features, that include the oviposition rate (e_v), pre-adult mosquito maturing rate (q), adult mosquito death rate (δ), and the virus incubation rate in mosquitoes (γ). Weekly mean temperature data in Kaohsiung (2001-2010) that has been utilized to perform the simulation. We applied L1 method to solve the model for

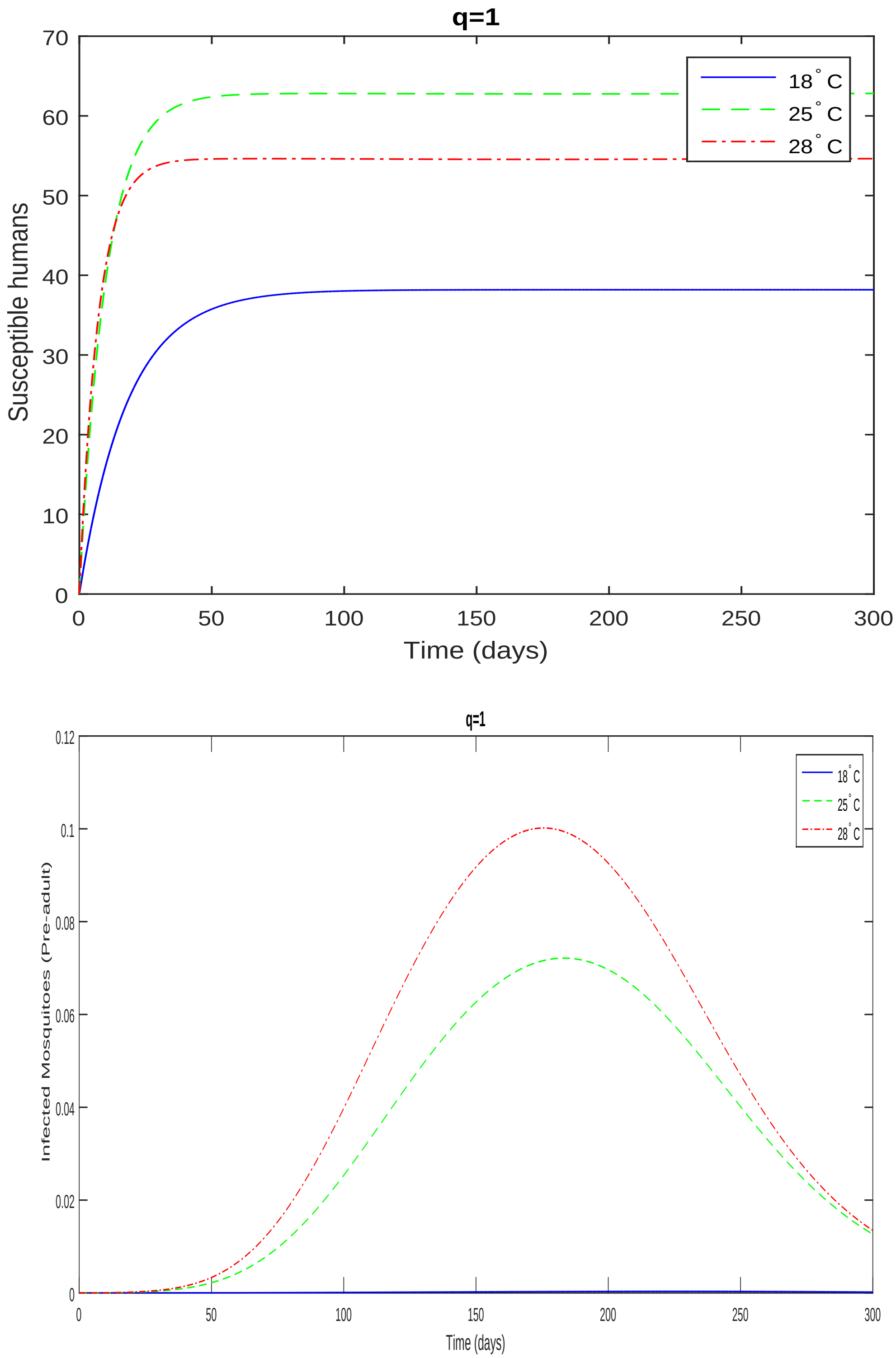

Figure 9: Simulation result pre-adult mosquito compartment at temperature 18° C, 25° C and 28° C with $q = 1$.

