Compatibility of $r - n_s$ Trajectories with Planck Data for G-inflationary Model

By

Rida khan

CIIT/FA17-RMT-028/LHR

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Name	Registration Number
Rida khan	CIIT/FA17-RMT-028/LHR

Supervisor

Dr. Rabia Saleem
Assistant Professor
Department of Mathematics,
COMSATS University Islamabad (CUI)
Lahore Campus
June, 2019

Final Approval

This thesis titled

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Has been approved

For the COMSATS University Islamabad, Lahore

External Examiner : ______________________

Supervisor : ______________________

(Dr. Rabia Saleem)

HoD/Incharge : ______________________

(Dr. Sarfraz Ahmad)

(Mathematics Department, CUI, Lahore Campus)

<u>Declaration</u>

I, **Rida khan** with registration number **CIIT/FA17-RMT-028/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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(Rida khan)

(CIIT/FA17-RMT-028/LHR)

Certificate

It is certified that Rida khan with CIIT/FA17-RMT-028/LHR has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore campus and the work fulfills the requirement for award of MS degree.

Dated :____________________ Supervisor:____________________

(Dr. Rabia Saleem)
Assistant Professor

Head/Incharge Of Department

____

(Dr. Sarfraz Ahmad)
Associate Professor
Department of Mathematics

DEDICATED TO

My loving Parents, Teachers, My Family,
and Friends who always encouraged
me to work hard and guided me towards
the right way and destination.

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Lahore ***Rida khan***

June, 2019 ***CIIT/FA17-RMT-028/LHR***

ABSTRACT

The main goal of this thesis is to explore a model with inflation inspired by Galileon inflaton field and how this model is modified by introducing a generalized coupling form $G(\phi, X) = \frac{\phi^{2p+1}X^q}{M^{6q}}$, in the action. The analytical solutions of the inflaton field and corresponding potential under slow-roll approximation are obtained using specific evolution of the isotropic scale factor, *i.e.*, intermediate, logamediate and exponential. Consistency of the predictions of the model with observations is being done by computing the scalar-tensor power spectra, the scalar spectral index as well as the tensor-scalar ratio. For all the three models with inflation, we constraint the involved model parameters for which its predictions lie inside the allowed region from *Planck* data.

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Chapter 1

Introduction

A comprehensive study of cosmic evolution is dubbed as “cosmology". Now a days, “Big Bang Theory (BBT)" is the utmost leading theory about universe’s origin and its developments. This theory explains the adventure of universe happened in the very beginning. Based on this theory, universe was of approximately zero size with infinite large density at the core after which it has been cooling and expanding from that point onward. Cosmology is established on the “cosmological principle": nobody is at the center of the universe and the cosmos viewed from any point looks the same as from any other point. In 19th century, *Albert Einstein* proposed his new “general theory of relativity (GR)", situated on two hypothesizes which are, “principle of equivalence" and “general covariance". *Alexander Friedmann* in 1922, discovered the analytical solutions for Einstein field equations (EFE’s) that reveal the non static nature of the universe as it is getting bigger with respect to time. Afterwards in 1927, *George Lemaitre* estimated that our cosmos must have begun with single atom outburst if it is inflated. The *Hubble* perception of expanding universe is the most convincing proof of BBT.

A great thrill to “modern cosmology" is to considerate the cosmic expansion. Even after an unceasing analysis with different observational objectives, there are so many questions linked to different aspects of the universe; that are still confidential. In appropriate manner, at present time, its initial and belated enlargements have been a great transaction. One can find two famous and well accepted theories by looking at the literature; namely the “quintessence" (the late-time cosmic evolution) and the “cosmic inflation" (the early cosmic evolution) [1].

