Two Population Neural Field Model With Gaussian External Input

By

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Declaration

I, **Sana Javed** with registration number **CIIT/FA17-RMT-006/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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Certificate

It is certified that **Sana Javed** with **CIIT/FA17-RMT-006/LHR** has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS Institute of Information Technology, Lahore campus and the work fulfills the requirement for award of MS degree.

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ABSTRACT

Abstract

In this, we have explored Gaussian spatio-temporal external input on persistent activity states. The aim of work was to build up a numerical scheme of action for two population model with Gaussian external input. We breakup the external input into 3 parts i.e., amplitude, the spatial part and the temporal part . In temporal part, we use Gaussian type external input. We have explored the boundedness, uniqueness for the present choice of work. We check the emergence then we examine the stationary solution called "bumps". This model is the continuation of Patrick et al [37] by extending the spatio-temporal Gaussian type external input. We have check thoroughly the broad bump solution. We also check the activity will converge, when the broad bump is less than τ_{cr}. The present choice of $H(x,t)$ produce better result as compare to Yousaf et al.

Chapter 1

Introduction

0.1 Nervous system

The nervous system is the set of nerves and neurons.Neurons can be separate from other cells in a number of ways. Neurons convey the signals between different parts of the body.

0.2 Human nervous system

Human nervous system basically consists of CNS and PNS.CNS functions to receive and send gestures from one part of the body to the others part of the body.[?][?] These two systems collaborate and obtain information from inside and outside of the environment.[4]

0.2.1 Human Brain

The human brain contains billions of nerve fibers (axon and dendrites) and nerve cells (neurons). It weighs about 1.5 kilograms.[5]

The human brain expands from three sections known as fore-brain, mid-brain and hind-brain.[5] [6]

- Forebrain
- Midbrain
- Hindbrain

1- The Forebrain

The fore-brain is the most recently flourished portion of our brain. It is the biggest part of the brain , most of which is cerebrum. Fore-brain consists of the thalamus , the hypothalamus and the limbic system. Cerebral cortex is the most important and complicated structure in our brain. It is grayish in colour. It is larger in size and have so many wrinkles.[3] Cerebrum is divided into two cerebral hemispheres associated by

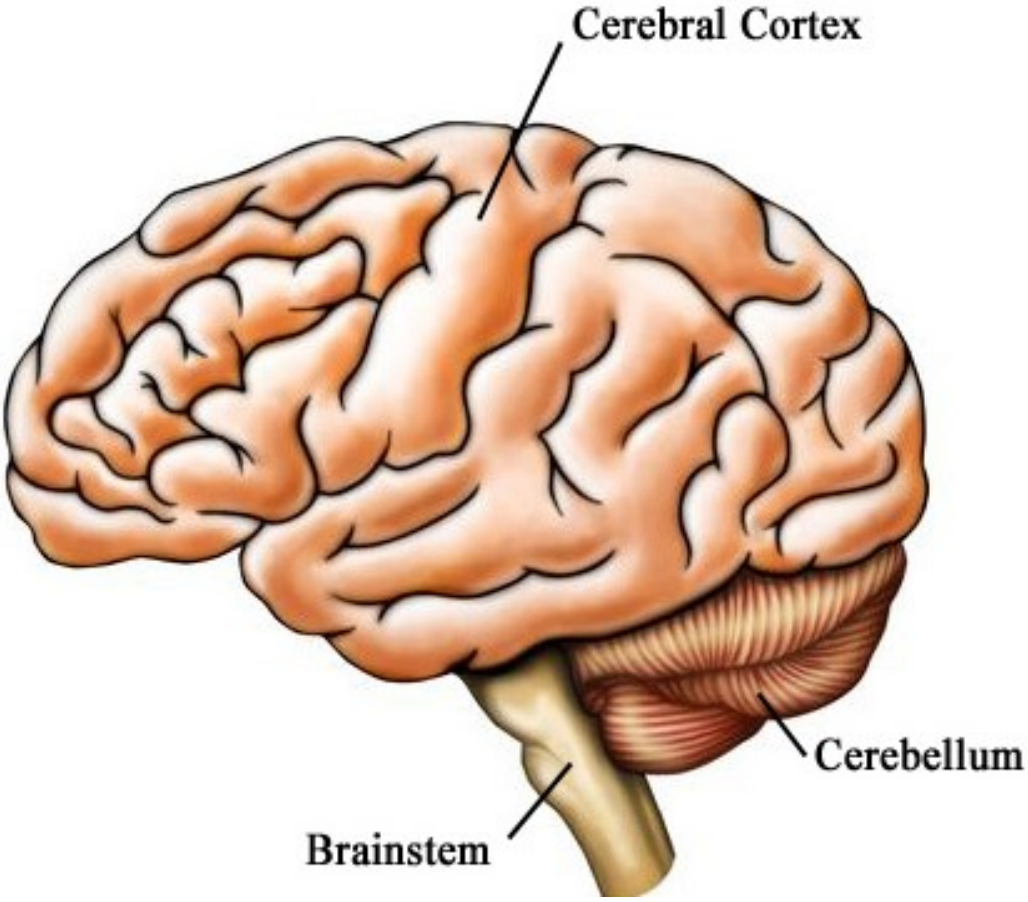

Figure 1: Brain Structure [8]

a mass of white matter. The outer surface of cortex is about 2 to 4 millimeters thick and it is enclosing round about 1.5 square meter surface area. Its work is combined with thought, realization, perception, awareness and remembrance. [10] [3]