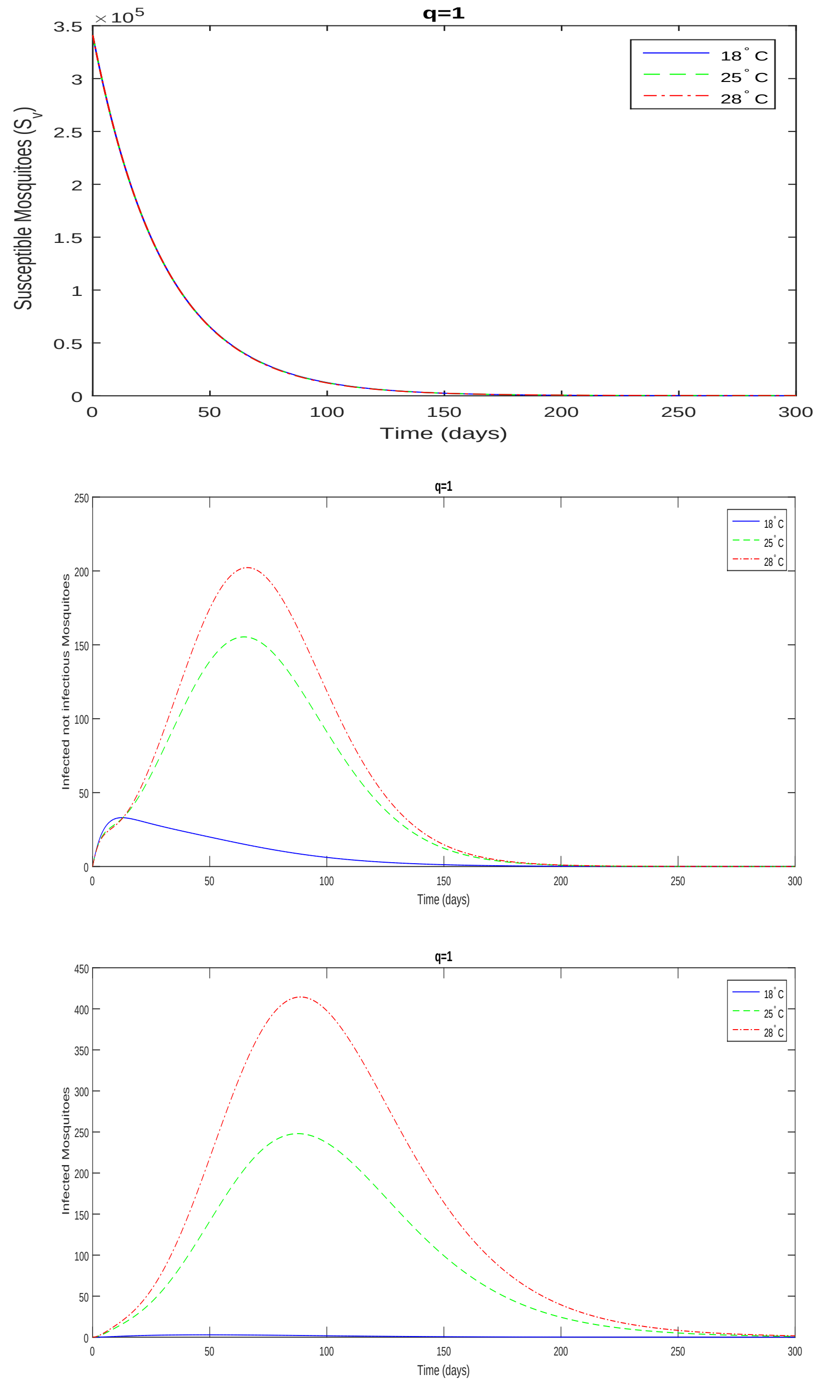

Figure 10: Simulation result adult mosquito compartment at temperature 18° C, 25° C and 28° C with $q = 1$.