Observational cosmology is presently in its “Golden Age". A worth of new experimental results are being discovered. The “cosmic microwave background radiation (CMBR)" has been measured to high accuracy, the dispersion of visible (ordinary matter) and of invisible (“dark matter (DM)") is being charted out to greater and greater depths. New experimental windows to detect the structure of the universe are opening. To explain these experimental results, it is essential to consider events, which happened in the very beginning of early universe. The observations of CMBR are strongly interpreted by standard universe model, but there are still some unsolved puzzles including "horizon, flatness, magnetic monopole" and "origin of fluctuations".

"Cosmic inflation", the term coined by *Alan Guth* [2], can explain some of the distract, which the earlier paradigm; standard BB cosmology could not lodging, although inflationary cosmology gave jump to the first justification for the generation of inhomogeneities in the cosmos based on causal physics. The inflationary pattern is precisely an accelerating stage of the ancient universe that perseveres for an intensely tiny span of time [3]-[6]. The potentiality of inflationary theory was directly perceived due to its capability to demonstrate the source of inhomogeneities existed in the cosmos [7]-[11].

One is former to identify the principal source of field matter that cause inflation, even if various feasible models have been determined. In various models, inflation is conducted by a minimal coupled "scalar (inflaton) field" [12]. In terms of scalar field Lagrangian formalism, one can explain the inflationary scheme. Quantum inconstancy of the scalar field gave a depiction of anisotropy in CMBR map and contribute the causal interpretation of the origin of "large scale structure (LSS)" distribution. This distribution foretell the detailed shape of the angular power spectrum of CMBR anisotropies; a prognosis, which was proved observationally many years later. The precise Planck measurements of the CMB anisotropies, covering the unified sky and over a broad magnitude of scales, contribute a strong probe on inflationary cosmology.

Inflationary models are restrained into "slow-roll (SR)" as well as "reheating" epochs. During first period, potential energy (PE) influence the kinetic energy (KE) and synergism between inflaton and some other fields is eliminated, thus the universe inflates [13]. After this stage, KE became proportionate to PE when the universe entered into reheating era where inflaton begins oscillatory motion about least value of potential by losing its energy to other existed fields in the background. After this era, the universe is dominated by radiation. Hence, it is quite impressive to note, the theory that came at the inception of eighties, is still remaining quite well with the modern observational data. Furthermore, the simplest feasible theory that approximately illustrate the ancient universe in a correct way of understanding with the modern observations is the "Theory of Inflation". For the first time in history, the scientific paradigm, which was about to be established provided mathematical tools, produced a consistent and rigorous study of the geometry and dynamics of the universe as a whole.

During exponential growth, the isotropic scale factor $(a(t))$ grows by many orders of implication. The inflationary models are classified into "power-law (PL), intermediate, logamediate" and "exponential models" by choosing various forms of $a(t)$ [14]. A model with inflation has many compelling definite solutions using a potential of exponential type, generally known to be a "PL model" where $a(t) = t^p;\ p > 1$ [15]. The logamediate form is derived from some scalar-tensor theories by implementing weak energy condition on the infinitely expanding models of cosmos. Among various models with inflation, logamediate inflation has not been explored greatly. On the other hand, *Myrzakulov* and *Sebastiani* [16] elaborate an exact inflationary solution of Hubble parameter (H), instead of parameterizing $a(t)$. In contrast to the models with intermediate/logamediate inflation, this parametrization of $H(t)$ has the novelty of explaining the termination of inflation.

It is commonly believed that scalar field (ϕ) is the basic ingredient to drive inflation, known as an inflaton. On the primary level, this fundamental ingredient can be interacted with the gravity and other existed fields, thus to consider the inflaton having non-minimal coupling (NMC) with gravity is quite natural. In different circumstances, instead of a "standard canonical inflation", a "non-canonical model", containing terms of higher order derivatives in the Lagrangian, has been considered to be a topic of theoretical and observational interest. This type of inflationary model yields a non-trivial sound speed and non-Gaussianities (NG) on either low or high scale.