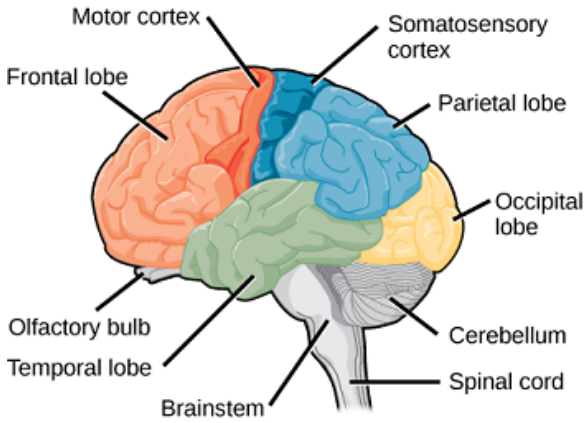

Figure 2: Cerebral Cortex

Each hemisphere is split into four lobes which are followings:

- Frontal lobe
- Perietal lobe
- Temporal lobe
- Occipital lobe
- Frontal lobe

The frontal lobe is part of the brain's 'cerebral cortex'. Individually, the paired lobes are acknowledged as the 'left and right' frontal cortex. The 'frontal lobe' is based near the 'front of the head', under the frontal 'skull bones' and near the 'forehead'. It commands important human skills like language 'comprehension', 'memory', 'decision making' etc. [11]

Most neuroscientists break the frontal lobe into four marked regions, each consists of a number of vital structures. Those include:

- Medial frontal lobe
- Lateral frontal lobe
- Polar region
- Orbital frontal lobe
- Perietal lobe

The 'parietal lobes' are arranged behind the frontal lobes and above the 'temporal lobes'. The parietal lobes are superior to the 'occipital lobes' and 'posterior' to the frontal lobes.It deliver message to other parts within seconds.[12]

- Temporal lobe

The 'temporal lobe' is based behind the left side of other lobes and in bottom middle part of 'cortex'. Its work is purely linked with 'hearing, listening and understanding sounds' having different pitches. A person without 'temporal lobe' cannot appreciate a meaningful talk. So this is very important lobe of 'human brain' due to which we are able to 'listen', 'understand' and 'talk about everything'.[13]

- Occipital lobe

The last one is 'occipital lobe' that is present in the 'back of head'. It makes a man to being able to 'correctly understand' and 'seeing things'. Related to the 'temporal lobe', makes sense of aural information, this makes sense of visual information.[14]

2- The Midbrain

The mid brain is based at the 'center of brain' between 'fore brain' and 'hind brain'. It is composed of brain stem which is base of the brain associated to 'spinal cord'. It plays important role in conveying message from brain to all other parts of body. It controls most important functions such as 'breathing', 'temperature', 'blood pressure', 'listening' and 'viewing'. [17] [18]

3- The Hindbrain

The hindbrain is one of the three major area of our 'brains', located at the lower back part of the brain.There are three main parts of the 'hindbrain - pons', 'cerebellum', and 'medulla oblongata'. The pons is the network between 'cortex and cerebellum'. The transmission of messages in all around body is possible due to 'pons'. Although it is very small in size but without pons, all body movements would not be possible.[28] [20]

Infect our 'body movements' is all due to 'brain'. It controls our 'thoughts', 'plans', actions even our body temperature and breathing. Its work is identify with 'communication of signals', comprehension that and after then decision making. [22]

0.2.2 'Brain Cells'

The main organ of 'human nervous system' is 'brain' and 'spinal cord'. Brain cells can be branched into 'two parts':

a. 'Neuroglia'

b. 'Neurons'

0.3 'Neuroglia'

Your 'central' and 'PNS' count on certain cells. These are sort of the unsung heroes of the 'nervous system'.There are four types of glial cells: 'astrocytes', 'oligodendrocytes', 'microglia', and 'ependymal cells'.These four essential functions of 'neuroglia' is to hold and surround neurons, supply 'nutrients' and oxygen to 'neurons', control

the ionic composition of extracellular space and to play the role of insulator between 'neurons' and remove the 'dead neurons'.[22]

0.4 Neurons

The 'human brain' is the main exchanging information center and this communication is only attainable due to nerve cells that are called 'brain cells' or 'neurons'. A 'human brain' consist of billions 'nerve cells' that have a work in conveying messages from one part of body to another. 'Nerve cells' are the fundamental units of the 'brain' and 'nervous system'.Neurons are cells within the nervous system that communicate information to other 'nerve cells', 'muscle', or 'gland cells'.Some are the 'sensory neurons' whose work is associated with 'touch', 'sound' and 'light'.[?][?] A neuron can be divided into three segments:

a. 'Dendrites'

b. 'Soma'(cell body)

c. 'Axon'

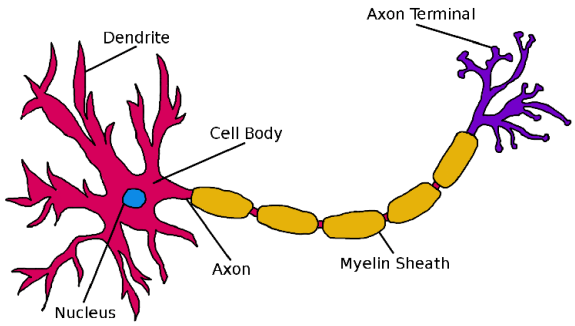

Figure 3: Neuron cell

Dendrites are bulges of a 'neuron' that receive signals (information) from other 'neurons'.Dendrites are not just the same in structure.They design directly from the

'soma' and branch extensively.The soma or the cell body is the 'non-process portion' of the neurons or the other brain cell types,containing the 'cell nucleus'. Soma is also called 'processing unit'. If the collected 'electrical potential in soma overshoots 'threshold value, it fires or generates an 'electrical signal' (called spike), which then moves along the axon to a connected target neurons.[24]

'Neurons' bring message to other part of body through junctions called 'synapses'.Synapse is a small gap contained three parts. The first one is 'presynaptic terminal' which contains 'neurotransmitter', that produces energy in the cell and cell bodies called 'organelles'. The second part is 'postsyanaptic terminal' that contains receptors for 'neurotransmitter'. The last one is the space between 'presynaptic' and 'postsyanaptic terminal'. Thus the presynaptic terminal of one neuron is connected to postsyanaptic terminal of other neuron through this synaptic gap. [26]

Myelin is the necessary coating surrounded axon and helps neuron in transmitting signals quickly through long distances. [?]