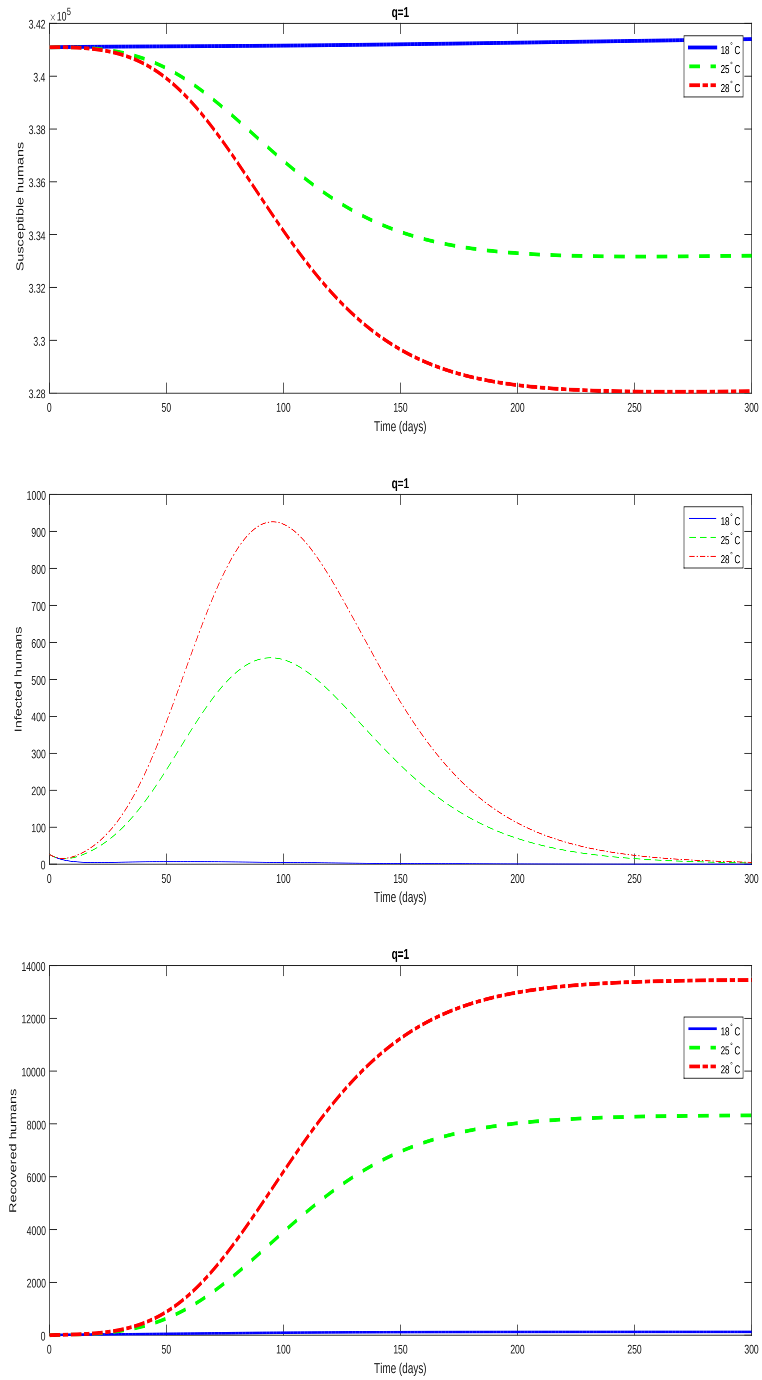

Figure 11: Simulation result host compartment at temperature 18° C, 25° C and 28° C with $q = 1$.

