Cosmological Implications of Equation of State Parametrizations

By

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I, **Sidra Saleem** with registration number **CIIT/FA17-RMT-051/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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Certificate

It is certified that **Sidra Saleem** with **CIIT/FA17-RMT-051/LHR** has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS Institute of Information Technology, Lahore campus and the work fulfills the requirement for award of MS degree.

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DEDICATED TO

My Family

Table of Contents

List of Figures

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Abstract

In this thesis, we assume flat FRW model to discuss cosmic evolution of the accelerating universe in the framework of dynamical Chern-Simons modified gravity in interacting scenario. We analyze the interaction term which calculates the action of interaction between dark matter and dark energy. We examine thermodynamically an extra driving term for a flat universe by applying Sharma Mittal entropy rather than Bekenstein-Hawking entropy to Padmanabhan's holographic equipartition law. Deviating from the Bekenstein-Hawking entropy by using holographic equipartition law, we generate an extra driving term in the acceleration equation.

We construct two models by using parametrization of equation of state parameter in the redshift. This parametrization may be a Taylor series extension in the redshift, a Taylor series extension in the scale factor or any other general parametrization of ω. We explore various cosmological parameters such as deceleration parameter, squared speed of sound, Om-diagnostic and statefinder via graphical behavior.

Chapter 1
Introduction

The branch of astronomy that is concerned with the origin, evolution and ultimate fate of the universe from the big bang to this day and endlessly into the future is known as cosmology. The big bang is the most common theory of the universe which states that the universe has not consistently existed. Earlier the universe was hot and dense, but now with the passage of time the universe cooled, expended and become slighter dense. Besides various laws of physics related to the evolution of the universe have been measured in cosmology. In cosmology we can't tweak, repeat or even try to change the variables in a managed way, we can simply observe.

The experiments was initiated long ago. Instead of using the laws of physics we try to make models that have characteristics matching with those we can notice in the universe. Thus, almost every prediction construct in cosmology is one of the model dependent. Fortunately, our models are tackling to match with the observable universe slightly well. At present, the two most favored models are apparently the big bang universe and the steady state universe. Neither of these two theories describe the universe that we observe but both are continually being reviewed as the new data and suggestions strengthen them.

It is believed that in present day cosmology, one of the most important discoveries is the acceleration of the cosmic expansion [1]-[10]. It is observed that the universe expands with repulsive force and is not slowing down under normal gravity. This unknown force called dark energy (DE) and is responsible for current cosmic acceleration. In physical cosmology and astronomy, DE is a mysterious procedure of energy which is assumed to pervade all of space which tends to blast the extension of the universe. However the nature of DE is still unknown which requires further attention [11]. In the standard ΛCDM model of cosmology, the whole mass energy of the cosmos includes 4.9% of usual matter, 26.8% of dark matter and 68.3% of a mysterious form of energy recognized as DE. In astrophysics, dark matter (DM) is an unknown form of matter which appears only participating in gravitational interaction, but does not emit nor absorb [12]. The nature of DM is still unknown but the existence of the cosmos is proved by astrophysical observation [13]. The majority of dark matter is to be thought non-baryonic in nature [14].

On the other hand, different modified theories of gravity have been proposed in order to explain cosmic acceleration. The dynamical Chern-Simons modified gravity has been recently proposed [15] which is motivated from string theory and loop quantum gravity [16, 17]. In this gravity, Jawad and Rani [18] investigated various cosmological parameters and planes for pilgrim DE models in that FRW universe. Jawad and Sohail [17] explored different cosmological planes as well as parameters for modified DE.

Numerous Observations suggest an accelerated extension of the universe [19]-[21] which can be explained by a positive cosmological continual Λ in standard ΛCDM models [19, 20]. However, it is noted that the observational measurement of Λ is much slighter than the hypothetical value of quantum field theory [22]. In order to resolve this problem of cosmological constant Λ, different cosmological models [23]-[26] have been suggested. Particularly most of the thermodynamic scenarios have been examined by the holographic principles, which assumes that the evidence of bulk is stowed over the horizon [27]. For instance, cosmological equations from the lookout of entropic forces [28]-[30] in FRW universe [31, 32] have been studied, whereas frequently neglected surface terms over the horizon which are assumed by cosmological entropy have been proposed [33]-[37]. In these studies, we use Bekenstein-Hawking entropy [38] for the replacement of the horizon of a black hole with the horizon of the universe. It has been perceived that a combine entropy exists which appears to generalize both of these entropies [39]. This two parametric entropy known as Sharma Mittal entropy [40].

Recently, another attracted thermodynamic situation known as holographic equipartition law has been suggested [41]. In this model, for a flat FRW universe the cosmological equations were effectively derived from the extension of cosmic space due to the modification between the degrees of freedom over the surface and in the bulk by using the Bekenstein-Hawking entropy. The development of cosmological equations has been evaluated from different viewpoints [42]-[47]. However the Bekenstein-Hawking entropy can not directly evaluate an extra driving term that is related to Λ and Λ(t), here Λ(t) is a time varying [48]. It is considered that the Bekenstein-Hawking entropy additively based on Boltzmann-Gibbs statistcs. Though, self-gravitating schemes represent peculiar structures, like nonextensive statistics [49]. Thus, for the black-hole entropy the Tsallis-Cirto [50] and a revised Renyi entropy [51, 52] have been suggested. In this framework, for a flat FRW universe we observe thermodynamically in cosmological equations an extra driving term by applying the Sarma Mittal entropy to the holographic equipartition law. The value obtain from the driving term is presumed to be constricted by the generalized second law of thermodynamics.

In order to explain DE, a large number of models have been suggested. Among all of these, the simplest is the cosmological constant model and this model is compatible with observations [1]. In the cosmological framework, the equation of state (EoS) parameter, ω, gives the relation among energy density and pressure [53]. This is a dimensionless parameter and indicates the two stages of the cosmos [54]. The EoS parameter might be used in FRW equations to define the evolution of an isotropic universe filled with a perfect fluid. The EoS parameter governs not only the gravitational properties of DE but also its progression. The EoS parameter may be a continual or a time dependent function [1]. It is observed that this parameter gives constant ranges by using various observational schemes. For deviating DE a parameterized formation of ω is supposed. Parametrization may be a Taylor series extension in the redshift, a Taylor series extension in the scale factor or any other parametrization of ω [55]-[70]. Using different considerations of parametrization, the cosmological parameters can be constrained [71]-[73].

By following the works of Tripathi, A., et al. [1] and Elizaldel, E., [74] the cosmological models in dynamical Chern-Simons modified gravity will be developed in the presence of parameterized formation of ω. We will also check the viability of the constructed models through different cosmological parameters and planes. The thermodynamics with different corrected entropies will also be investigated of the constructed models.

- In chapter 2, we provide the basic definitions and formalism to understand the thesis work.

- Chapter 3 is devoted to the reconstruction of some CPL-parameterizations models in dynamical Chern-Simons modified gravity. We construct some cosmological parameters taking parameterizations of equation of state parameter.

- In chapter 4, we construct different models through different corrected entropies and also check the behavior of the models through different cosmological parameters.

- In the last chapter, we summarize the results.

Chapter 2
Preliminaries

This chapter has the basic concepts for understanding the thesis work. The basic definitions are very useful for the calculations.

2.1 Evolution of the Universe

Cosmology is an analysis of the nature of the universe as an entire operation. The word cosmology is acquired from the Greek word Kosmos which means that the order or harmony. Cosmologist are focused on the future, evolution, formation and constituents of the universe. Most objects such as stars, planets, galaxies, cluster of galaxies and even superclusters we can perceive with telescopes are vast or exist at maximum distances. Majority of the cosmologists view that all these entities were founded after a beginning, extremely hot and dense emergence event around fourteen Giga years ago that has generated and continuous to generate the space that we see around us.

Humanity's perception of the universe has developed notably over time. In the advance history of astronomy, earth was considered as the interior of all the things, with stars and planets orbiting it. In the 16^{th} century, Nicolaus Copernicus a Polish scientist suggested that the earth and the other planets in the solar system actually orbited the sun, creating an intense shift in the understanding of the universe. In the late 17^{th} century, Isaac Newton evaluated how the forces between the planets particularly the gravitational forces interchanged. The dawn of the 20^{th} century present further perception into understanding the vast universe. Albert Einstein suggested the unification of time and space in his general theory of relativity.

In the early 1900s, scientists were discussing either the milky way accommodate the whole universe within its period or it was simply one of many crowd of stars. Edwin Hubble evaluated the distance of a fuzzy nebulous particle in the sky and calculated that it situate outside of the milky way, showing our galaxy to be a little drop in the extensive universe. Using general relativity to deposit the framework, Hubble measured other galaxies and decided that they were shooting away from us and concluded that the universe was not static but expanding.

2.2 Accelerated Expansion of the Universe

The accelerated expansion was discovered in 1998 by two unconventional projects, the high-Z supernova search team and the supernova cosmology project. Both used distant type Ia supernova to calculate the acceleration. Between the two distant departs of the universe with time the expansion of the universe is the increase of the distant. It is a permanent expansion by which the scale of space itself varies. The universe does not demand space to live outside it and does not expand into anything. It is thought that about 5 billion years ago the accelerated expansion of the universe have begun when the universe entered in the DE dominated era.

In the framework of general relativity the accelerated expansion of the universe can be considered by a positive value of the cosmological constant Λ which is equivalent to the presence of a positive vacuum energy known as DE. The accelerated expansion of the cosmos is that velocity at which the distance galaxy is decreasing from the viewer is steadily increasing with time.

2.3 Cosmological Contents

2.3.1 Dark Energy

It is observed that the universe expands with repulsive force and is not slowing down under normal gravity. This unknown force called DE and is responsible for current cosmic acceleration. DE was discovered in 1998 by two international teams that included American astronomers Riess and Perlmutter and Australian astronomer Schmidt. In physical cosmology and astronomy, DE is a mysterious procedure of energy which is assumed to pervade all of space which tends to blast the extension of the cosmos. One explanation for DE is that it is a property of space. Another explanation for DE is that it is a new type of dynamical energy fluid, something whose effect on the expansion of the universe is opposite to that of matter and normal energy although it fills all of space.

It is thought that DE permeate the universe, despite its low energy density and responsible for the accelerating expansion of the cosmos. Billions of years ago the mystery of this millennium is that the cosmos was nothing but an extremely small particle. Then in very less time than the glimmer of an eye the cosmos expanded and increased in size by a factor of 10^{50}. Some scientists propose that an unknown force, called DE, may be the source of this accelerated expansion. DE and DM refers to the invisible energy and matter constituents of the universe.

2.3.2 Dark Matter

The majority of DM is thought to be non-baryonic in nature probably being composed of some undiscovered infinitesimal particles. Normal matter counting all the perceivable stars, galaxies and planets makes up less than 5% of total mass of the cosmos. Astronomers cannot perceive DM directly but can analyze its effects. They can see that light bent from gravity of invisible objects which is called gravitational lensing. DM is composed of such particles that do not reflect, absorb or emit light. Therefor, they cannot be recognized by observing the electromagnetic radiation. DM is that material which cannot be seen precisely.