Synapses have two forms i.e 'excitatory and inhibitory'. Excitatory synapses brings positive action in whole body while inhibitory decrease the firing action and cause negative action in cells. Excitatory neurons are make up of round about 80% of our settlement and try to bring the affected neurons nearby to the action potential while the remaining 20% are inhibitory and they take away neurons from action potential. [27]

0.5 Modeling Approaches

Brain gives signals from one part of brain to other parts. This process is interpreted by different mathematical models. There are many models available in literature .

A model is a mathematical state planned to conclude and explain the basic biological process. A mathematical model precise the signal transformation of neuron.Use of mathematical methods with large estimating power of computer in exploring the biological systems. There are many models available in literature.

0.6 Compartmental modeling

Compartmental modeling is a technique usually more combined with the electrical properties of neuron rather than chemical aspects. In this modeling the neuron is branched into different small subdivision. The compartmental model frequently used to call the transformation. Each compartment acts like a small circuit, in which current is carried by the ions. The current moving through the compartments (the ion channels) is proportional to the difference in voltage times the conductance. The following Fig 4 shows construction of compartmental modeling.

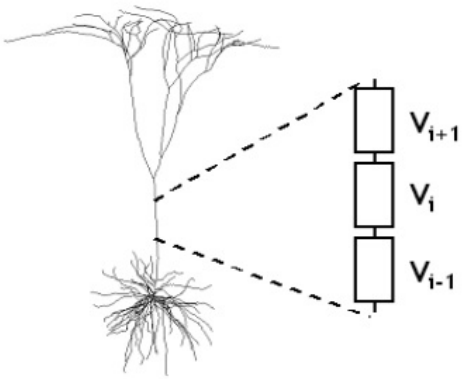

Figure 4: Construction of compartmental model for a neuron [22]

Mathematical equation for nth compartments is given from Kirchhoff's law of current as

$$\mathbf{g_{n,n+1}}(\mathbf{V_{n+1}}-\mathbf{V_n})-\mathbf{g_{n-1,n}}(\mathbf{V_n}-\mathbf{V_{n-1}})=\mathbf{k_n}\frac{\mathbf{dV_n}}{dt}+\sum_t \mathbf{I}_n^t+\sum_j \mathbf{I}_n^j \qquad (1)$$

Where the left term of equation is due to the ohmic current between nth and the other two neighboring sections. The first right term allows the membrane capacitance while the other term shows the current injected from the pre-synaptic neurons and the last term is due to the leak ion channels. From end to end these channels current depends on the absorption of certain ions within and outer of the membrane. [19] [29]

0.7 Point Neuron Models

Compartmental modeling have many disadvantages due to use of no. of parameters and pricey calculations. This made difficult to compute and rush these parameters. Then to overcome this intricacy, the model is reduced to a single compartment by removing a no. of compartments. In this way dendrites and soma distorted into a single point and neuron becomes a dimensionless component. In this model,their is no information of time period of action potential. Also, In this model their is no need of calculation of time course [19] [30]

It is a simple form for repetitive firing in neurons. It is just a point and when threshold is approached it fires spike of constant width and amplitude. [30]

Point neuron model is the simple point model that target on the sub threshold action and output of neuron. Since all action potentials of the neuron have same behavior with almost same amplitude and the time. [32].

The sub-threshold action and output of the neuron are focal point of this model. In a single section model, action potential sub-threshold dynamics of the membrane

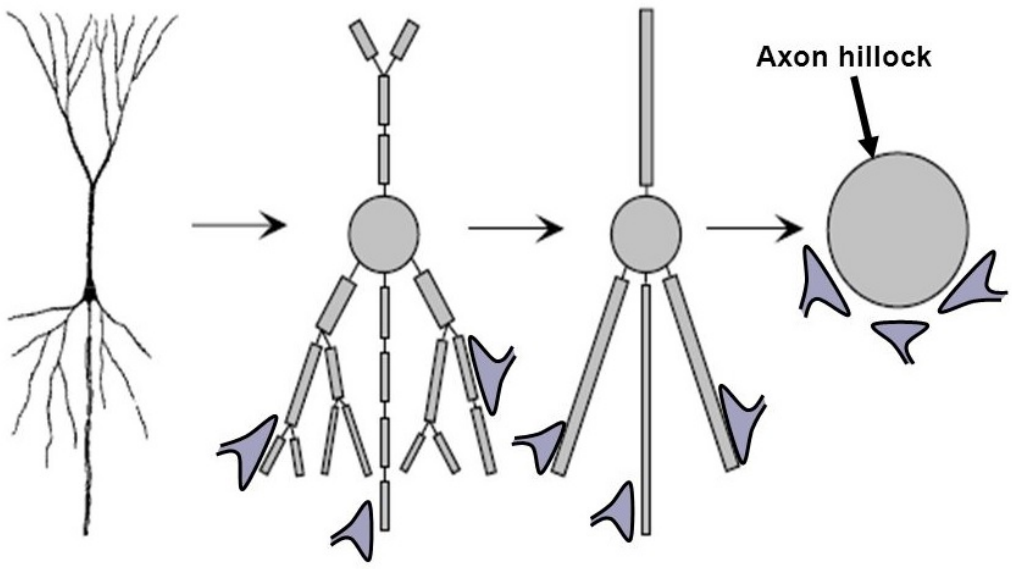

Figure 5: Conversion from compartmental models to point neurons [31]

is given as [33]

$$\mathbf{k}\frac{\mathbf{dV}}{dt} = -\mathbf{g_L}(\mathbf{V} - \mathbf{E_L}) - \sum_t \mathbf{I}^t \tag{2}$$