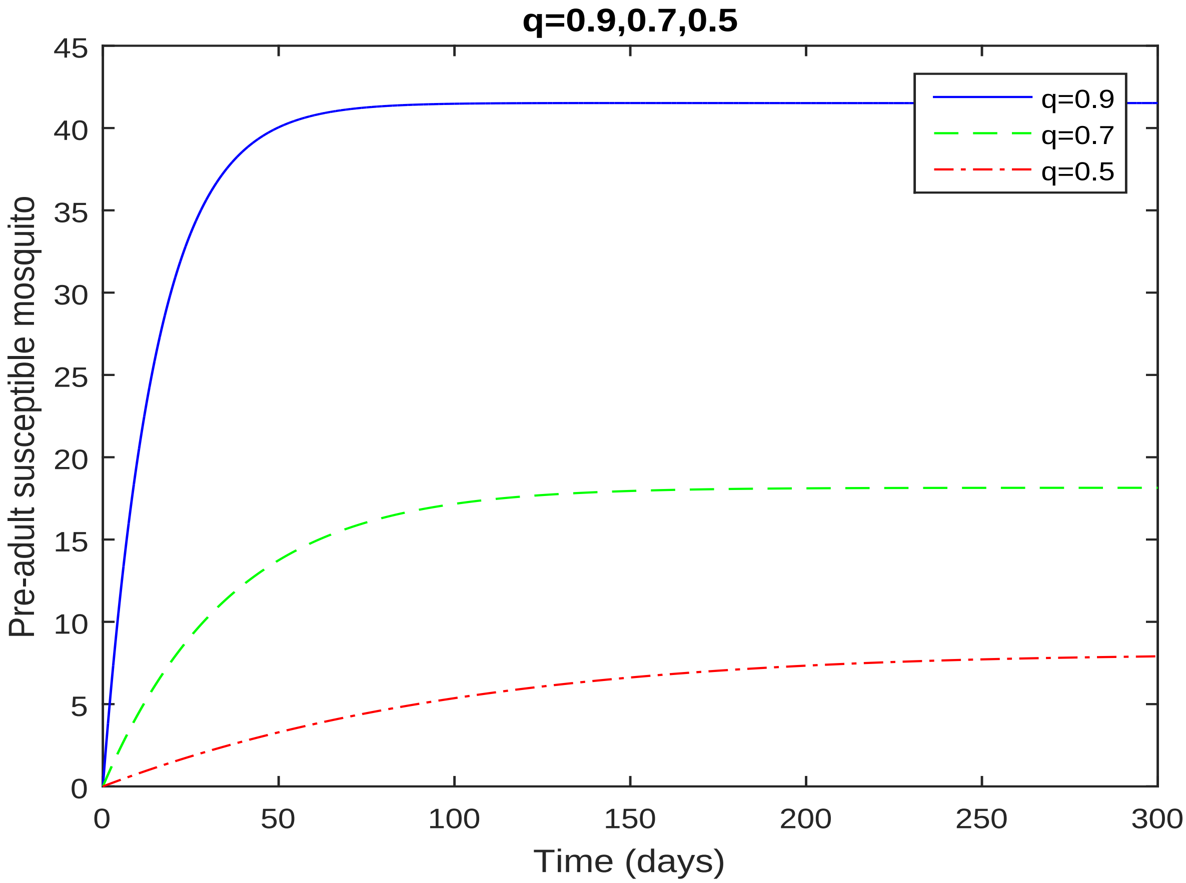

Figure 12: Simulation of the pre-adult susceptible mosquito with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