DM is an unseen form of material that is thought to prevail because scientists have perceived its visible gravitational consequences on galaxies and galaxy clusters. DM is an invisible non-baryonic matter assumed to explain the phenomena including galactic rotation curves and gravitational lensing.

2.4 FRW Universe

The Friedmann-Robertson-Walker (FRW) metric is an exact solution of Einstein's field equations of general relativity. It describes an isotropic, homogeneous and expanding universe which is path connected but not simply connected. It is given by

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2\big(d\theta^2 + \sin^2\theta d\phi^2\big)\right), \tag{2.4.1}$$

here k represents curvature which involves three cases as follow

- $k = -1$ represents the negative curvature (hyperboloid),
- $k = 0$ corresponds to no curvature (flat surface),
- $k = 1$ gives the positive curvature (for the 3-sphere).

2.5 Scale Factor

In cosmology, scale factor is the parameter that tells how the size of the cosmos is changing with respect to its size at current time. This parameter is the ratio of real distance between two objects at some time t to the real distance between the two objects at some related time t_0. It is represented by $a(t)$. Since, the latest cosmic observations have shown that universe is accelerating so $a(t) > 0$.

2.6 Hubble Parameter

The Hubble parameter indicates the rate at which the cosmos is currently expanding. It is called a constant and it changes with time. Mathematically, it can be written as

$$v = H(t)r, \tag{2.6.1}$$

here r and v indicate the radius and the recessional velocity respectively, $H(t)$ is Hubble parameter depends upon time. Hubble parameter is defined by using scale factor as

$$H = \frac{\dot{a}(t)}{a(t)}, \tag{2.6.2}$$

where dot represents time derivative.

- $\ddot{a}(t) > 0 \quad \Rightarrow$ accelerated expansion of the universe,
- $\ddot{a}(t) < 0 \quad \Rightarrow$ decelerated expansion of the universe.

2.7 Parameterizations

Parameterizations is a specification of one or more variables which allows us to choose values in defined specific range. This process is used for the differentiation of data sets. The EoS parameter is used to categorize the decelerated and accelerated phases of the universe. This parameter is defined as $\omega = \frac{p}{\rho}$ where p represents the pressure and ρ expresses the density of the universe. The decelerated phase of the universe involve following eras:

- $0 < \omega < \frac{1}{3} \quad \Rightarrow$ radiation era,
- $\omega = 0 \quad \Rightarrow$ cold dark matter era.

The DE dominated phase has following eras:

- $\omega = -1 \quad \Rightarrow$ cosmological constant,
- $-1 < \omega < \frac{-1}{3} \quad \Rightarrow$ quintessence,
- $\omega < -1 \quad \Rightarrow$ phantom.

2.8 Interaction Term

Interaction is an idea of two way action that occur when two or more objects have effect on each other. The interaction term Q calculates the action of interaction between the DM and DE. Basically, Q tells about the rate of energy exchange between DM and DE [75].

- $Q > 0 \quad \Rightarrow$ energy is being converted from DE to DM.
- $Q < 0, \quad \Rightarrow$ energy is being converted from DM to DE.

2.9 Redshift Parameter

Blueshift and redshift describe how light moves toward longer or shorter wavelengths as objects like stars or galaxies in space move farther or closer away from us. The concept is a key of charting the expansion of universe. Redshift is mainly an effect of the expansion of space in the widely obtained cosmological model constructed from general relativity. Astronomers use redshift to estimate approximate distances from very detached galaxies. The farther away an object the more it will be redshifted. It is denoted by z and is defined through the following relation.

$$1 + z = \frac{a_0}{a}, \tag{2.9.1}$$

wherea_0 is the present-day value of scale factor.

2.10 Dynamical Chern-Simons Modified Gravity

The dynamical Chern-Simons modified gravity is given by the following action

$$\begin{aligned} S &= \frac{1}{16\pi G}\int_V d^4x[\sqrt{-g}R + \frac{l}{4}{}^*R^{\rho\sigma\mu\nu}R_{\rho\sigma\mu\nu}\theta - \frac{1}{2}g^{\mu\nu}\nabla_\mu\theta\nabla_\nu\theta + V(\theta)] \\ &+ S_{mat}, \end{aligned} \tag{2.10.1}$$

here R is the Ricci scalar, ${}^*R^{\rho\sigma\mu\nu}R_{\rho\sigma\mu\nu}$ is the topological invariant called the pontryagin term, l is the coupling constant, θ is the dynamical variable, S_{mat} is the action of matter and $V(\theta)$ is the potential. Now in case of string theory, we take $V(\theta) = 0$. The variation of Eq.(2.10.1) corresponding to metric $g_{\mu\nu}$ and scalar field θ, respectively, give the following field equations

$$
\begin{aligned}
G_{\mu\nu} + lC_{\mu\nu} &= 8\pi G T_{\mu\nu}, && (2.10.2)\\
g^{\mu\nu}\nabla_\mu\nabla_\nu\theta &= -\frac{l}{64\pi}{}^{*}R^{\rho\sigma\mu\nu}R_{\rho\sigma\mu\nu}, && (2.10.3)
\end{aligned}
$$

where $G_{\mu\nu}$ is known as Einstein tensor and $C_{\mu\nu}$ appears as Cotton tensor which is defined as

$$
C_{\mu\nu} = -\frac{1}{2\sqrt{-g}}((\nabla_\rho\theta)\varepsilon^{(\rho\beta\tau^{\mu}_{\nabla_\tau}R^{\nu}_{\beta})} + (\nabla_\sigma\nabla\rho\theta)^{*}R^{\rho(\mu\nu)\sigma}. \quad (2.10.4)
$$

The energy momentum-tensor related to scalar field and matter are given by

$$
\begin{aligned}
\hat{T}^{\theta}_{\mu\nu} &= \nabla_\mu\theta\nabla_\nu\theta - \frac{1}{2}g_{\mu\nu}\nabla^\rho\theta\nabla_\rho\theta, && (2.10.5)\\
T_{\mu\nu} &= (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, && (2.10.6)
\end{aligned}
$$

here $\hat{T}^{\theta}_{\mu\nu}$ shows the scalar field contribution and $T_{\mu\nu}$ represents the matter contribution. Also, $u_\mu = (1, 0, 0, 0)$ is the four velocity.

2.11 Thermodynamics

The branch of the physical science that deals with the associations between heat and other formations of energy like mechanical, electrical, or chemical energy is known as thermodynamics. Nicolas Lonard Sadi Carnot with his various scientific contribution is often known as the father of thermodynamics. Thermodynamics is a vary important part of chemistry, physics, and engineering. Therefore, it is an analytical area of study in science and technology. Thermodynamics also perceives importance in energy, ecology and other studies. The are four laws of thermodynamics.

2.11.1 Generalized Second Law of Thermodynamics

In standard thermodynamics, the generalized second law of thermodynamics requires that in consequence of general transformations the entropy of a closed system shall commonly increase and never decrease. However this law might hold good for a system containing a black hole that is not instructive in its genuine form. For example, we can see that for an exterior observer the ordinary entropy becomes invisible if an ordinary system drop into a black hole. Hence from the observer viewpoint saying that the increase in ordinary entropy does not provide any vision that the ordinary second law is exceeded. The more useful law the generalized second law of thermodynamics can be given by containing the black hole entropy in the entropy journal which states that as a consequence of generic transformations of the black hole the sum of the ordinary entropy outside the black hole and the total black hole entropy never decreases and typically increases such that

$$\triangle S_0 + \triangle S_{BH} \geq 0, \tag{2.11.1}$$

where S_0 is the ordinary entropy of the system and S_{BH} is the total entropy of the black hole.

2.11.2 Bekenstein-Hawking Entropy

The Bekenstein-Hawking entropy is also known as black hole entropy. It is the aggregate of entropy that must be allocated to a black hole in order for it to observe with the constitutions of thermodynamics as they are illuminated by observers outermost to that black hole. For the first and second laws this is specially true. It is an idea with geometric root but with multiple physical consequences. It ties simultaneously conceptions from thermodynamics, gravitation and quantum theory. Thus it is regarded as a window into the as still mainly hidden world of quantum gravity. The formula of Bekenstein-Hawking entropy is given as

$$S_{BH} = \frac{K}{H^2}, \tag{2.11.2}$$

where K is the positive constant which is defined as $K = \frac{\pi k_b d^2}{L_p^2}$. The Planck length L_p is interpreted by $L_p = \sqrt{\frac{\hbar G}{c^3}}$, where k_b is the Boltzmann constant, d is the speed of light, G is the gravitational constant and $\hbar$ is the reduced Planck constant [20] which is described as $\hbar \equiv \frac{h}{2\pi}$, where h is the Planck constant.

2.11.3 Sharma Mittal Entropy

It has been perceived that a combine entropy exists which appears to generalize both of these entropies [21]. This two parametric entropy known as Sharma Mittal entropy [22]. This is a generalized entropy introduced by Sharma Mittal which is basically defined as

$$S_{SM} = \frac{1}{R}\left((1 + \frac{\delta A}{4})^{\frac{R}{\delta}} - 1\right), \tag{2.11.3}$$

where δ and R are two free unknown parameters, the surface area A of the sphere is defined as $A = 4\pi r^2$ and the radius r is given by $r = \frac{d}{H}$.

2.12 Holographic Equipartition Law

The accelerated expansion of the universe can be illuminated as a liability to assure the holographic equipartition. The holographic equipartition law can be stated as, the increase in dV of the cosmic volume for an infinitesimal interval dt can be written as

$$\frac{dV}{dt} = L_p^2(N_{sur} - \epsilon N_{bulk}) \times c, \tag{2.12.1}$$

here N_{sur} and N_{bulk} are the number of degrees of freedom of radius r on a spherical surface and the number of degrees of freedom of the bulk respectively [23], V represents the Hubble volume in Planck units and t shows the cosmic time in Planck units. In the present thesis, d is not 1, so the right hand side of above relation includes d and ϵ is a parameter discussed below.

Chapter 3
Cosmic Analysis using Parameterizations of Equation of State Parameter

In this chapter, we provide the field equations for dynamical Chern-Simons modified gravity taking FRW spacetime. In this framework, we reconstruct different models using parameterizations of equation of state parameter. We discuss some cosmological parameters like, deceleration parameter, squared speed of sound, Statefinder parameters and Om-diagnostic.