Here gL and EL depicts conductance and resting potential of the neuron, when input current is absent. In this model currents omitted between sections of a single neuron, instead we aim over only the ohmic leak current. But after converting compartmental modeling into point neuron, it is still not easy to come across critical result for the full dynamics of system for such simple spiking neurons, or even just the settlement rate dynamics. [19] [34]

0.8 Firing Rate Models

The spike sequence achieved by the neuron can be described by the neural functions, which is composed of sum of Dirac delta functions, located at the times when a neuron fires action potential. In firing rate models the exact description of activity level is

characterized by firing rate function in change of the neural response function r(t), where r(t) is the probability density of firing action potential and is obtained from the neural response function by averaging over trials.

A neural tissue has a large dynamics which repeatedly studied by a firing rate models. Firing rate model is actually analyzes that how group of neurons give firing response to the input signals and what effect produced to other neurons after this response. These models have been commonly applied in the examination of the network properties of the strongly interlinked cortical networks. [35].

1.) These models are grown in form of integro-differential and equations. It makes easier to study the properties of the solutions by using different mathematical techniques developed in applied mathematics.

2.) These models have less free parameters than spiking models, appropriate tuning of these free parameters parameters can be difficult.

3.) It is easier to work with these models, because in experiments spike trains vary.

Using these models make mathematical analysis of network behavior transparent and easier. [35].

1 Population Models:

- One population model
- Two population model

1.1 One population model :

The voltera form of one population model is

$$l = \alpha * \omega \bigotimes M(u - \theta) \tag{3}$$

then it is transformed into integro-differential equation i.e.,

$$\frac{\partial l_e}{\partial t} = -l_e(x',t) + \int_{-\infty}^{\infty} \omega(x'-x) M(u(x,t)-\theta) dx \tag{4}$$

Here 'l' is the activity level and 'P' is firing rate function . θ is the threshold value for firing potential . ω is the distance-dependent connectivity function .

1.2 Two population model:

The voltera form for two population excitatory and inhibitory neurons are as

$$l_e = \alpha_{ee} * \omega_{ee} \bigotimes M_e(l_e - \theta_e) - \alpha_{ie} * \omega_{ie} \bigotimes M_i(l_i - \theta_i) \tag{5}$$

$$l_i = \alpha_{ei} * \omega_{ei} \bigotimes M_e(l_e - \theta_e) - \alpha_{ii} * \omega_{ii} \bigotimes M_i(l_i - \theta_i) \tag{6}$$

with $\omega_{ee} \bigotimes M_m(l_m - \theta_m)$ defined as the 'spatial convolution integral'

$$\omega_{mn} \bigotimes M_m(l_m - \theta_m)(x,t) = \int_{-\infty}^{\infty} \omega_{mn}(y-x) M_m(l_m(y,t) - \theta_m)) dy, m, n = e, i. \tag{7}$$

and $\alpha * n$ defined as the temporal convolution integral

$$(\alpha_{mn} * n)(x,t) = \int_{-\infty}^{t} (\alpha_{mn}(t-s) n(x,s) ds, m, n = e, i. \tag{8}$$

TPM is expressed in detail in the next section.

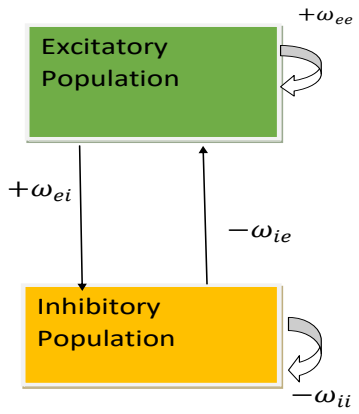

Figure 6: Two population model without external input

Chapter 2

Two Population Neural Field Model with Gaussian External Input

2 Two Population Model with Gaussian External input (TP-MGEI)

Neurons can be restricted as 'excitatory' and 'inhibitory' neurons. The right balance between 'excitatory' and 'inhibitory' cells in the brain is built. In simple terms there are two types of 'neurons' in the brain: those which 'increase activity' in other cells (excitatory neurons) and those which 'decrease activity' (inhibitory neurons)."Blomquist et al" acknowledges "TPM" comparable to "Wilson-Cowan type model" on the form [36] 12

$$\frac{\partial s_e}{\partial t} = -s_e + \omega_{ee} \bigotimes \mathrm{Z}_e(s_e - \theta_e) - \omega_{ie} \bigotimes \mathrm{Z}_i(s_i - \theta_i) + J_e \tag{9a}$$

$$\tau \frac{\partial s_i}{\partial t} = -s_i + \omega_{ei} \bigotimes \mathrm{Z}_e(s_e - \theta e) - \omega_{ii} \bigotimes \mathrm{Z}_i(s_i - \theta_i) + J_i \tag{9b}$$