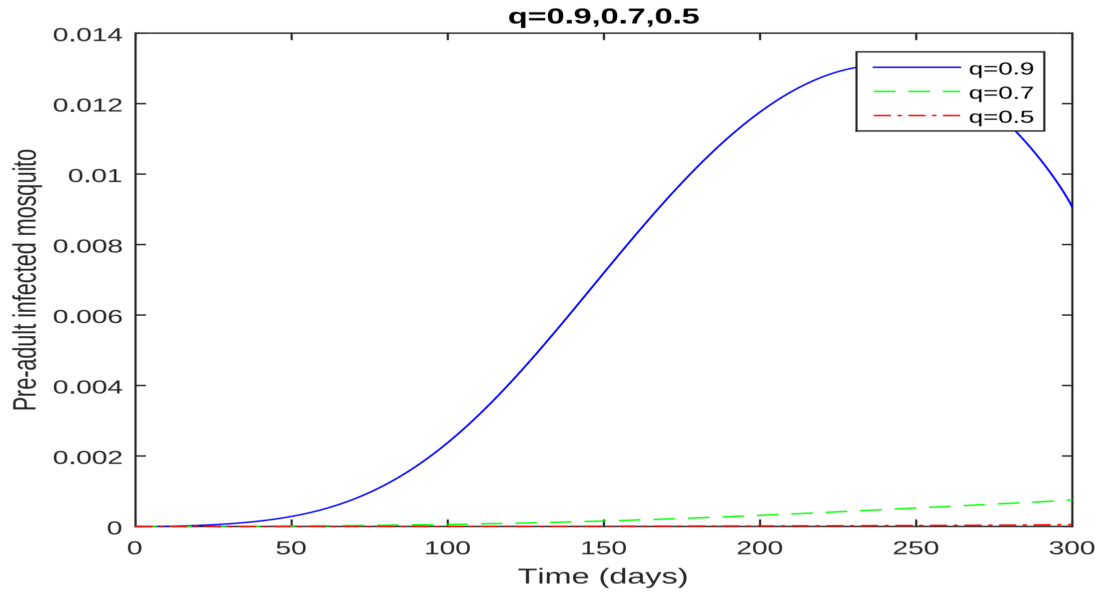

Figure 13: Simulation of the pre-adult infected mosquito with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

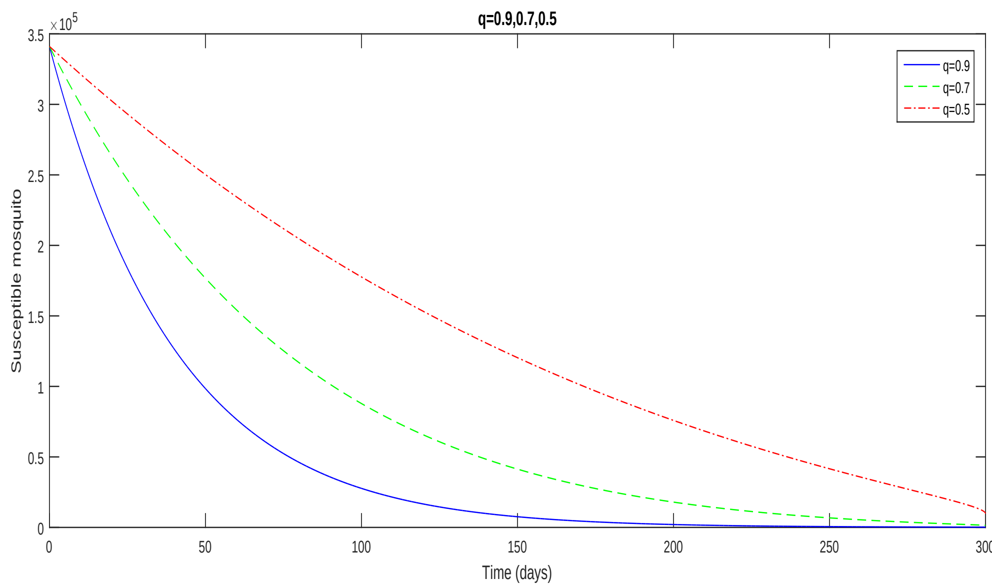

Figure 14: Simulation of the adult susceptible mosquito with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

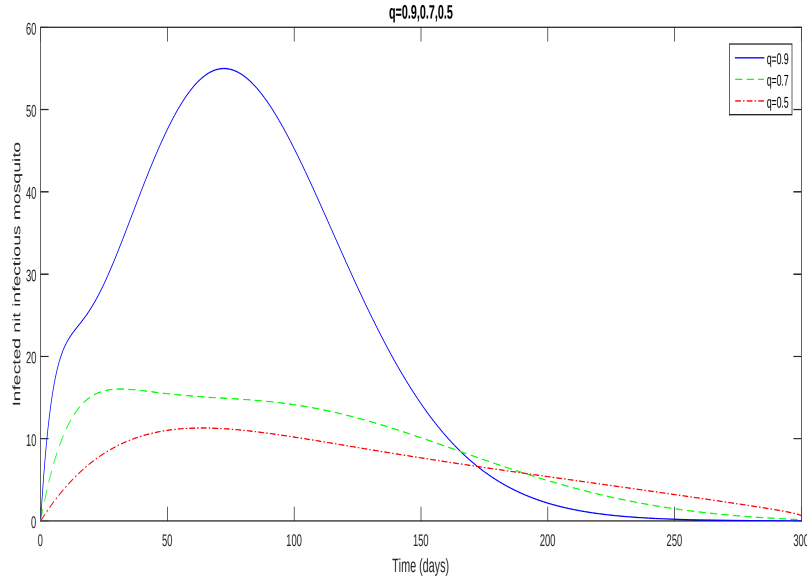

Figure 15: Simulation of the adult infected but not infectious mosquito with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