In case of flat FRW universe, first Friedmann equation for dynamical Chern-Simons modified gravity becomes

$$H^2 = \frac{1}{3}(\rho_m + \rho_d) + \frac{1}{6}\dot{\theta}^2, \tag{3.0.1}$$

where $\rho = \rho_m + \rho_d$ is the effective density and $8\pi G = 1$. We assume $p_m = 0$ then for ordinary matter, the conservation equations are given as

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{3.0.2}$$

$$\dot{\rho}_d + 3H(\rho_d + p_d) = 0. \tag{3.0.3}$$

For FRW universe the ponytrying term vanishes, so the scalar field in Eq.(2.10.3) reduces to the following form

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\theta = g^{\mu\nu}(\partial_\nu\partial_\mu\theta) = 0. \tag{3.0.4}$$

By taking $\theta = \theta(t)$, we get the following equation

$$\ddot{\theta} + 3H\dot{\theta} = 0. \tag{3.0.5}$$

The solution of this equation for $\dot{\theta}$ is $\dot{\theta} = ba^{-3}$ where b is an integration constant. Using this solution in Eq.(2.10.6), we have

$$H^2 = \frac{1}{3}(\rho_m + \rho_d) + \frac{1}{6}b^2a^{-6}. \tag{3.0.6}$$

Taking into account the equation of continuity Eq.(3.0.2), Eq.(3.0.6) takes the form

$$-2\dot{H} - 3H^2 - \frac{1}{6}b^2a^{-6} = p_d. \tag{3.0.7}$$

Eq.(3.0.6) can be re-written as

$$E^2(z) = \frac{1}{3H_0^2}(\rho_m + \rho_d) + \frac{1}{6H_0^2}b^2a^{-6}, \tag{3.0.8}$$

where $E(z) = \frac{H}{H_0}$ is a normalized Hubble parameter. The continuity equations for energy densities are defined as

$$\begin{aligned} {\rho_m}' - \left(\frac{3}{1+z}\right)\rho_m &= -\frac{Q}{H_0 E(z)(1+z)}, & (3.0.9) \\ {\rho_d}' - 3\left(\frac{1+\omega_d}{1+z}\right)\rho_d &= \frac{Q}{H_o E(z)(1+z)}. & (3.0.10) \end{aligned}$$

Here prime denotes the derivative with respect to the redshift. In the preceding prospectus of DE, HDE is one of the sensational attempts to analyze the nature of DE in the frame of quantum gravity. The HDE is based on holographic principle which states that all information relevant to a physical system inside a spatial region can be observed on its boundary instead of its volume. The relation of ultra-violet UV (Λ) and infra-red IR (L) is introduced by Cohen et. al [76] which plays a key role in the construction of HDE model [77]. The relation is about the energy of vacuum of a system with particular size whose maximal quantity should not be greater than the black hole mass of the similar size. This can be indicated as $L^3\rho_d \leq LM_p^2$, here $M_p^2 = (8\pi G)^{-1}$ and L represents the reduced Planck mass and IR cutoff respectively [78]. From the above inequalities, the HDE density takes the following form

$$\rho_d = 3c^2 H_0^2 E^2(z), \tag{3.0.11}$$

where c is the constant parameter of the dimensionless HDE and it lies in the interval $0 < c^2 < 1$ and L is taken as Hubble horizon. Inserting Eqs.(3.0.9), (3.0.10) and (3.0.11) in (3.0.8), we get the following expression

$$\omega_d = (1+z)\frac{dE^2(z)}{3c^2E^2(z)dz} - \frac{b^2(1+z)^6}{6c^2H_0^2E^2(z)} - \frac{1}{c^2}, \tag{3.0.12}$$

after some calculation we get the following result

$$\frac{dE^2(z)}{dz} - (3 + 3c^2\omega_d)\left(\frac{E^2(z)}{1+z}\right) = \frac{b^2(1+z)^5}{2H_0^2}. \tag{3.0.13}$$

3.1 Parameterizations of Equation of State Parameter

We construct two different models; one with a constant EoS parameter and other with a dark fluid in the existence of DM [1, 74]. By using a function of redshift the variation of EoS parameter can be estimated and many parameterizations have been suggested so far. We use the following parameterizations

$$\omega_{1d} = \omega_0, \tag{3.1.1}$$

$$\omega_{2d} = \omega_0 + \omega_1 q. \tag{3.1.2}$$

At present, ω_0 is the value of EoS parameter and ω_1 is the parameter of the model that is determined by using the observational data [74]. Inserting Eq.(3.1.1) in (3.0.13), we have

$$E^2(z) = \frac{b^2(1+z)^6}{2H_0^2(3 - 3c^2\omega_0)} + A(1+z)^{3+3c^2\omega_0}, \tag{3.1.3}$$

where A is a constant of integration. Similarly by inserting Eq.(3.1.2) in (3.0.13), we get the following result

$$E^2(z) = \frac{b^2}{2H_0^2\left(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}\right)}(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}} + B(1+z)^{\frac{3+3c^2\omega_0-3c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}. \tag{3.1.4}$$

3.2 Analysis of Interaction Term

In this section, we analyze the interaction term Q for the chosen parametrization of EoS parameter. Using Eqs.(3.0.8) and (3.0.11), we obtain the energy density of DM in the following form

$$\rho_m = 3H_0^2E^2(z)(1-c^2) - \frac{b^2}{2}(1+z)^6. \tag{3.2.1}$$

Taking derivative with respect to z of Eq.(3.2.1) along with ρ_m in the continuity equation related to DM, we have

$$(1+z)\frac{d}{dz}\ln(E^2(z)) = 3 + \frac{b^2(1+z)^6}{2H_0^2E^2(z)(1-c^2)} - \frac{Q}{3H_0^3E^3(z)(1-c^2)}. \tag{3.2.2}$$

Using Eqs.(3.0.8), (3.0.9) and (3.0.10), we get the following result

$$\frac{(1+z)}{3}\frac{d}{dz}\ln(E^2(z)) = 1 + \frac{\omega_d}{1+r+\frac{b^2(1+z)^6}{2}} - \frac{b^2(1+z)^6}{2H_0^2E^2(z)}, \tag{3.2.3}$$

where r is the coincidence parameter which is defined as $r = \frac{\rho_m}{\rho_d}$ with the help of Eqs.(3.0.11) and (3.2.1). Comparing the above equations, it yields

$$\begin{aligned}\frac{Q}{9H_0^3E^3(z)(1-c^2)} &= -\frac{\omega_d}{1+r+\frac{b^2(1+z)^6}{2}} + \frac{b^2(1+z)^6}{2H_0^2E^2(z)} \\ &\times \left(1+\frac{1}{3(1-c^2)}\right).\end{aligned} \tag{3.2.4}$$

At present time, the above equation becomes

$$Q_0 = -9(1-c^2)\left(\frac{\omega_{d,0}}{1+r_0+\frac{b^2}{2}}\right) + \frac{9b^2(1-c^2)}{2}\left(1+\frac{1}{3(1-c^2)}\right). \tag{3.2.5}$$

It is significant to express that the value of Q-term predicts the rate at which the universe expands and coincidence parameter decreases. Using the positivity condition of the Q-term at present time, Eq.(3.2.5) takes the following form

$$\omega_{d,0} < (1+r+\frac{b^2}{2})\left(\frac{3b^2(1-c^2)+b^2}{b(1-c^2)}\right). \tag{3.2.6}$$

The normalized Hubble parameter in terms of coincidence parameter is obtained by the ratio of Eqs.(3.0.11) and (3.2.1) such that

$$E^2(z) = -\frac{b^2(1+z)^6}{6H_0^2c^2(r(z)-r_c)}, \tag{3.2.7}$$

where $r_c = \frac{(1-c^2)}{(c^2)}$ is a constant quantity. This parameter shows the singular behavior at $r(z) = r_c$. At present time, $r_c = \frac{r_o + \frac{b^2}{6}}{1-\frac{b^2}{6}}$, where $c^2 = \frac{1-\frac{b^2}{6}}{1+r_0}$. For the coincidence parameter, we can consider a CPL-type parametrization form [47] $r(z) = r_o + \epsilon_o \frac{z}{1+z}$, where $\epsilon_o = r'_o$. We can notice that above parametrization becomes singular at $z = -1$ and it has a linear behavior and bounded nature for low and high value of redshift respectively. Taking into account the above parametrization, we get the value of redshift z_s, such that $z_s = -\frac{r_o - r_c}{\epsilon_o(1+\frac{(r_o-r_c)}{\epsilon_o})}$. For the singular behavior, we have the condition $-1 < z_s < 0$. After some manipulation, we obtain

$$r(z) - r_c = \epsilon_o(\frac{(z - z_s)}{(1 + z_s)(1 + z)} \geqslant 0 \Longrightarrow z \geqslant z_s, \tag{3.2.8}$$

which yields $\frac{(-\epsilon_o z_s)}{(1+z_s)} \geq 0$ at present time. Substituting these results in Eq.(3.2.8), it can be written as

$$E^2(z) = -\eta b^2 \frac{(1 + z)^7}{6H_0^2(z - z_s)}, \tag{3.2.9}$$

here $\eta := \frac{(1+z_s)}{c^2 \epsilon_o} > 0$ since $\epsilon_o > 0$. Moreover, we define the function $\theta(z) := \frac{(1+z)}{(z-z_s)}$ and substitute Eq.(3.2.9) in (3.2.3), we get the following result

$$1 + \frac{\omega_d}{1 + r + \frac{b^2(1+z)^6}{2}} = \frac{9 + \eta\theta(7 + \theta)}{3\eta\theta}. \tag{3.2.10}$$

Using above result Eq.(3.2.4) for the Q-term can be written as

$$\begin{aligned} \frac{Q}{9H_0^3E^3(z)} &= (1 - c^2)\left(\frac{9 - \eta\theta(4 + \theta)}{3\eta\theta}\right) + \frac{b^2(1 + z)^6(1 - c^2)}{2H_0^2E^2(z)} \\ &+ \frac{b^2(1 + z)^6}{6H_0^2E^2(z)}. \end{aligned} \tag{3.2.11}$$

For ω_{1d} the expression for the Q-term takes the following form

$$\begin{aligned} Q_1 &= 9H_0^3\left(\frac{b^2(1 + z)^6}{2H_0^2(3 - 3c^2\omega_0)} + A(1 + z)^{3+3c^2\omega_0}\right)^{\frac{3}{2}}(1 - c^2) \\ &\times \Bigg(- \frac{\omega_0}{1 + r_o + \epsilon_o\frac{z}{1+z} + \frac{b^2(1+z)^6}{2}} + \frac{b^2(1 + z)^6}{2H_0^2(\frac{b^2(1+z)^6}{2H_0^2(3-3c^2\omega_0)} + A(1 + z)^{3+3c^2\omega_0})} \end{aligned}$$

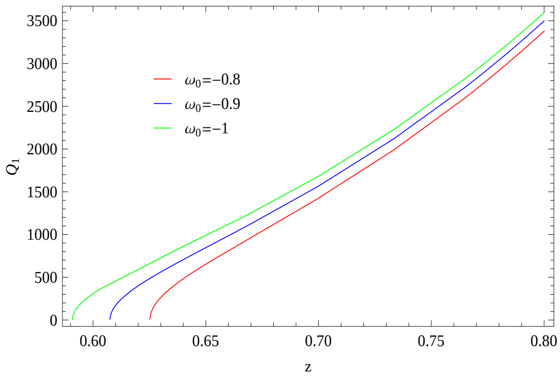

Figure 3.1: Plot of Q_1 corresponding to z for ω_{1d}.