Here the operation $\bigotimes$ determine the spatial convolution integral,likely as

$$\omega_{mn} \bigotimes \mathrm{Z}_m(\omega_m - \theta_m)(t,x) = \int_{-\infty}^{\infty} \omega_{mn}(x-x') Z_m(s_m(x',t) - \theta_m) dx' \tag{10}$$

After generalizing , we will get

$$\frac{\partial s_e}{\partial t} = -s_e(t,x) + \int_{-\infty}^{\infty} \omega_{ee}(x-x') Z_e(s_e(t,x') - \theta_e) dx' - \int_{-\infty}^{\infty} \omega_{ie}(x-x') Z_i(s_i(t,x') - \theta_i) dx' + J_e(t,x) \tag{11a}$$

$$\tau \frac{\partial s_i}{\partial t} = -s_i(t,x) + \int_{-\infty}^{\infty} \omega_{ei}(x-x') Z_e(s_e(t,x') - \theta_e) dx' - \int_{-\infty}^{\infty} \omega_{ii}(x-x') Z_i(s_i(t,x') - \theta_i) dx' + J_i(t,x) \tag{11b}$$

The parametres 'ω_e' and 'ωi' are the 'firing rate' .The function $J_m(m = i,e)$ model the 'spatiotemporal EI, the ratio between 'excitatory' and 'inhibitory time constants' is τ. The parameter θ_e and θ_i depicts threshold value for 'excitatory' and 'inhibitory' population.The functions P_m(m=i,e) are limited by a 'positive steepness specification' β_m(m=e,i).The functions 'ω_{mn}'(m,n=e,i) are labeled as 'connectivity functions'.

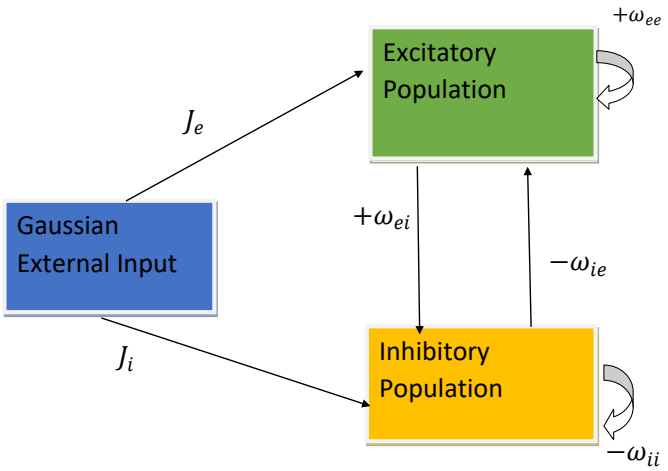

Figure 7: Two population model

These functions are 'positive', 'real valued','symmetric','normalized' and 'bounded' ($\int_{-\infty}^{\infty} \omega_{mn} = 1$). Z_e and Z_i are the 'firing rate function' for 'inhibitory' and 'excitatory population'. ($\int_{-\infty}^{\infty} h_m(t)dt = 1$)

We have

$\mathrm{Z}_m(u) = \frac{1}{2}(1 + \ tanh(\beta_m(u))$

For 'β_m' approaches to ∞ , the 'firing rate function' Z_m reaches the unit step function. In the present work, the model considered by "Blomquist et al" [37] with 'constant EI', Schematically the process is shown in the Fig. 7

The actual model reads

$$\frac{\partial s_e}{\partial t} = -s_e + \omega_{ee} \otimes \mathrm{Z}_e(s_e - \theta_e) - \omega_{ie} \otimes \mathrm{Z}_i(s_i - \theta_i) + \ J_e \tag{12a}$$

$$\tau \frac{\partial s_i}{\partial t} = -s_i + \omega_{ei} \otimes \mathrm{Z}_e(s_e - \theta e) - \omega_{ii} \otimes \mathrm{Z}_i(s_i - \theta_i) + \ J_i \tag{12b}$$

Here 'J_e, J_i' are the 'spatiotemporal EI' to the 'inhibitory' and 'excitatory populations' commonly, as long as the remaining variables are as defined previously. Categorization of the 'spatiotemporal EI' is $\mathrm{J}_m(t,x) = C_m\ S_m(x)\ h_m(t-t_{mo})$ where m=e,i. Here the function $J_m(m=i,e)$ represents the 'temporal dependence', S_m serve as the 'spatial dependence' and C_m the amplitudes.

"Yousaf et al" [22] talk about the TPM under 'no EI' and find the 'stability approaches' by using "Amari's" approach and the linearized stability analysis.They also examined existence of bumps as a function of 'amplitude' and fixed width parameter. "Blomquist et al" [37] and "Yousaf et al" [22] talk about the complication of singularity and existence of 'stationary symmetric solutions' and their stability properties by using both stability analysis.

"Yousaf et al" investigated the TPM with "transient spatio-temporal EI" and by using stability properties they found different results. They also investigate the number of bump pair as a function of 'amplitude'. We are approaching the work of "Yousaf et al" [22] about TPMGEI with "Gaussian temporal part". In this work, we will study the same problems within the framework of the model 12 with "Gaussian EI" under the condition of "Gaussian EI functions" J_e, J_i.