An appropriate class of similar models designate as *Galileon inflationary models*, were influenced by theories dubbed as *Galilean symmetry* [17]-[19]. Moreover, the corresponding derived set of FE's are still up to second order, getting rid of ghosts [17], but this characterization remains valid only for Minkowsi space-time [18]. Despite the fact, the second order equation of motion (EoM) is obtained via "Galileon covariantization", but it no longer remain invariant under *Galilean symmetry* [18, 19]. Following the literature [20], this mentioned theory is well consistent with HT [21], which is known to be the most familiar scalar-tensor theory having FE's up to second-order. It is observed that a vast model space of HT is removed to current era. Particularly, those factors that proceed to NM kinetic coupling are eliminated and remaining terms are "k-essence, cubic Galileon" and "NMC sectors" that are crucial for the construction of theory. One can see the references [20], for a classical work on the topic of Galileon inspired inflation.

In modern years, different authors showed desirous interest to assess inflationary models in different scenarios. Alongside the development of inflationary cosmological theory, more interesting inflationary models are coming into view, with several theoretical discoveries and observational point of interest as well [22]. For FRW space-time, *Setare* and *Kamali* [23] performed the calculations to discuss "intermediate/logamediate warm inflation (WI)" inspired by *vector fields* and later with *non-Abelian gauge fields*. Through the perturbed parameters, they verified the consistency of their attained results with "WMAP7" observations. *Sharif* and *Saleem* [24] explored WI inspired by both above mentioned fields using an anisotropic universe model. They discussed this topic using generalized dissipation coefficient and obtained well consistent results with “Planck", “WMAP9" and “BICEP2" astrophysical data.

To inspect the models with inflation inspired by tachyon field, a great amount of work has been done. *Sharif* and *Saleem* [25] analyzed inflationary dynamics inspired by “GCCG (generalized cosmic Chapygin gas)" along with standard/tachyon inflaton fields under “intermediate/logamediate" evolutions. A comprehensive analysis on the dynamics of WI with viscosity taking isotropic and an anisotropic universe described by “Bianchi type I model" is presented in [26]. They studied the model for various types of Ω (dissipation parameter) and ξ (bulk parameter) and reported the well fitted range of scalar spectral index (n_s) for less e-folding number (N). In reference [27], the polynomial WI is investigated and confirmed the consistency of attained results with recent astrophysical data. Polytropic inspired inflation on the brane obtaining precise expressions for physical parameter, tensor-scalar ratio (r), scalar power spectrum (P_s), n_s and its running (α_s) in terms of the polytropic parameters is studied in [28]. They achieved a new constraint on the energy scale of inflation and the brane tension using the “WMAP9 data".

Chen [29] searched in specific aspects the n_s, its α_s and r explicitly on the potential driven cases. *Sadjadi* and *Goodarzi* [30] discussed oscillatory model of inflation with NMC of the kinetic term as a resolution of few e-folding number elaborated in [31]. Extension of [31] is done by *Saleem* [32], she examined the compatibility of the anisotropic oscillatory inflation model with Planck 2015 data. *Taotao Qiu* [33] studied the *DINKIC* inflationary model with NMC of the kinetic term and a non-linear correction term is included in the action via *DBI-TYPE* action’s form. The proposed model tells about the ghost free property of Horndeski/Galileon models [34, 35]. *Kousar*, *Saleem* and *Zubair* explored inflation in $f(R, \phi)$ gravity with exponential model [36].

Teimoori and *Karami* [37] explored the viability of an inflationary model inspired by Galileon inflation field (GIF) of cubic order as $G(\phi, X)\Box\phi \propto X^n\Box\phi$. By developing this model into intermediate regime, the authors found the constraints on this model to be well fitted with Planck 2015 data. Moreover, it is proved that to get the current observational data, the power n plays a crucial role in cosmological parameters. A work on G-inflation has been done in [38], considering a linear coupling function $G(\phi, X) \propto \phi X$. During intermediate era, an extension of this work is given in [37], taking coupling parameter just a function of X as $G(\phi, X) \propto X^n$. Utilizing the framework of the cubic Galileon, *Herrera et al.* [39] further examined the observational consequences of the intermediate/logamediate and exponential inflationary models. They checked that how these cosmic models are being modified with the coupling $G(\phi, X) \propto \phi^\nu X^n$. *Motaharfar et al.* [40] studied a recently proposed WI phenomenon inspired by GIF, whose KE immediately dissipates into radiation fluid throughout inflation and cosmos smoothly enters into a radiation dominated phase without going through the reheating epoch.