$$\times \quad \left(1+\frac{1}{3(1-c^2)}\right)\Bigg). \tag{3.2.12}$$

Similarly for ω_{2d}, the Q-term is reduced in the following relation

$$\begin{aligned} Q_2 \;=\; & \left(\frac{b^2}{2H_0^2(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)})}(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}+B(1+z)^{\frac{3+3c^2\omega_0-3c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}\right)^{\frac{3}{2}} \\ \times \;& 9H_0^3(1-c^2)\Bigg(-\frac{\omega_0+\omega_1 q}{1+r_o+\epsilon_o\frac{z}{1+z}+\frac{b^2(1+z)^6}{2}}\left(1+\frac{1}{3(1-c^2)}\right) \\ + \;& \left(b^2(1+z)^6\right)\Bigg(2H_0^2\Big(\frac{b^2}{2H_0^2(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)})}(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}+B \\ \times \;& (1+z)^{\frac{3+3c^2\omega_0-3c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}\Big)\Bigg)\Bigg)^{-1}. \end{aligned} \tag{3.2.13}$$

In Figure 3.1, the plot of Q_1 as a function of z is expressed for three different values of $\omega_0=-0.8,-0.9,-1$. The specific values for the other constants are $b=3$, $H_0=67$, $c=0.8$, $A=-0.002$, $\epsilon_o=0.1$ and $r_o=0.43$. We can observe that Q_1 inclines the positive trajectory. It is mentioned [79] that the interaction term must not change its sign during cosmic evolution and is observationally verified. The plot of Q_2 versus z for ω_{2d} as shown in Figure 3.2. The particular values of other constants are $\omega_1=-0.2,-0.5,-0.8, B=0.002$ and remaining are same as in the above case. It can be seen that Q_2 gives the positive behavior for all epochs

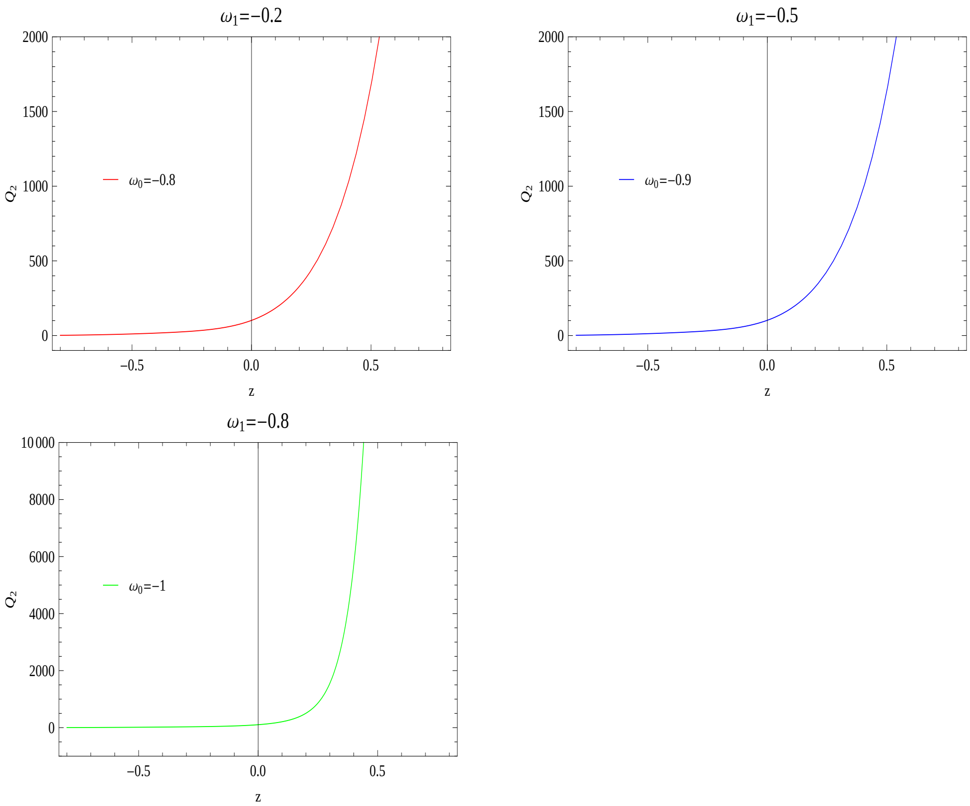

Figure 3.2: Plot of Q_2 corresponding to z for ω_{2d}.

related to z.

3.3 Cosmological Parameters

In this section, we construct some cosmological parameters such as the deceleration parameter, stability analysis, statefinder and Om-diagnostic corresponding to parameterizations of EoS parameter in the presence of dynamical Chern-Simons modified gravity.

3.3.1 Deceleration Parameter

The deceleration parameter can be described as follows

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{3.3.1}$$

This parameter characterizes the accelerated as well as decelerated phases of the universe.

- $q \in [-1, 0) \quad \Rightarrow \quad$ accelerated phase
- $q \geq 0 \quad \Rightarrow \quad$ decelerated phase

The time derivative of Hubble parameter gives the following relation in terms of redshift function

$$\frac{\dot{H}}{H^2} = -\frac{(1+z)}{E}\frac{dE}{dz}. \tag{3.3.2}$$

Inserting Eq.(3.3.2) into (3.3.1), we have

$$q = -1 + \frac{(1+z)}{E}\frac{dE}{dz}. \tag{3.3.3}$$

The deceleration parameter for ω_{1d} can be evaluated by using Eqs.(3.1.3) and (3.3.3) such that

$$\begin{aligned} q_1 &= -1 + \frac{(1+z)}{2(\frac{b^2(1+z)^6}{2H_0^2(3-3c^2\omega_0)} + A(1+z)^{3+3c^2\omega_0})} \\ &\times \left(\frac{3b^2(1+z)^5}{H_0^2(3-3c^2\omega_0)} + A(1+z)^{2+3c^2\omega_0}(3+3c^2\omega_0)\right). \end{aligned} \tag{3.3.4}$$

The plot of this equation in shown in Figure 3.3 (left) versus z for three different values of ω_0. The particular values of other constants are same as in above case. For $z > 0$, the deceleration parameter transits towards the range for accelerated phase. For present and future epochs, this parameter represents the accelerated phase of the evolving universe. Substituting the

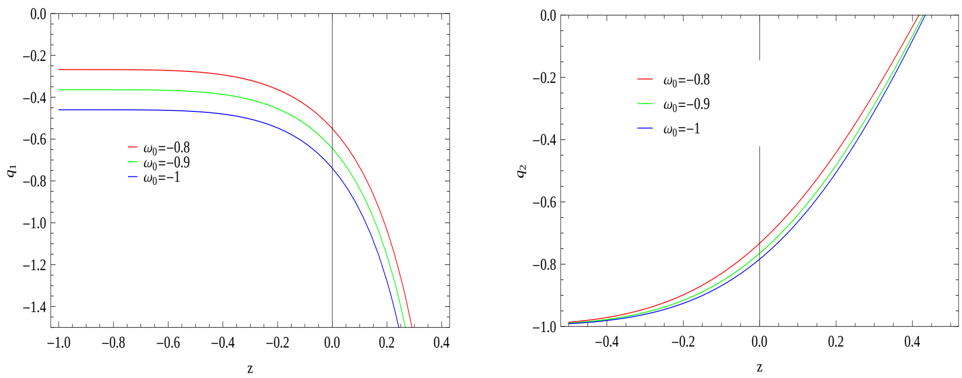

Figure 3.3: Plot of q_1 for ω_{1d} and q_2 for ω_{2d} corresponding to z taking $(\omega_0, \omega_1) = (-0.8, -0.2), (-0.9, -0.5), (-1, -0.8)$.

Eq.(3.1.4) into (3.3.3), the expression of deceleration parameter for ω_{2d} takes the following form

$$\begin{aligned} q_2 &= -1 + (1+z)\left(\frac{b^2(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}}{H_0^2\left(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}\right)} + 2B(1+z)^{\frac{3+3c^2\omega_0-3c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}\right)^{-1} \\ &\times \left[\frac{(3-3c^2\omega_1+3c^2\omega_0)B(1+z)^{-1+\frac{3-3c^2\omega_1+3c^2\omega_0}{1-\frac{3}{2}(c^2\omega_1)}}}{1-\frac{3}{2}(c^2\omega_1)} + \frac{6-11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}\right. \\ &\times \left.\frac{b^2(1+z)^{-1+\frac{6-11c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}}}{2H_0^2\left(3-\frac{8c^2\omega 1}{1-\frac{3}{2}(c^2\omega_1)}-3c^2\omega_0\right)}\right]. \end{aligned} \tag{3.3.5}$$

For ω_{2d}, we plot the deceleration parameter q_2 as shown in Figure 3.3 (right) for same parametric values. In this scenario, the deceleration parameter exhibits accelerated phase of the universe since it remains between -1 and 0 for all values of (ω_0, ω_1).

3.3.2 Stability Analysis

To examine the behavior of DE models, there is another cosmological parameter which is known as squared speed of sound. It is denoted by v_s^2 and is calculated by the following formula

$$v_s^2 = \frac{dp}{d\rho}. \tag{3.3.6}$$

The stability of the model can be checked by it.

- $v_s^2 \geq 0 \quad \Rightarrow \quad$ stability of model
- $v_s^2 < 0 \quad \Rightarrow \quad$ instability of model

Inserting Eqs.(3.0.6), (3.0.7) and (3.1.3) in (3.3.6), we obtain the squared speed of sound for ω_{1d} as follows

$$\begin{aligned} v_{s1}^2 &= \Big(-3b^2(1+z)^5 + H_0^2(1+z)\big(\frac{15b^2(1+z)^4}{H_0^2(3-3c^2\omega_0)} + A(1+z)^{1+3c^2\omega_0} \\ &\times (2+3c^2\omega_0)(3+3c^2\omega_0)\big) - 2H_0^2\big(\frac{3b^2(1+z)^5}{H_0^2(3-3c^2\omega_0)} + A(1+z)^{2+3c^2\omega_0} \\ &+ (3+3c^2\omega_0)\big)\Big)\Big(3AH_0^2(1+z)^{2+3c^2\omega_0}(3+3c^2\omega_0) + \frac{3b^2(1+z)^5c^2\omega_0}{1-c^2\omega_0}\Big)^{-1}. \end{aligned} \tag{3.3.7}$$

Taking into account Eq.(3.1.4), the relation of squared speed of sound for ω_{2d} takes the following form

$$\begin{aligned} v_{s2}^2 &= \Big[-3b^2(1+z)^5 + \frac{3b^2(1+z)^{5-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2(3-3c^2\omega_0-\frac{8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})} + 3BH_0^2 \\ &\times (1+z)^{2+3c^2\omega_1-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(3+3c^2\omega_1-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})\Big]^{-1}\Big[-\frac{b^2}{2}(1+z)^6 \\ &- 3BH_0^2(1+z)^{3+3c^2\omega_1-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}} - \frac{3b^2(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}}{2(3-3c^2\omega_0-\frac{8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})} \\ &+ \frac{b^2(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{(3-3c^2\omega_0-\frac{8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})} + 2BH_0^2(1+z)^{3+3c^2\omega_1-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}} \\ &\times (3+3c^2\omega_1\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}) - \frac{b^2(1+z)^{5-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2(3-3c^2\omega_0-\frac{8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})} \end{aligned}$$