2.1 'Boundedness of the solutions'

Let us now assume that the 'external inputs' J_e and J_i are bounded. Bounding inequalities are

$$-s_e-1+\ J_e \leq \frac{\partial s_e}{\partial t} \leq\ -s_e+1+\ J_e \tag{13a}$$

$$-s_i-1+\ J_i \leq \tau\frac{\partial s_i}{\partial t} \leq s_i+1+J_i$$

for rate of change of s_e and s_i. Computation discloses the bounds which are

$$d_e(t,x) \leq\ s_e(t,x) \leq\ D_e(t,x) \tag{14a}$$

$$d_i(t,x) \leq\ s_i(t,x) \leq\ D_i(t,x) \tag{14b}$$

for s_e and s_i . So, the 'upper and the lower bounding functions' are:

$$d_e(t,x) = \alpha_e(t)(\ X_e(x)+1) - 1 + [\alpha_e * \ J_e](t,x) \tag{15a}$$

$$D_e(t,x) = \alpha_e(t)(\ X_e(x)-1) + 1 + [\alpha_e * \ J_e](t,x) \tag{15b}$$

$$d_i(t,x) = \tau\alpha_i(t)(\ X_i(x)+1) - 1 + [\alpha_i * \ J_i](t,x) \tag{15c}$$

$$D_i(t,x) = \tau\alpha_i(t)(\ X_i(x)-1) + 1 + [\alpha_i * \ J_i](t,x) \tag{15d}$$

and here X_e and X_i are conditions of the system and the 'temporal convolution' $\alpha_m * J_m$ is

$$[\alpha_m * \ J_m](t,x) = \int_0^t \alpha_m(t-s)\ J_m(x,s)\ ds \tag{16}$$

and

$$\alpha_m(t) = \frac{1}{\tau_m}\exp(-\frac{t}{\tau_m}) \tag{17}$$

where τ_m is

$$\tau = \begin{matrix} , & \text{m=e} \\ \tau, & m = i \end{matrix}$$

when J_m is given as

$$[\alpha_m * J_m](x,t) = C_m S_m(x) \int_0^t \alpha_m(t_{m0}-s) j_m(s-t_{mo}) ds \tag{18a}$$

$$[\alpha_m * J_m](x,t) = C_m S_m(x) \alpha_m(t-t_{m0}) \gamma_m(t-t_{mo}) \tag{18b}$$

where γ is

$$\gamma_m(t) = \frac{1}{\rho(\sqrt{2}\pi)} e^{-0.5(\frac{t-\mu}{\rho})^2} \tag{19}$$

After some calculations, we will get

$$\alpha_m * J_m(x,t) = \frac{1}{(\rho)(\sqrt{2\pi})} te^{(\frac{s}{t} + \frac{x^2}{\rho_m})} + C \tag{20}$$

After applying limits , we will get

$$\alpha_m * J_m(x,t) = -\frac{1}{\rho_m \sqrt{2\pi}} (e^0 - e) te^{\frac{x^2}{\rho_m}} \tag{21}$$

An valuable conclusion is that the 'bounding inequalities' 2.1 can be transformed to

$$s_e(x,t) - S_e(x,t)| \leq Q(t) \tag{22}$$

$$s_i(x,t) - S_i(x,t)| \leq Q(\frac{t}{\tau}) \tag{23}$$

and F is characterized as

$$F(t) = 1 - exp(-t) \tag{24}$$

S_e and S_i are given as

$$S_e(t,x) = \alpha_e(t) X_e(x) + [\alpha_e * J_e](t,x) \tag{25}$$

$$S_i(t,x) = \tau \alpha_i(t) X_i(x) + [\alpha_i * J_i](t,x) \tag{26}$$

Let us assume that

$$\lim_{t \to \infty} [\alpha_m * J_m(x,t)| = 0 \tag{27}$$

We conclude that $(S_e, S_i) \implies (0,0)$ as $t \implies \infty$ uniformly in x. We get the asymptotic bounds

$$|s_m(x,t) \implies \infty| \leq 1. \tag{28}$$

This indicates that the "initial value problem" is well-posed.

Chapter 4

"Numerical Results"

3 "Numerical Results":

We examine the "emergence of activity" in feedback of of external input in TPMGEI. For this choice of the parameters there take place two stationary symmetric solutions (bumps) , the first is broad and the second one is narrow.[2], [1]. The stability of these bumps are rely on 'relative inhibition time' constant τ.

Firstly, we study the appearance of the activity.

In the first part if there is zero external input. The process of non-zero excitatory EI can stimulate the activity, even if the EI is turned off.The activity still remains stable.This picture shows the "emergence of activity" It is rely upon the 'spatio-temporal EI'.

3.1 'Reliance on temporal function'

We first check into thoroughly the reliance of the 'temporal part' $h_m(t)$,

The 'emergence' of the activity is in figure.

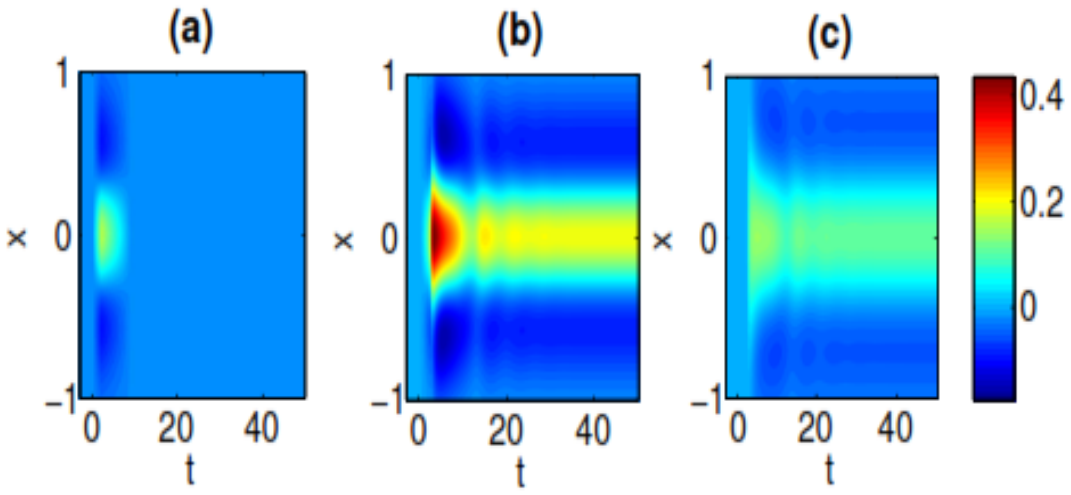

Figure 8: Appearance of the activity in result of the external input. The spatial function is fixed.