The main goal of this thesis is to explore a model with inflation inspired by GIF and to examine how this model is changed by introducing a generalized coupling form in the action. Consistency of the undertaken model with observations is being done by computing the perturbed parameters. For intermediata, logamediate and exponential models with inflation, we constraint the involved model parameters for which the $r-n_s$ trajectories falls inside the allowed region from Planck data.

The thesis is arranged on the following patron: In **chapter 2**, a brief review of relevant basic concepts has been provided in order to carry out the systematic analysis. **Chapter 3** is devoted to evaluate the G-inflationary model and to check the effects of some new parameters involved in generalized coupling form. Finally, the results are summarized in **chapter 4**.

Chapter 2

Preliminaries

This chapter comprised of some fundamental concepts relevant to our research work.

2.1 *Historical Background:*

The one of the most important endeavors of modern physics is the study of ancient universe, not only because of its indications for few of the earliest "big questions" about our influence and existence in the universe, but also as a result of the primary universe, is an absolute system to test ideology of particle physics at the maximal energies. At a time, when usually more definite cosmological observations are available, a solid understanding of the implications of new physics in the early universe, to gain knowledge of cosmic history and of physics as a whole is of the ultimate importance. With the initiation of GR, the development of modern cosmology lead ahead. When, in 1917, *Einstein* wrote his "cosmological considerations in GR", he was still arguably more interested in using cosmological constant as added constraints on GR (leading to the infamous "blunder" of the introduction of Λ) than the other way round. Nonetheless, the door was finally open for a fruitful symbiosis between cosmology and relativity.

The first to take full advantage of this new relationship was the theoretical meteorologist *Alexander Friedmann*, who in 1922 put forth the possibility of an expanding (or contracting) universe allowed by GR. Unfortunately, *Einstein* did not appreciate the physical relevance of *Friedmann's* contribution (even after retracting an initial attack on his calculations) and despite *Friedmann's* own international connections, his work seems to not have reached that many outside of the "*Union of Soviet Socialist Republics*". In 1927, two years after *Friedmann's* death, *Georges Lemaitre* independently re-derived *Friedmann's* expanding solution and predicted a linear relation between distance and recessional velocity for extragalactic nebulae. This relation was observationally demonstrated two years later by *Edwin Hubble*, and thus became famous as "*Hubble*'s law". This development crucially contributed to the growing acceptance of *Lemaitre's* approach, notably by *Einstein* (who had initially opposed it as he had *Friedmann's*).

Not long after, in 1931, *Lemaitre* once again espoused controversy by suggesting that the universe may have had a beginning "a little before the beginning of space and time". At first, this was not a popular assumption, with even *Eddington*, the most influential proponent of *Lemaitre's* earlier work on expanding cosmology (under whom *Lemaitre* had worked in Cambridge), finding the basic notion "repugnant" despite his own refractions on the nature of time and the "end of the world" providing the seed for *Lemaitre's* original argument. This hypothesis, which came to be known as the BBT, would remain an important bone of contention for decades.

Although initially viewed with cautious suspicion (which *Lemaitre's* status as a Catholic priest did little to dispel), the idea survived through the 1930's proliferation of alternative cosmologies and eventually found it is most persistent contender in *Fred Hoyle's post-war* (1948) "the steady state theory" (which assumed particle creation made up for the then-established expansion so that the universe would look the same at any given time). It was also around that time when *Friedmann's* former student, *George Gamow*, started working on cosmology on the side of the BBT. Drawing from his background in nuclear physics, *Gamow* investigated how chemical elements could be produced in a hot expanding universe.