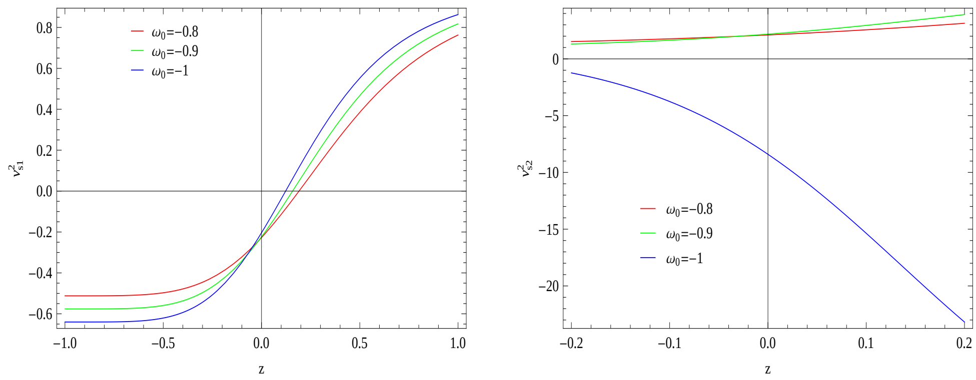

Figure 3.4: Plot of v_{s1}^2 for ω_{1d} and v_{s2}^2 for ω_{2d} corresponding to z taking $(\omega_0, \omega_1) = (-0.8, -0.2), (-0.9, -0.5), (-1, -0.8)$.

$$
\begin{aligned}
- \quad & H_0^2 B(1+z)^{2+3c^2\omega_1 - \frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(3 + 3c^2\omega_1 - \frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}) - 3b^2(1+z)^5 \\
+ \quad & \frac{b^2(1+z)^{5-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(5 - \frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(6 - \frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{(3 - 3c^2\omega_0 - \frac{8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})} + 2BH_0^2 \\
\times \quad & (1+z)^{2+3c^2\omega_1 - \frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(2 + 3c^2\omega_1 - \frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}) \\
\times \quad & (3 + 3c^2\omega_1 - \frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})\Big]. \qquad (3.3.8)
\end{aligned}
$$

The plot of v_{s1}^2 is expressed in Figure 3.4 (left). It can be seen that the trajectories of squared speed of sound show positive behavior for positive range of z (except some values) which gives stability of the model. However, for a small range of positive values of z, $z = 0$ and $z < 0$, the model expresses unstable behavior. In Figure 3.4 (right), the graph of squared speed of sound versus redshift parameter is given. The trajectories for $\omega_0 = -0.8, -0.9$ give the positive behavior for all values of z while for $\omega_0 = -1$, the squared speed of sound represents negative behavior for all values. This shows the stable behavior in first case while in latter case, the model is unstable.

3.3.3 Statefinder Parameters

The statefinder parameters (r, s) are two new cosmological parameters introduced by Sahni [48] which are defined for flat universe model as

$$s = \frac{r-1}{3(q-\frac{1}{2})}, \quad r = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}, \tag{3.3.9}$$

These parameters differentiate the various DE models, their behavior and cosmological evolution at present time.

- $(r, s) = (1, 0) \quad \Rightarrow \quad \Lambda$CDM limit
- $(r, s) = (1, 1) \quad \Rightarrow \quad$ CDM limit
- $s > 0,\ r < 1 \quad \Rightarrow \quad$ phantom and quintessence
- $r > 1,\ s < 0 \quad \Rightarrow \quad$ Chaplygin gas

We can obtain statefinder parameters (r, s) for ω_{1d} by using Eqs.(3.1.3) and (3.3.2) in (3.3.9), such that

$$\begin{aligned}
r_1 &= 1 + \frac{3(1+z)}{2\left(\frac{b^2(1+z)^6}{2H_0^2(3-3c^2)\omega_0} + A(1+z)^{3+3c^2\omega_0}\right)} \times \left(\frac{9b^2(1+z)^5}{H_0^2(3-3c^2\omega_0)} + 3A\right. \\
&\times \left.(1+z)^{2+3c^2\omega_0}(3+3c^2\omega_0)\right) - \left(\frac{1}{2(\frac{b^2(1+z)^6}{2H_0^2(3-3c^2)\omega_0} + A(1+z)^{3+3c^2\omega_0})}\right) \\
&\times \left(\frac{3b^2(1+z)^6}{H_0^2(3-3c^2\omega_0)} + A(1+z)^{3+3c^2\omega_0}(3+3c^2\omega_0)\right)\left(\frac{15b^2(1+z)^6}{H_0^2(3-3c^2\omega_0)}\right. \\
&+ \left.A(1+z)^{3+3c^2\omega_0}(3+3c^2\omega_0)(2+3c^2\omega_0)\right),
\end{aligned} \tag{3.3.10}$$

$$\begin{aligned}
s_1 &= \left[\frac{3(1+z)}{\frac{b^2(1+z)^6}{H_0^2(3-3c^2)\omega_0} + 2A(1+z)^{3+3c^2\omega_0}} \times \left(\frac{9b^2(1+z)^5}{H_0^2(3-3c^2\omega_0)} + 3A\right.\right. \\
&\times \left.(1+z)^{2+3c^2\omega_0}(3+3c^2\omega_0)\right) - \left(\frac{1}{\frac{b^2(1+z)^6}{H_0^2(3-3c^2)\omega_0} + 2A(1+z)^{3+3c^2\omega_0}}\right) \\
&\times \left(\frac{3b^2(1+z)^6}{H_0^2(3-3c^2\omega_0)} + A(1+z)^{3+3c^2\omega_0}(3+3c^2\omega_0)\right)\left(\frac{15b^2(1+z)^6}{H_0^2(3-3c^2\omega_0)}\right.
\end{aligned}$$

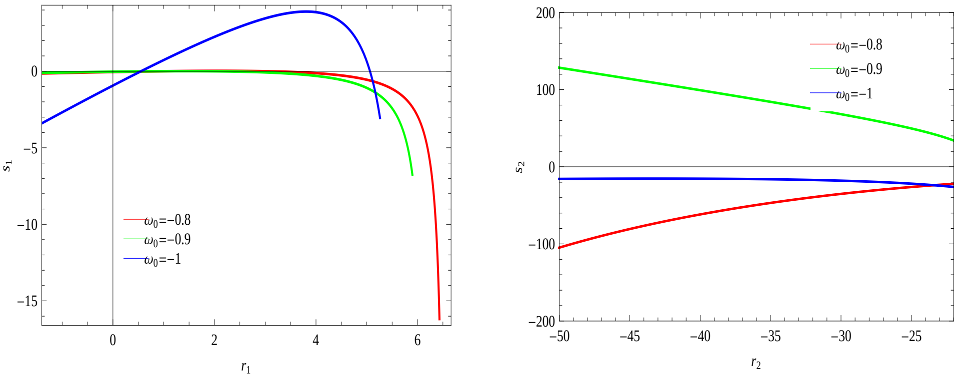

Figure 3.5: Plot of r_1 corresponding to s_1 for ω_{1d} and r_2 corresponding to s_2 for ω_{2d}.

$$\begin{aligned}
&+ \quad A(1+z)^{3+3c^2\omega_0}(3+3c^2\omega_0)(2+3c^2\omega_0)\Big)\Big]\Big[3\Big(-\frac{3}{2}-(1+z)\\
&\times \quad \Big(\frac{b^2(1+z)^6}{2H_0^2(3-3c^2)\omega_0}+A(1+z)^{3+3c^2\omega_0}\Big)^{-1}\times\Big(\frac{3b^2(1+z)^5}{H_0^2(3-3c^2\omega_0)}\\
&+ \quad A(1+z)^{2+3c^2\omega_0}(3+3c^2\omega_0)\Big)\Big)\Big]^{-1}. \qquad (3.3.11)
\end{aligned}$$

Inserting Eqs.(3.1.4) and (3.3.2) in (3.3.9), the statefinder parameters for ω_{2d} take the following form

$$\begin{aligned}
r_2 \quad &= \quad 1+3(1+z)\Big(\frac{b^2}{H_0^2}\big(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}\big)(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}+2B\\
&\times \quad (1+z)^{\frac{3+3c^2\omega_0-3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Big)^{-1}\Big[\frac{3b^2(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(1-\frac{3}{2}c^2\omega_1)(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2H_0^2(3-3c^2\omega_0-8c^2\omega_1)}\\
&+ \quad 3B(1+z)^{3+3c^2\omega_0-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Big]-\Big[\frac{b^2}{H_0^2}\big(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}\big)(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\\
&+ \quad 2B(1+z)^{\frac{3+3c^2\omega_0+3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Big]^{-1}\Big[\Big(\frac{b^2(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(1-\frac{3}{2}c^2\omega_1)(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2H_0^2(3-3c^2\omega_0-8c^2\omega_1)}\\
&+ \quad B(1+z)^{3+3c^2\omega_0-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Big)+\Big(B(1+z)^{3+3c^2\omega_0-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(3+3c^2\omega_0\\
&- \quad \frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(2+3c^2\omega_0-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})+\frac{b^2(1-\frac{3}{2}c^2\omega_1)(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2H_0^2(3-3c^2\omega_0-8c^2\omega_1)}
\end{aligned}$$

$$
\begin{aligned}
&\times \quad (5-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\bigg)\bigg], \qquad (3.3.12)\\
s_2 &= 3\Bigg(\frac{(1+z)}{\frac{b^2}{H_0^2}(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}+2B(1+z)^{\frac{3+3c^2\omega_0-3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}}\Bigg)\\
&\times \quad \Bigg[\frac{3b^2(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(1-\frac{3}{2}c^2\omega_1)(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2H_0^2(3-3c^2\omega_0-8c^2\omega_1)}+3B(1+z)^{3+3c^2\omega_0}\\
&\times \quad (1+z)^{-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Bigg]-\Bigg[\frac{b^2}{H_0^2}(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}+2B\\
&\times \quad (1+z)^{\frac{3+3c^2\omega_0+3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Bigg]^{-1}\Bigg[\Bigg(\frac{b^2(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(1-\frac{3}{2}c^2\omega_1)(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2H_0^2(3-3c^2\omega_0-8c^2\omega_1)}\\
&+ \quad B(1+z)^{3+3c^2\omega_0-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Bigg)+\Bigg(B(1+z)^{3+3c^2\omega_0-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}(3+3c^2\omega_0\\
&- \quad \frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(2+3c^2\omega_0-\frac{3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})+\frac{b^2(1-\frac{3}{2}c^2\omega_1)(6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})}{2H_0^2(3-3c^2\omega_0-8c^2\omega_1)}\\
&\times \quad (5-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(1+z)^{6-\frac{11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Bigg)\Bigg]\Bigg[3\Bigg(-\frac{3}{2}-(1+z)\\
&\times \quad \Bigg(\frac{b^2}{2H_0^2}(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1})(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}+B(1+z)^{\frac{3+3c^2\omega_0+3c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Bigg)^{-1}\\
&\times \quad \Bigg(\frac{3B(1+z)^{-1+\frac{3}{1-\frac{3}{2}c^2\omega_1}}}{1-\frac{3}{2}c^2\omega_1}+\Bigg(b^2(6-11c^2\omega_1)(1+z)^{-1+\frac{6-11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}\Bigg)\\
&\times \quad (1-\frac{3}{2}c^2\omega_1)H_o^2(3-\frac{8c^2\omega 1}{1-\frac{3}{2}c^2\omega_1}-3c^2\omega_0)\Bigg)\Bigg)\Bigg]^{-1}. \qquad (3.3.13)
\end{aligned}
$$

In Figure 3.5 (left), the graph of s_1 displayed against r_1 for three different values of ω_0. We can observe that the (r, s) parameters corresponds to Chaplygin gas behavior for the underlying scenario. However, the trajectory for $\omega_0 = -1$ does not yield any result for some region which is related to $r > 1, s > 0$. The model constitutes the ΛCDM limit $(r, s) = (1, 0)$ for the trajectories of $\omega_0 = -0.8, -0.9$. In the right side plot, we draw s_2 versus r_2 for ω_{2d} which gives the $r < 1$ and $s > 0$ for the trajectory $\omega_0 = -0.9, \omega_1 = -0.5$. This shows the DE eras, phantom and quintessence.