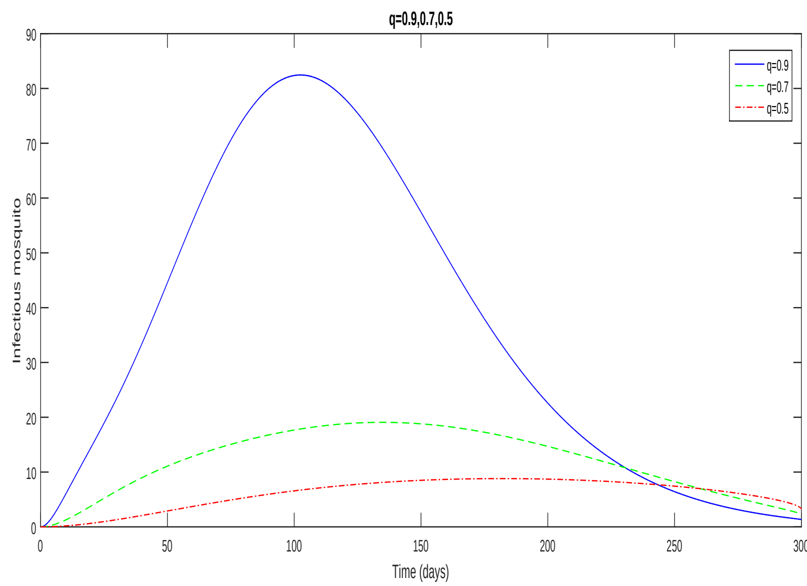

Figure 16: Simulation of the infectious mosquito with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

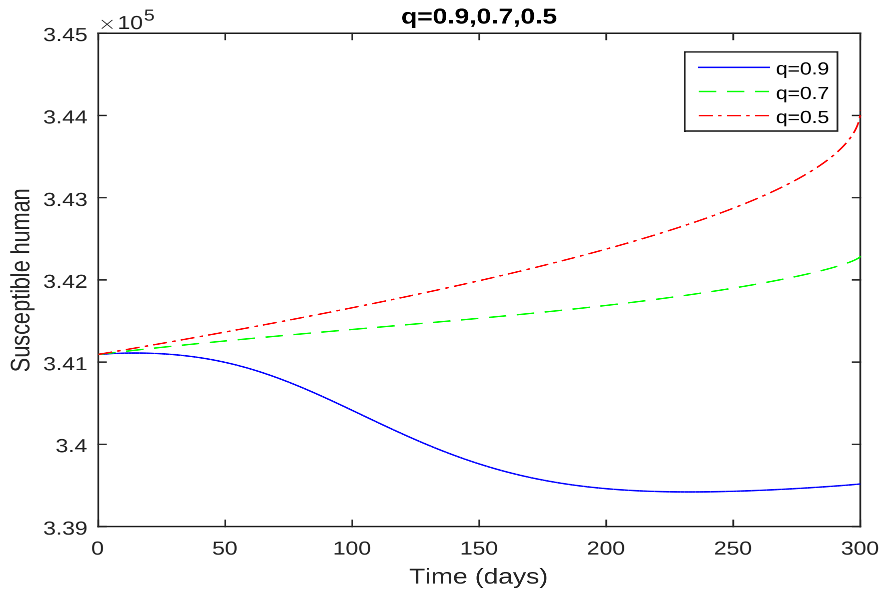

Figure 17: Simulation of the susceptible human with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