On the 1st of April of 1948, *Gamow* and his student *Ralph Alpher* (famously, *Hans Bethe's* name was added as an author to the paper for its comic potential) showed how light elements could be produced by neutron capture in the early universe, effectively introducing the field of BB nucleosynthesis. Although *Gamow's* and *Alpher's* original vision of all elements being produced in the hot early universe ended up being refuted by *Hoyle*'s (later in collaboration with *William Fowler* and *Margaret* and *George Burbidge*) work on stellar nucleosynthesis, the success of this approach at explaining the observed overabundance of helium (compared to what would be expected from just stellar nucleosynthesis) became an important argument in favour of the BB. The last conclusive piece of evidence that definitely established the BB as the dominant paradigm was the discovery of the CMB, just two years before *Lemaitre's* death. The CMB is a thermal black body spectrum left over from the time of recombination (when the first hydrogen atoms formed), which was initially predicted by *Ralph Alpher* and *Robert Herman* in 1948. This prediction was however only briefly mentioned in *Alpher* and *Herman's* paper and was thus easily overlooked. Having been independently re-predicted by *Robert Dicke* and *Jim Peebles* in the early 1960s, the CMB was first presented as a detectable relic by *Andrei Doroshkevich* and *Igor Novikov* in 1964.

Later that same year, *Robert Dicke*, *David Wilkinson* and *Peter Roll* set out to measure the CMB; just late enough for *Arno Penzias* and *Robert Wilson* to inadvertently do it first (for which they would receive part of the 1978 Nobel Prize in Physics). While alternative theories for a more local origin of the CMB meant this new detection did not change opinions on the issue of the BB as quickly as *Hubble*'s observations did on the issue of expansion, the following decade was marked by a steady shift in opinion as new measurements confirmed the black body nature of the observed spectrum. Nowadays, BB cosmology is almost universally accepted as the standard framework for dealing with the early universe with the important caveat that the initial state is taken to be slightly later than the actual singularity, thus for the time being keeping the question of whether the universe had a beginning outside of the realm of scientific inquiry. [41]

In the present-day physics, the cosmology is one of the pillars and gives the fortunate description of the universe. A fundamental factor of the present-day cosmology is the “inflationary theory", which shows the good understanding with the modern observational data of the ancient universe and different measurements have been examined to determined the models of inflation. We expect more precise and improved data by the forthcoming observations along high level of detect ability with observational technologies have been developing year by year.

2.2 *Standard Model of Big Bang:*

The general cosmological imitation is situated on two props: GR and cosmological convention. The universe, which is uniform and isotropic at biggest scales for that greater than 100Mpc, admits to “cosmological principle". “Isotropy" is the opinion that universe emerges likewise in all orders and remains constant on the bottom of rotations while “homogeneity" tells that at every point in space, the universe looks uniform. Literally, there remain no special direction or position in the universe. General relativity illustrate the fluctuations of the universe [42] and the EFE’s interpret the geometry of the spacetime. “These EFE’s equations are written as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu}, \tag{2.1}$$

here, $G_{\mu\nu}$ (*the Einstein tensor*), $R_{\mu\nu}$ (*the Ricci tensor*), R (*the Ricci Scalar*), $g_{\mu\nu}$ (*metric tensor*), $T_{\mu\nu}$ (*energy-momentum tensor*) along with coupling constant $\kappa = \frac{8\pi G}{c^4}$. The left expression in Eq.(2.1) expresses the spacetime’s deflection due to gravity and right side explains matter in the universe.