The remaining two trajectories do not give any fruitful results.

3.3.4 Om-diagnostic

The Om-diagnostic is another cosmological parameter. It is a tool which is used to differentiate different phases of the universe. It is calculated by the following formula

$$Om = \frac{(\frac{H}{H_0})^2 - 1}{(1+z)^3 - 1}. \tag{3.3.14}$$

- positive slope $\Rightarrow$ phantom
- negative slope $\Rightarrow$ quintessence

The Om-diagnostic for ω_{1d} and ω_{2d} can be obtained by substitution of Eqs.(3.1.3) and (3.1.4) in Eq.(3.3.14), such that

$$Om_1 = \frac{\frac{b^2(1+z)^6}{2H_0^2(3-3c^2\omega_0)} + A(1+z)^{3+3c^2\omega_0} - 1}{(1+z)^3 - 1}, \tag{3.3.15}$$

$$Om_2 = \frac{\frac{b^2(1+z)^{\frac{6-11c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}}}{2H_0^2\left(\frac{3-3c^2\omega_0-8c^2\omega_1}{1-\frac{3}{2}c^2\omega_1}\right)} + B(1+z)^{\frac{3+3c^2\omega_0-3c^2\omega_1}{1-\frac{3}{2}(c^2\omega_1)}} - 1}{(1+z)^3 - 1}. \tag{3.3.16}$$

In Figure 3.6, we draw the Om-diagnostic versus z for ω_{1d} in left plot and for ω_{2d} in right plot. It can be observed that the trajectories of Om-diagnostic for both cases represent the negative slopes at past epoch which implies the quintessence era while give positive slopes at future epoch which constitutes the phantom era of the universe.

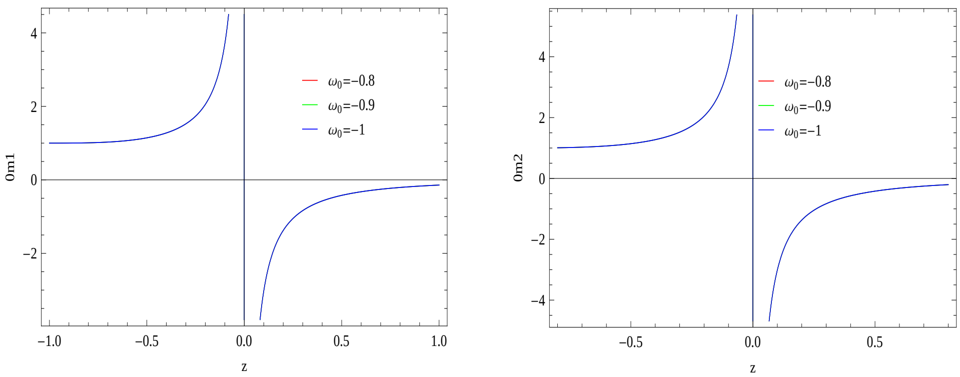

Figure 3.6: Plot of Om_1 for ω_{1d} and Om_2 for ω_{2d} corresponding to z.

Chapter 4
Cosmological Implications of Equation of State Parameter from Thermodynamical Constraints and Holographic Equipartition Law

This chapter contains the discussion on thermodynamics including holographic equipartition law and GSLT in the presence of Bekenstein-Hawking entropy and Sharma Mittal entropy.

4.1 Bekenstein-Hawking and Sharma Mittal Entropy

In this section, we discuss two different entropy. The Bekenstein-Hawking entropy [20] is basically used as a standard scale of a connected entropy on the Hubble horizon of the universe. The Bekenstein-Hawking entropy can be defined as

$$S_{BH} = \frac{k_b d^3 A}{4\hbar G}, \tag{4.1.1}$$

Substituting the value of A in Eq.(4.1.1), we get [17]-[19],[30]

$$S_{BH} = \frac{\pi k_b d^5}{\hbar G H^2} = \frac{K}{H^2}. \tag{4.1.2}$$

The rate of change of this entropy is given by

$$\dot{S}_{BH} = -\frac{2K\dot{H}}{H^3}. \tag{4.1.3}$$

From different observations, it can be seen that $\dot{H} < 0$ and $H > 0$ [55]. For the Bekenstein-Hawking entropy, the second law of thermodynamics must satisfy $\dot{S}_{BH} > 0$. Substituting the value of A in Eq.(2.11.3) and then by using $r = \frac{d}{H}$ and $r_0 = \frac{d}{H_0}$, we obtain

$$S_{SM} = \frac{1}{R}\left((1 + \frac{\psi}{H^2})^{\frac{R}{\delta}} - 1\right), \tag{4.1.4}$$

here ψ is a dimensionless parameter considered to be a constant and given by $\psi = \delta\pi r_0^2 H_0^2$. In this scenario, for an accelerating universe ψ is assumed to be positive. At present time, H_0 and r_0 are the Hubble parameter and radius respectively. Consequently, the entropy S_H on the horizon of the universe can be rewritten from Eq.(4.1.4), such that $S_H = S_{SM}$. We consider rate of change of S_H for the second law of thermodynamics. Differentiating above relation with respect to t and by using Eq.(4.1.3), we have

$$\dot{S}_H = \frac{\dot{S}_{BH}\psi}{K\delta}(1 + \frac{\psi}{H^2})^{\frac{R}{\delta}-1}, \tag{4.1.5}$$

here $\dot{S}_{BH} > 0$ as discussed above. Similarly for the satisfaction of $\dot{S}_H > 0$, we require

$$\frac{\psi}{\delta K}(1 + \frac{\psi}{H^2})^{\frac{R}{\delta}-1} > 0. \tag{4.1.6}$$

Substituting $\psi = \delta\pi r_0^2 H_0^2$ into Eq.(4.1.6) and then by using the value of r and r_0, we obtain an equivalent inequality

$$\frac{\pi r_0^2 H_0^2}{K}(1 + \delta\pi r^2)^{\frac{R}{\delta}-1} > 0. \tag{4.1.7}$$

In cosmological equations we can examine the order of an extra driving term by using these constraints.

4.2 Holographic Equipartition Law

In this section, we introduce Padmanabhan's holographic equipartition law [23]. We can write the Hubble volume $V = \frac{4\pi r^3}{3}$ by using value of r, such that

$$V = \frac{4\pi d^3}{3H^3}. \tag{4.2.1}$$

Taking derivative of above relation with respect to t, we have

$$\frac{dV}{dt} = -\frac{4\pi d^3 \dot{H}}{H^4}. \tag{4.2.2}$$

For the fulfilment of equipartition law of energy [23] the number of degrees of freedom can be considered as $N_{bulk} = \frac{|E|}{\frac{1}{2}k_B T}$, where $|E|$ is the Komar energy included inner the Hubble volume V is defined as

$$|E| = V|(\rho d^2 + 3p)| = -V\epsilon|(\rho d^2 + 3p)|, \tag{4.2.3}$$

where ρ is the mass density of the cosmological fluids and p is the pressure of the cosmological fluids [30]. In case of an accelerating universe the parameter $\epsilon = +1$ and $\rho d^2 + 3p < 0$ while in a decelerating universe $\epsilon = -1$ and $\rho d^2 + 3p > 0$ [23]. On the horizon the temperature T is written as

$$T = \frac{\hbar H}{2\pi k_B}. \tag{4.2.4}$$

Over the spherical surface the number of degrees of freedom can be defined as

$$N_{sur} = \frac{4S_H}{k_B}, \tag{4.2.5}$$

here S_H represents the entropy of the Hubble horizon [30]. For S_H different type of entropy can be used. In the following section, S_H is used as Sharma Mittal entropy which is given by Eq.(**??**). Now from the holographic equipartition law we calculate an accelerating equation. In this thesis, $\rho d^2 + 3p < 0$ is selected [23] so, $\epsilon = +1$ as discussed above. Using Eqs.(4.2.1), (4.2.3) and (4.2.4) in (4.2.6), we obtain

$$N_{bulk} = -\frac{16\pi^2 d^5}{3hH^4}\left(\rho + \frac{3p}{d^2}\right). \tag{4.2.6}$$

Additionally, by substituting $\epsilon = +1$, $L_p = \sqrt{\frac{hG}{d^3}}$ and Eqs.(2.10.4), (4.2.2) and (4.2.5) in (4.2.6) and then by solving the obtain resultant with regard to $\dot{H}$ [30], we have

$$\dot{H} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{d^2}\right) - \frac{S_H H^4}{K}. \tag{4.2.7}$$

Using above relation and $S_{BH} = \frac{K}{H^2}$ into $\frac{\ddot{a}}{a} = \dot{H} + H^2$ we get the accelerating equation in the following form

$$\frac{\ddot{a}}{a} = H^2\left(1 - \frac{S_H}{S_{BH}}\right) - \frac{4\pi G}{3}\left(\rho + \frac{3p}{d^2}\right). \tag{4.2.8}$$

The first term on the right-hand side of above relation is zero [23] when $S_H = S_{BH}$ and vise versa. Therefore an additional driving term appears. This driving term depends on the variation of S_H from S_{BH}.