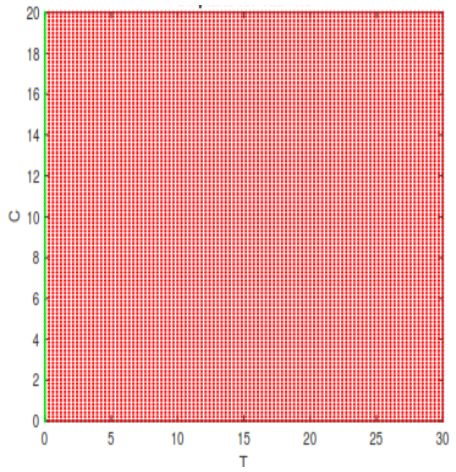

Figure 9: 'Appearance of activity for τ=1'

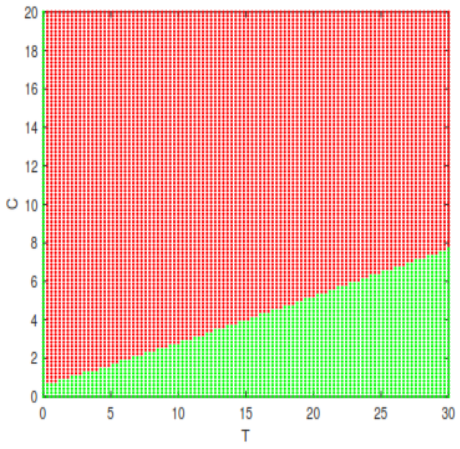

Figure 10: Appearance of activity for τ= 1.5

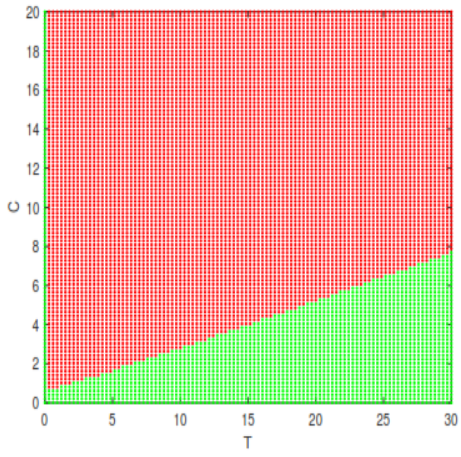

Figure 11: Appearance of activity for τ= 2

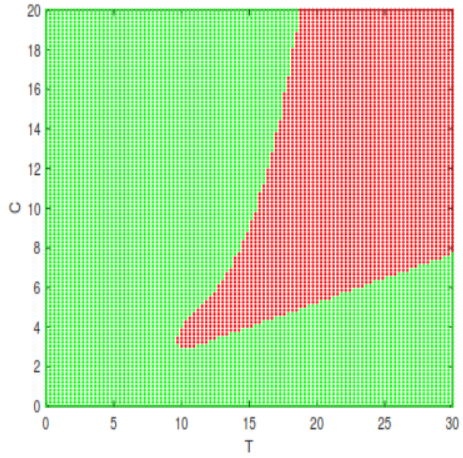

Figure 12: 'Generation of activity for τ= 2.5'

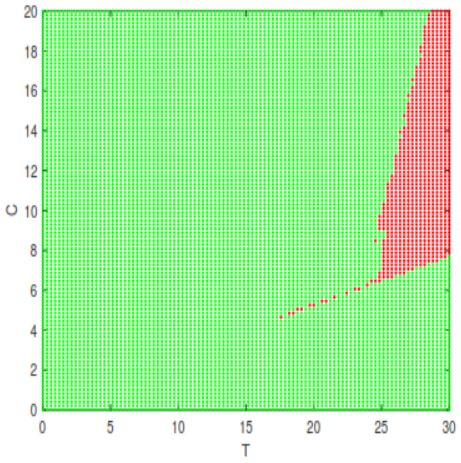

Figure 13: 'Generation of activity for τ= 3'

We noticed the 'strong dependence' for different values of relative tau , The 'parameter-plane' looks very much alike

When tau increases 2 to 2.5, there is a 'wide dissimilarity' in 'total duration plane' vs 'time duration' shown in Fig. For the 'time' T_e , there are three different C_e and from these three C_e , two of them leads to decline of 'bump-state generation'. and one intermediate C_e leads to successful generation and the successful generation will reduced, if come closer to τ_{cr}. The equivalent "pulse width coordinate planes" (PWC) planes are shown in Fig

In order to built a better insight, we plot the 'temporal progress' in the PWC plane. The 'red diamond' given the 'initial condition'. At this point when the 'inhibitory' is also stimulate. Then we notice the "inhibitory pulse width coordinate" increasing quickly . It moves towards the 'attractor' (red *). Then the established behaviour for the 'persistent activity' in the network and the slow inhibition for large value of τ shown in figure above. Our numerical imitations some of the results of "Yousaf et al".