The FRW metric illustrate an expansion of cosmos that is consistent with cosmological perception. First of all, *Alexander Friedmann* invented this model, then jointly in 1935, *Robertson* and *Walker* gave its altered form. “In spherical coordinates, FRW spacetime is given below [43]

$$ds^2 = -dt^2 + a(t)^2 \big[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\big],$$

with dimensional curvature k; the flat universe is attained for $k = 0$, $k = 1$ leads to closed and an open universe model is obtained for $k = -1$. The parameter $a(t)$ is the expansion factor known as the cosmic scale factor. The matter-energy contents, which are well fitted with above mentioned spacetime attributes like isotropy as well as homogeneity is given in the form of a "perfect fluid (a fluid without impurities such as heat conduction, viscosity *etc*).

The stress energy tensor for this type of fluid is given as below

$$\text{“}T_{\mu\nu} = (\rho + p)v_\mu v_\nu + P g_{\mu\nu}\text{”}, \tag{2.2}$$

with density (ρ), pressure (P) and $v_\mu = (1, 0, 0, 0)$ (co-moving four vector fluid's velocity). "The EFE's are rewritten as

$$H^2 = \frac{\kappa\rho}{3} - \frac{k}{a^2}, \quad \frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3P)\text{”}.$$

The parameter that is affiliate with the amount of cosmic expansion is named as "*Hubble parameter*" denoted as $H(t) = \frac{\dot{a}}{a}$ (dot is the cosmic time derivative). The conservation of $T_{\mu\nu}$ yields the following result

$$\text{“}\dot{\rho} + 3H(\rho + P) = 0\text{”}.$$

The amount of ρ that is comparable to the symmetrically uniform cosmos is denominate by "critical density (ρ_{crit})" and is determined as follows

$$\rho_{crit} = \frac{3H^2}{\kappa}.$$

Thus, the first field equation is rephrased as

$$|\Omega - 1| = \frac{k}{(aH)^2},$$

here $\Omega = \frac{\rho}{\rho_{crit}}$ known to be a dimensionless parameter of density. It consists of inputs same as parameters of matter density (Ω_m), cosmological constant (Ω_Λ) along with radiation (Ω_{rad}). The cosmos with $\rho > \rho_c$ re-collapses, on the other hand, it is concluded that cosmos with $\rho < \rho_c$ will be widen eternally. For flat geometry, the results are liable by "radiation dominant era", where $a(t) \propto t^{\frac{1}{2}}$ and radiation density $\rho_{rad} \propto a^{-4}$ and "matter dominant era", $a(t) \propto t^{\frac{2}{3}},\ \rho_m \propto a^{-3}$.

In the cases that are mentioned above, the inflation of the universe is decelerating, which means that $\ddot{a} < 0$. "Modern measurements support uniform cosmic expansion, leading to following spacetime

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)".$$

2.3 *Shortcomings of the Hot Big Bang:*

At the time, it became probable that although the victory of "Hot BB (HBB) model", depict the good old days of the cosmos, as a gradation of the time of radiation era, accelerating expansion era due to DE as well as matter influence, there were a few inconsistencies and problems that can not be sufficiently determined without understanding the physics. These are the following:

2.3.1 *The Horizon Problem:*

It is related to the uniformity of the CMBR temperature all over the sky. It can be further explained as if we detect CMBR by placing a radio antenna in the east and then turning its direction towards west, we will come to this conclusion that CMBR is exactly at the same temperature in both directions. The radiation coming from west and the east are apart from each other by 28BLY (billion light years). It is not possible for information to travel with a speed greater than that of speed of light so the radiations in the west can not be contacted thermally with that of east. Also, the regions that are source of radiations have not been ever in communication. If any two regions are separated at long distance then there is no enough time for light to travel through them. The only possibility for two regions to be in thermal contact is, the distance between them must be sufficiently small so that the information can be exchanged. Thus, how the thermal equilibrium was maintained between these regions, if there exists no causal connection?