4.3 Holographic Equipartition Law with Sharma Mittal Entropy

In this section, the Sharma Mittal entropy given by Eq.(**??**) is applied to holographic equipartition law. Substitution of Eq.(2.12.1) in (4.2.8) gives the following relation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{d^2}\right) + H^2 - \frac{H^4}{K}\left(\frac{1}{R}(1 + \frac{\psi}{H^2})^{\frac{R}{\delta}} - 1\right). \tag{4.3.1}$$

The second term on the right hand side is an extra driving term and proportional to H. For an accelerating universe a positive second term is required. Similarly $\psi > 0$ is needed because for an expanding universe $H > 0$ is required. In this thesis, within time varying $\Lambda(t)$ cosmologies [30] present model is assumed to be a special case of $\Lambda(t)$CDM models. Similarly the Friedmann and acceleration equations can be written as

$$H^2 = \frac{8\pi G\rho}{3} + f(H), \tag{4.3.2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{d^2}\right) + f(H). \tag{4.3.3}$$

Substitution of Eq.(4.3.2) in (4.3.3) give us the following continuity equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{d^2}\right) = -\frac{3\dot{f}(H)}{8\pi G}, \tag{4.3.4}$$

where $f(H)$ is an extra driving term defined as

$$f(H) = H^2 - \frac{H^4}{K}\left(\frac{1}{R}(1+\frac{\psi}{H^2})^{\frac{R}{\delta}} - 1\right). \tag{4.3.5}$$

For CDM models when $f(H)$ is constant than the right hand side of Eq.(4.3.4) becomes zero. On the other hand if $f(H)$ varies with time than the right hand side of Eq.(4.3.4) becomes nonzero. According to the holographic principle the nonzero right hand side implies a type of energy transfer between the boundary and the bulk. Using $\frac{\ddot{a}}{a} = \dot{H} + H^2$ and $p = \omega\rho$ and Eq.(4.3.5) in (4.3.3), we get

$$\begin{aligned}\frac{dH}{dz} &= \frac{3}{2d^2(1+z)}\left(\omega H + d^2 H - \left(H - \frac{H^3}{K}\left(\frac{1}{R}(1+\frac{\psi}{H^2})^{\frac{R}{\delta}} - 1\right)\right)\right.\\ &\times \left.(3\omega + \frac{3}{2d^2})\right). \end{aligned} \tag{4.3.6}$$

4.4 Cosmological Parameters

Substituting the parameterizations defined in above section in Eq.(4.3.6) we obtain the following equations for model 1 and 2 respectively

$$\begin{aligned}\frac{dH}{dz} &= \frac{3}{2d^2(1+z)}\left(\omega_0 H + d^2 H - \left(H - \frac{H^3}{K}\left(\frac{1}{R}(1+\frac{\psi}{H^2})^{\frac{R}{\delta}} - 1\right)\right)\right.\\ &\times \left.(3\omega_0 + \frac{3}{2d^2})\right), \end{aligned} \tag{4.4.1}$$

$$\begin{aligned}\frac{dH}{dz} &= \frac{3}{2d^2(1+z)}\left(d^2 H - \left(H - \frac{H^3}{K}\left(\frac{1}{R}(1+\frac{\psi}{H^2})^{\frac{R}{\delta}} - 1\right)\right)\right.\\ &\times \left.(3\omega_0 + 3\omega_1 q + \frac{3}{2d^2}) + (\omega_0 + \omega_1 q)H\right). \end{aligned} \tag{4.4.2}$$

Next, we establish some cosmological parameters (deceleration parameter, stability analysis, statefinder parameters and Om-diagnostic) by using a parameterizations of ω.

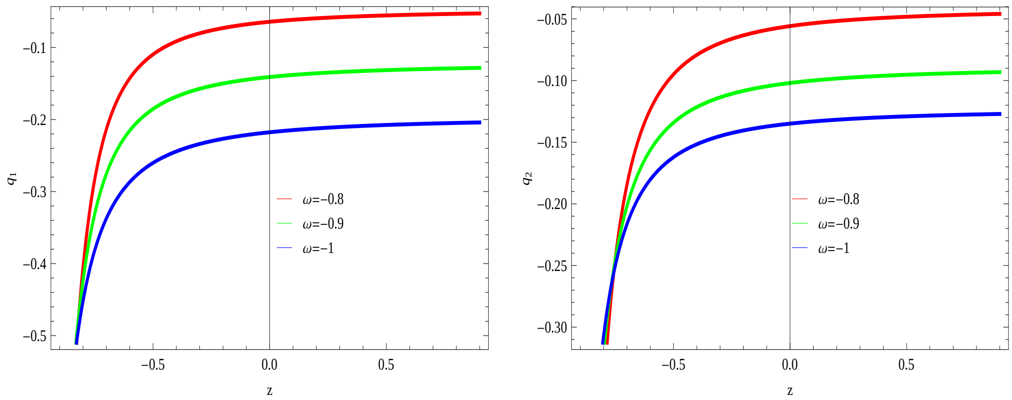

Figure 4.1: Plot of q_1 for model 1 and q_2 for model 2 corresponding to z taking $(\omega_0, \omega_1) = (-0.8, -0.2), (-0.9, -0.5), (-1, -0.8)$.

4.4.1 Deceleration Parameter

Inserting $\dot{H} = -H(1+z)\frac{dH}{dz}$ in Eq.(3.3.1), we have

$$q = -1 + \frac{(1+z)\frac{dH}{dz}}{H}. \tag{4.4.3}$$

Substituting Eqs.(4.4.1) and (4.4.2) in above relation we get the deceleration parameter for model 1 and 2 respectively.

• Figure 4.1 (left) represents the plot of deceleration parameter versus z for model 1 at different values of $\omega_0 = -0.8, -0.9, -1$. The specific values for the other constants are $K = 1,\ H_0 = 65,\ d = 2,\ \delta = -100,\ r_o = 0.01$ and $R = 1$. The graph describes that all values of ω_0 give accelerated region as $q \in [-1, 0)$ in all epochs.

• The plot of deceleration parameter for model 2 is shown in Figure 4.1 (right) corresponding to z for three different values of ω_0. The particular values of other constants are same as in above case. In this case, the deceleration parameter for $z > 0$ exhibits accelerated phase. This parameter constitutes the accelerated phase for future and past epochs.

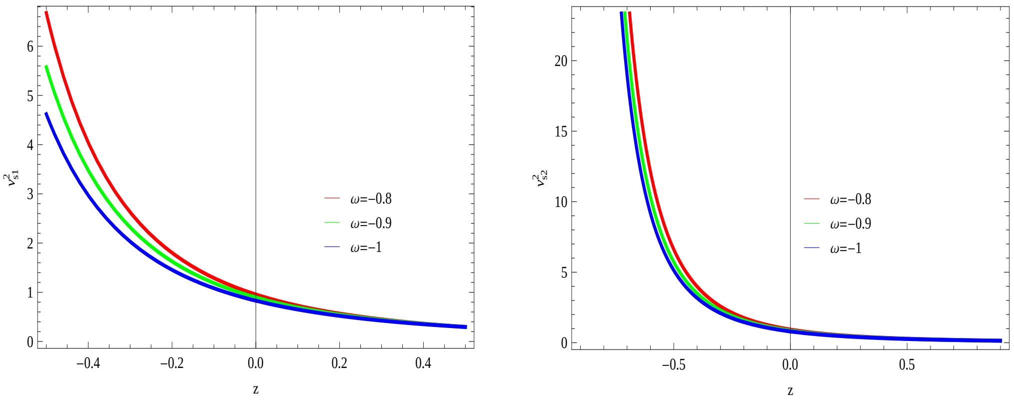

Figure 4.2: Plot of v_{s1}^2 for model 1 and v_{s2}^2 for model 2 corresponding to z taking $(\omega_0, \omega_1) = (-0.8, -0.2), (-0.9, -0.5), (-1, -0.8)$.

4.4.2 Squared Speed of Sound

Inserting Eqs.(4.3.2) and (4.3.3) along with derivative with respect to z in Eq.(3.3.6), we have the squared speed of sound as follows

$$\begin{aligned}
v_s^2 &= \Bigg[8c^2k\psi(1+\frac{\psi}{H^2})^{\frac{R}{\delta}-1}\frac{dH}{dz}\Bigg(H^2+3\Bigg(H^2-\frac{(-1+\frac{(1+\frac{\psi}{H^2})^{\frac{R}{\delta}}}{R})H^4}{K}\Bigg)\Bigg)\\
&+ 2H(1+z)\frac{dH}{dz}\Bigg]\Bigg[3\delta H^7\Bigg(-1+\frac{(1+\frac{\psi}{H^2})^{\frac{R}{\delta}}}{R}\Bigg)^2\Bigg]^{-1}\Bigg[-16c^2k\frac{dH}{dz}\Bigg(H^2\\
&+ 3\Bigg(H^2-\frac{(-1+\frac{(1+\frac{\psi}{H^2})^{\frac{R}{\delta}}}{R})H^4}{K}\Bigg)+2H(1+z)\frac{dH}{dz}\Bigg)\Bigg]\Bigg[(-1+\frac{(1+\frac{\psi}{H^2})^{\frac{R}{\delta}}}{R})\\
&\times 3H^5\Bigg]^{-1}+\frac{4c^2k}{3H^4\Bigg(-1+\frac{(1+\frac{\psi}{H^2})^{\frac{R}{\delta}}}{R}\Bigg)}\Bigg[4H\frac{dH}{dz}+2(1+z)\Bigg(\frac{dH}{dz}\Bigg)^2\\
&+ 6H\frac{dH}{dz}+\frac{6\psi H\frac{dH}{dz}(1+\frac{\psi}{H^2})^{\frac{R}{\delta}-1}}{K\delta}+2(1+z)H\frac{d^2H}{dz^2}-\Bigg(12H^3\frac{dH}{dz}(-1\\
&+ \frac{(1+\frac{\psi}{H^2})^{\frac{R}{\delta}}}{R})\Bigg)(K)^{-1}\Bigg].
\end{aligned} \tag{4.4.4}$$

Using Eqs.(4.4.1) and (4.4.2) in above relation we get the squared speed of sound for model 1 and 2 respectively.

• The plot of v_s^2 for model 1 is shown in Figure 4.2 (left). It can be seen that the trajectories of squared speed of sound shows the positive behavior

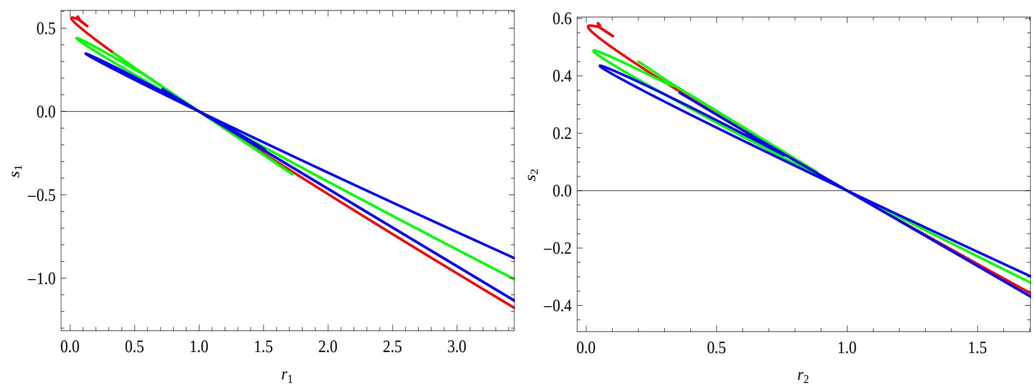

Figure 4.3: Plot of r_1 corresponding to s_1 for model 1 and r_2 corresponding to s_2 for model 2.

for all value of z which gives the stability of model.

• The plot of v_s^2 for model 2 is shown in Figure 4.2 (right). We can observe that all the trajectories constitute positive behavior for all epochs which implies the stability of model.