Chapter 5

Conclusion and Discussion

4 Conclusion and Discussion

In this analysis,"Wilson Cowan type model' 12 stretched out 'spatio-temporal EI'.The limitations for this study are same as used by "Blomquist et al".We are in the parameter management where we know that there are 'narrow' and 'broad bump' pairs in the network. At first, there is no activity in the network. Later on, "Yousaf et al"[22] investigated same population model with 'spatial EI' . Also proved that both stability approaches does not generate same result.They also generated the number of bumps and found at-most four bump pair. The "present work"is the continuation for more 'smooth temporal function' Here, we have analyzed for 'Gaussian temporal function'. The conclusions of the present work can be compiled as follows:

1. We find out the TPNFM (12), under the effect of 'Gaussian EI'.
2. We will check thoroughly the 'emergence of persistent activity' as a function of TPNFMGEI .
3. We will check thoroughly the emergence of persistent activity as a function of Gaussian EI.
4. Emergence also depends on τ.
5. We have investigated the broad bump solution behaviour. We also investigate that the activity will converge, when the broad bump is less than τ_{cr}.
6. As we are increasing the value of τ, emergence is becoming a bit difficult to evoke the activity.
7. As we are increasing the total duration, we need larger values of amplitude.
8. We have investigated that as much as the activity is narrow or the external input is narrow, the EI will become stronger.
9. The present choice of H(x,t) produces better results as compare to Yousaf et al as we can see in table .

Chapter 4

"Appendix"

4.1 "Appendix A". "Runge–Kutta split step method"

We arrange a code for the 'initial value problem'. Here we will make use of the 'fourth order Runge–Kutta method'. Announce the 'sequence' $[t_j]_{j=0}^{N}$ where $t_0 \equiv 0$ and the 'time stepping length'. $\delta v_j \equiv v_{j+1} v_j$. Let D_j and $E_{(D_j)}$ be the '2D vector fields' give description as

Where h_j and i_j are given as

$$h_j = -p_e(x, v_j) + \phi \tag{29}$$

$$i_{j]=\frac{1}{\tau} p_1(x,v_j)+\frac{1}{\tau}\Psi_j} (30)$$

$$\phi_j = \Omega_{ee}(d(v_j)+x) + \Omega_{ee}(d(v_j)-x) - \Omega_{ie}(e(v_j)+x) - \Omega_{ie}(e(v_j)-x) + J_e(x,t) \tag{31}$$

$$\Psi_j = \Omega_{ei}(a(t_j)+x) + \Omega_{ei}(a(t_j)-x) - \Omega_{ii}(b(t_j)+x) - \Omega_{ii}(b(t_j)-x) + J_i(x,t) \tag{32}$$

and

$$\Omega_{mn}(x) = \int_0^x \omega_{mn}(y)dy = \int_0^{\frac{x}{\sigma_{mn}}} \phi_{mn}(y)dy \tag{33}$$

$$\omega_{mn}(x) = \sigma_{mn}^{-1}\phi_{mn}(\xi_{mn}), \xi_{mn} = \frac{x}{\sigma_{mn}} \tag{34}$$

The $d(v_j)$ and $e(v_j)$ approaching ϕ_j and Ψ_j satisfy the 'constraints'

$$p_e(d(v_j), v_j) = \theta_e, p_i(e(v_j), v_j) = \theta_i, \tag{35}$$

The 'Runge–Kutta code' is as follows:

1. Estimate the 'iteratively give description vector fields' $\overrightarrow{l_n}$,n= $1,2,3,4$

$$\overrightarrow{l_1} = \Delta v_j . \overrightarrow{E(D_j)} (36)$$

$\overrightarrow{l_2} = \Delta v_j . \overrightarrow{E}\,(\overrightarrow{D_j} + \frac{1}{2}\overrightarrow{L_1})(37)$

$\overrightarrow{l_3} = \Delta v_j . \overrightarrow{E}\,(\overrightarrow{D_j} + \frac{1}{2}\overrightarrow{L_2})(38)$

$\overrightarrow{l_3} = \Delta v_j . \overrightarrow{E}\,(\overrightarrow{D_j} + \overrightarrow{L_3})(39)$

2. Then $\overrightarrow{D_{j+1}}$ is given as

$\overrightarrow{D_{j+1}} = \overrightarrow{D_{j+1}} + \frac{1}{6}\overrightarrow{L_1} + \frac{1}{3}\overrightarrow{l_2} + \frac{1}{3}\overrightarrow{l_3} + \frac{1}{6}\overrightarrow{l_4}(40)$

Observe that iterates $p_n(x, v_j)$ and the widths $(d(v_j), e(v_j))$ in the "time stepping process'. The iteration action is as follows: • Calculate the initial widths $(d(0), e(0))$ and resolve

$$P_e(d(0)) = \theta_e, P_i(e(0)) = \theta_i, \tag{41}$$

• Decide emphasize $(p_e(x, v_1), p_i(x, v_1))$ Estimate the 'width coordinates' (d(v1), e(v1))

$$p_e(d(v_2), v_2)) = \theta_e, P_1(e(v_2), v_2)) = \theta_i, \tag{42}$$

etc., . . .

• Find the iterates $(p_e(x, v_j), p_i(x, v_j))$ • Compute the width coordinates $(d(v_j), e(v_j))$

$$p_e(d(v_j), v_j)) = \theta_e, p_1(e(v_j), v_j)) = \theta_i, \tag{43}$$

• Find the iterates $(p_e(x, v_{j+1}),\ p_i(x, v_{j+1}))$ Observe that the 'split step iteration scheme' The 'numerical code' is also acceptable providing 'pulse amplitudes'

Chapter 6

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