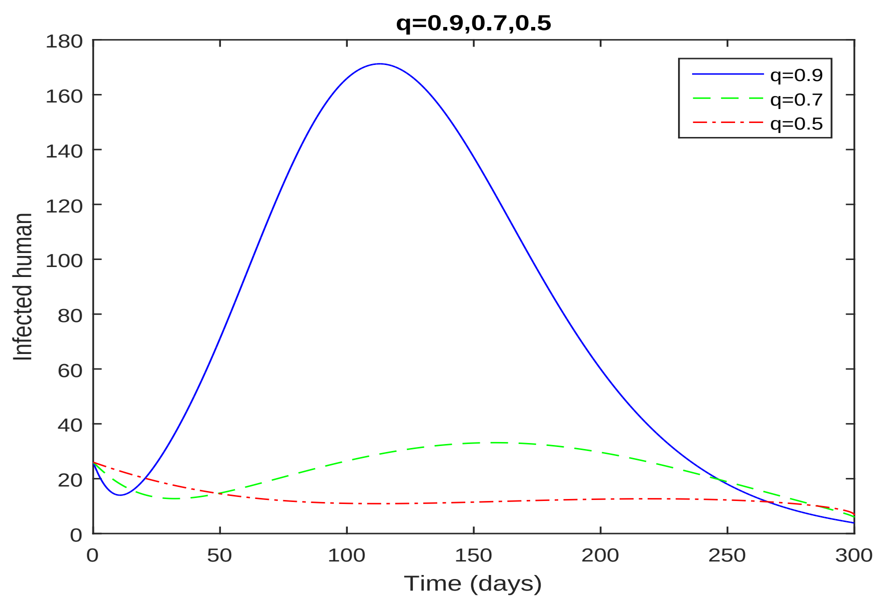

Figure 18: Simulation of the infected human with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

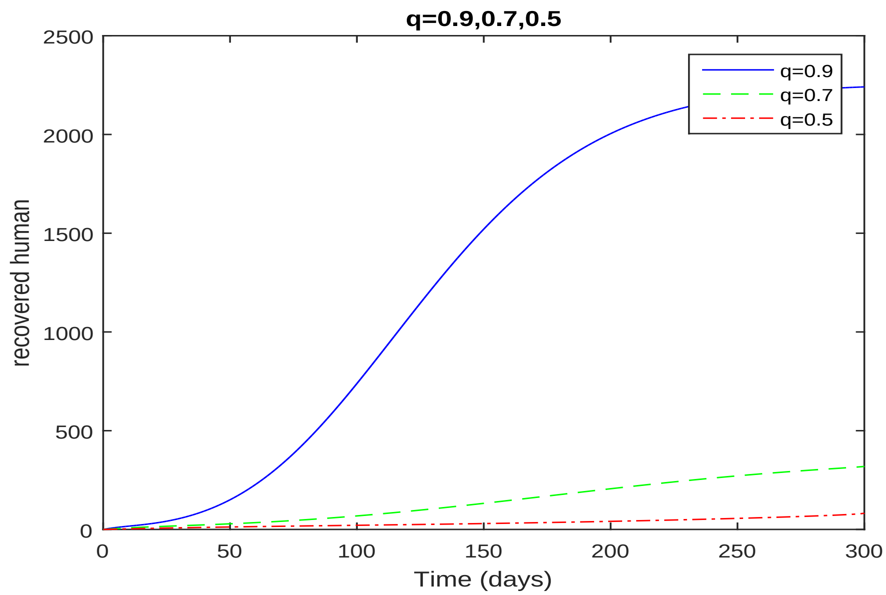

Figure 19: Simulation of the recovered human with fractional order $q = 0.9, 0.7$ and $q = 0.5$.

various fractional order and temperatures. Further basic reproduction number is derived of the fractional order dengue model. Simulation results refers that the highest danger of dengue transmission exists at temperature 28° C.

Chapter 4

Conclusion

4 Conclusion

In this thesis, we study fractional order dengue models. First, we study the fractional order differential equation of the dengue epidemic system based on the susceptible-infected-recuperated (SIR) model. The threshold quantity value R_0 similar to the basic reproduction number is obtained using the next-generation matrix approach. It can be shown that if R_0 is less than unity then the disease can be eradicated. During the study it is found that local asymptotically endemic equilibrium can be stable when $R_0 > 1$. Secondly, we investigate dengue model with temperature effect. Equilibrium points of the dengue model and the Basic reproduction number are obtained. Real data of dengue fever in subtropical Taiwan by the adding temperature-dependent entomological parameters is used to calculate the parameter values. By using L1 method we solve the model numerically and present the simulations. Our simulations depicted that dengue fever dynamics modeling is affected by the temperature variation. Further it is observed that the 28° C temperature is highest risk of dengue spread. These results are very useful to control the dengue spread and future planning to eradicate this disease. In the light of all these mentioned facts and figure it is found fractional order can be better approach for modeling the better dengue transmission. In the research it is proposed that there must be clear and the better estimated value of fractional order q which should be match with the real data and may be much helpful to improve the results and research.

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