4.4.3 Statefinder Parameters

Substituting $\dot{H} = -H(1+z)\frac{dH}{dz}$ and $\ddot{H} = H(1+z)^2(\frac{dH}{dz})^2 + H^2(1+z)\frac{dH}{dz} + H^2(1+z)^2(\frac{d^2H}{dz^2})^2$ into Eq.(3.3.9), we get the following results

$$r = 1 - \frac{3(1+z)\frac{dH}{dz}}{H} + \frac{(1+z)^2(\frac{dH}{dz})^2}{H^2} + \frac{(1+z)\frac{dH}{dz}}{H} + \frac{(1+z)^2(\frac{d^2H}{dz^2})^2}{H}, \tag{4.4.5}$$

$$s = \left(-\frac{3(1+z)\frac{dH}{dz}}{H} + \frac{(1+z)^2(\frac{dH}{dz})^2}{H^2} + \frac{(1+z)\frac{dH}{dz}}{H} + \frac{(1+z)^2(\frac{d^2H}{dz^2})^2}{H}\right)\left(+\frac{3(1+z)\frac{dH}{dz}}{H} - \frac{9}{2}\right)^{-1}. \tag{4.4.6}$$

Inserting Eqs.(4.4.1) and (4.4.2) in above relation we get the statefinder parameters for model 1 and 2 respectively.

• In Figure 4.3 (left), the graph of s_1 displayed against r_1 for three different values of ω_0. We can notice that the (r, s) parameters corresponds to

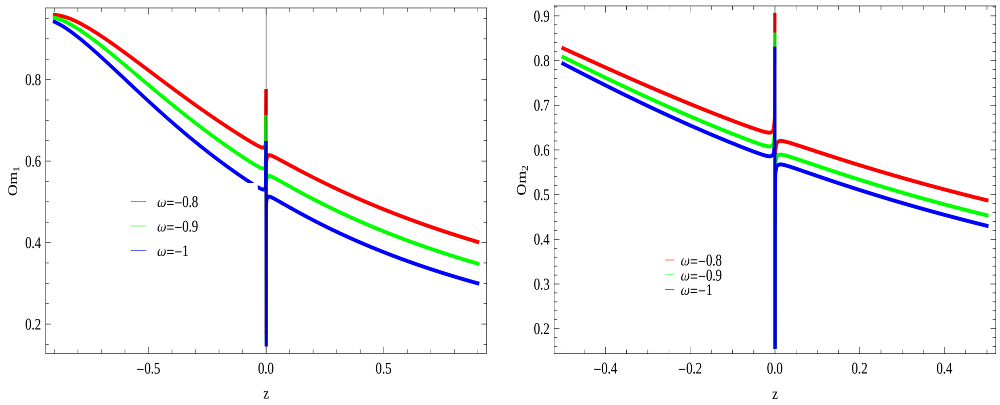

Figure 4.4: Plot of Om_1 for model 1 and Om_2 for model 2 corresponding to z.

Chaplygin gas behavior for all trajectories. At $(r, s) = (1, 0)$ these parameter represent the ΛCDM limit. These parameters also constitute phantom and quintessence regions.

• In Figure 4.3 (right), the graph of s_2 displayed against r_2 for three different values of ω_0. We can observe that the (r, s) parameters corresponds to Chaplygin gas behavior for the underlying scenario. Also these parameters constitutes the phantom and quintessence regions and ΛCDM limit.

4.4.4 Om-diagnostic

Using Eqs.(4.4.1) and (4.4.2) in Eq.(3.3.14), we get the Om-diagnostic for model 1 and 2 respectively.

• In Figure 4.4 (left), we draw the plot of Om-diagnostic for model 1. We can observe that all the trajectories represent positive slopes for all epochs which constitutes the phantom era of the universe.

• The plot of Om-diagnostic for model 2 is drawn in Figure 4.4 (right). It can be observed that the trajectories of Om-diagnostic represent positive slopes at all epochs which implies the phantom era of the universe.

4.5 Generalized Second Law for Present Model

In this section, we analyze for the present model the generalized second law of thermodynamics. For this both entropy are considered [58] i.e, S_m, the entropy of matter inner the horizon and S_H, the Sharma Mittal entropy over the Hubble horizon. Therefore the total entropy S_t can be written as

$$S_t = S_m + S_H. \tag{4.5.1}$$

The rate of change of S_H is given by Eq.(4.1.5), whereas for the present model the rate of change of S_m can be evaluated from the first law of thermodynamics. For this we observe change of entropy of matter inner the horizon. Firstly we assume a closed system carrying a continual number of molecules in volume V. First law of thermodynamics states that dQ is the flow of heat over a region through a time interval dt, it can be written as

$$dQ = dE = pdV, \tag{4.5.2}$$

where dE represents the change of internal energy E and dV shows the change of the volume of region [59]. Differentiating above relation with respect to t and calculating different operations [1], we get

$$\frac{dQ}{dt} = (\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + \frac{p}{c^2}))d^2(\frac{4\pi r^3}{3}). \tag{4.5.3}$$

Additionally, $\frac{dQ}{dt}$ is considered to be connected to reversible entropy [60] and dS the change of entropy can be expressed as

$$dS = \frac{dQ}{T}. \tag{4.5.4}$$

We get continuity equation from Eq.(4.1.5), given by $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + \frac{p}{d^2}) = 0$, if we assume the adiabatic process. In present scenario, we assume the flow of energy over the Hubble horizon since present model is assumed to be a special case of $\Lambda(t)$CDM models so the continuity equation is given by Eq.(4.1.5). When the extra driving term $f(H)$ is constant like then energy flows slowly. From Eq.(4.1.5), it is assumed that the temperature of the Hubble horizon T is equivalent to temperature of matter T_m [58]. Since the flow of energy across the region are assumed to be small. So the rate of change of entropy can be written by using Eqs.(4.5.3) and (4.5.4), such that

$$\dot{S}_m == \frac{1}{T}\left(\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + \frac{p}{d^2})\right)d^2(\frac{4\pi r^3}{3}). \tag{4.5.5}$$

From section 4, it can be observed that in $\Lambda(t)$CDM models, trivial flows of energy are accommodated [6, 8]. Therefore the flow of small energy assumed here is relatable with those works. Substitution of $r = \frac{d}{H}$ and Eqs.(2.10.4), (4.2.4) and (4.3.4) in above relation give us the following result

$$\dot{S}_m = -\frac{\dot{f}K}{H^4}. \tag{4.5.6}$$

Additionally by using Eq.(4.1.3) and (4.3.5) in above relation we get

$$\dot{S}_m = \dot{S}_{BH} - 4\frac{\dot{H}}{H} + (1 + \frac{\psi}{H^2})^{\frac{R}{\delta}}(\frac{4\dot{H}}{RH} + \frac{2\dot{H}\psi}{\delta H^3}). \tag{4.5.7}$$

It is observed that by using the Sharma Mittal entropy we examine modified Friedmann equations.

$$\begin{aligned}\dot{S}_t &= \dot{S}_H + \dot{S}_m = \frac{\dot{S}_{BH}\psi}{K\delta}(1 + \frac{\psi}{H^2})^{\frac{R}{\delta}-1} + \dot{S}_{BH} - 4\frac{\dot{H}}{H} + (1 + \frac{\psi}{H^2})^{\frac{R}{\delta}} \\ &\times (\frac{4\dot{H}}{RH} + \frac{2\dot{H}\psi}{\delta H^3}) \end{aligned} \tag{4.5.8}$$

Substituting Eqs.(4.4.1) and (4.4.2) in above relation we get the $\dot{S}_t$ for model 1 and 2 respectively.

- In Figure 4.5(left), the graph of $\dot{S}_{1t}$ for model 1 is drawn. Since the

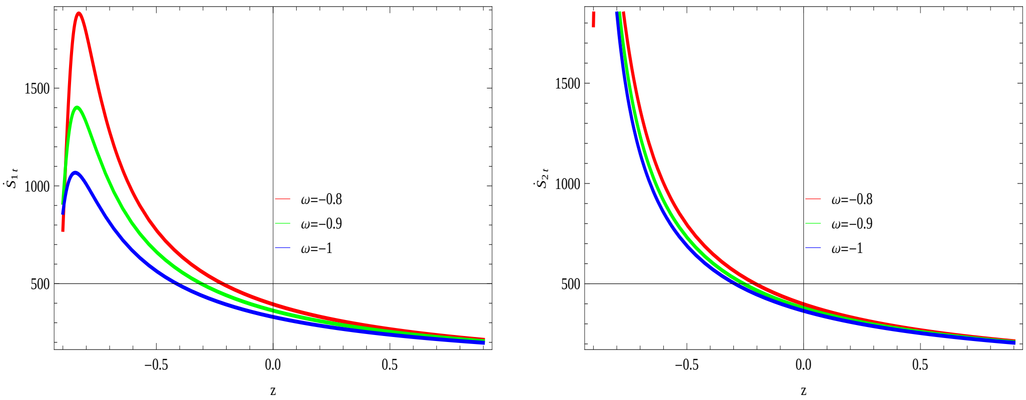

Figure 4.5: Plot of $\dot{S}_{1t}$ for model 1 and $\dot{S}_{2t}$ for model 2 corresponding to z taking $(\omega_0, \omega_1) = (-0.8, -0.2), (-0.9, -0.5), (-1, -0.8)$.

trajectories of $\dot{S}_t$ shows positive range for all epochs i.e, $\dot{S}_t > 0$, which implies the existence of model.

- The graph of $\dot{S}_{2t}$ for model 2 is displayed in Figure 4.5(right). The trajectories of the graph constitutes positive range which shows that model 2 holds.

Chapter 5
Summary

In this thesis, we have assumed the flat FRW spacetime and discussed different DE models by using a collection of observations at low redshift in the framework of dynamical Chern-Simons modified gravity. We have taken the parameterizations of EoS parameter to explore the cosmic evolution of accelerating universe in interacting scenario. We have evaluated the different cosmological parameters such as deceleration parameter, squared speed of sound, Om-diagnostic and statefinder parameters. The trajectories of the constructed models have been plotted with different constant parametric values.

We have examined thermodynamically an additional driving term in the cosmological equations for a flat FRW universe at ancient eras by applying the Sharma Mittal entropy on the Padmanabhan's holographic equipartition law. From the holographic equipartition law an additional driving term in the acceleration equation can be extracted because of a deviation from the Bekenstein-Hawking entropy. Interestingly, the extracted term obtained from the acceleration equation is initiate to be constricted by the second law of thermodynamics. To discuss thermodynamic constraints we have used generalized second law of thermodynamics i.e, $\dot{S}_t = \dot{S}_m + \dot{S}_H$. However, an identical constraint can be prevailed directly from $\dot{S}_H > 0$, without using $\dot{S}_t$.

In chapter 1, the interaction term represented the positive behavior and is observationally verified that the interaction term must not change its sign during cosmic evolution. The deceleration parameter indicated the consistent result while squared speed of sound expressed some stable solutions. The statefinder parameters for first parametrization represented Chaplygin gas model behavior and met the ΛCDM limit for specific choice of parameters and DE era is obtained for second choice of parametrization. The Om diagnostic parameter indicated the phantom and quintessence eras of the universe.

In chapter 2, the deceleration parameter indicated the consistent result while squared speed of sound expressed stable solutions. The statefinder parameters for first parametrization represented Chaplygin gas model behavior, phantom and quintessence regions and met the ΛCDM limit for specific choice of parameters and DE era , ΛCDM limit and Chaplygin gas model is obtained for second choice of parametrization. The Om diagnostic parameter indicated the phantom era of the universe.

Chapter 6
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