The Hubble radius $(H^{-1}(t))$ is related to physical wavelength (al) by following expression, ($H^{-1}(t) > al$). During BB cosmology, the evolution of $a(t)$ is $a \propto t^p$, $0 < p < 1$. Here, the wavelength progress as $al \propto t^q$ and the Hubble radius evolves to $H^{-1}(t) = H_0 t$. With the passage of time, the quantity $H^{-1}(t)$ evolves much larger than al. Therefore, the causally connected regions ultimately become the only small fraction of $H^{-1}(t)$. Since the development of the cosmos, a distance that light has covered is known as *particle horizon* (L_H), which is described as

$$L_H = a d_H, \quad d_H = \int_{t_i}^{t} \frac{dt}{a},$$

with d_H, co-moving distance at t_i, which represent the time at beginning of the cosmos. At the instant of decoupling, the L_H is given by "$L_H(t_d) = a(t_d) d_H(t_d)$", represented a space with causal connection among photons. At that time, the emission of photons is observed in the CMBR map. The fraction of $d_H(t_d)$ and the present *particle horizon* $d_H(t_0)$ is shown as

$$“\frac{d_H(t_d)}{d_H(t_0)} \approx 10^{-2}”,$$

implying tiny causally connected regions of photons. But it is observed that the photons in CMBR have the same temperature in all regions. This is well-known *horizon problem*.

2.3.2 *The Flatness Problem:*

The measurements of cosmic curvature density parameter signify that it is presently invariable with 0 ± 10^{-2} [44]. Essentially, the cosmos can be entirely plane with $k = 0$. As a matter of choice, wherever $k \neq 0$, then this parameter became immediately non-zero and on each count in the cosmos's history, it will not be uniformly zero.

It is described that for a flat universe, Ω remains unity throughout the time span. Even for very short curvature, the time-dependence of the term $\Omega - 1$ is different at a considerable extent. Since, the term "$(aH)^2$" decreases always in the standard BBT where $\ddot{a} < 0$, which implies the deviation of $\Omega(t)$ from unity because of cosmic expansion. However, $\Omega(t)$ is order of unity according to current observational data, so in the past, it must be approached to unity. Consider that the EFE's are applicable up to the *Planck* era where the temperature is "$T_{Pl} \sim 10^{19} GeV$" and the value of Ω is "$|\Omega(t_{Pl})| < \mathcal{O}(10^{-64})$" whereas "$|\Omega(t_{nucleo})| < \mathcal{O}(10^{-16})$". It is known to be an excellent adaptation of primordial constraints. By choosing the incorrect initial values, the problem can be occurred that either the quick cosmic expansion before the structure formation or the universe rapidly converge to a single point. It is dubbed as the *flatness inconsistency*.

2.4 *Cosmic Inflation:*

Cosmic microwave background is inherited by the era of recombination, which possesses a mess of information about BBT. The early cosmic phase contains hot plasma made up of photons, baryons and electrons, the mean free path of a photon was very short in it because of constant interactions between photons and plasma. However, expansion of the universe is responsible to drop in temperature, electrons combined with protons to make "hydrogen atoms" and hence photons are free to travel through space. *Penzias* and *Wilson* observed the same faint background glow through radio telescope all over the sky. This light is not coming from any star, galaxy or any stellar object and remains within the microwave region of the radio spectrum. Actually, the CMBR has been detected. One would expect the form of CMBR by considering that the earth was bounded in a box or in a black body cavity at $2.7K$. However, an inverse relation exist between cosmic area and CMBR temperature because the electron frequency was red-shifted through cosmic expansion [45].

$$``\frac{T}{T_0} = \frac{A_0}{A}."$$

Our present cosmos has $2.725K$ temperature, which provides an estimate to predict the temperature at beginning of cosmic era and also the speed of cosmic evolution. Though, existed anisotropies in CMBR could not be described at this temperature. Initially, "*Cosmic Background Explorer (COBE)* satellite (1992)" noticed initial anisotropies lies in CMBR map that gives way to observational cosmology. It wa observed that the primordial radiation was isotropic, almost 1 part in 10^5 that provides a notion of almost homogeneous early universe with perturbations of small amplitude. The perturbations are found to be the origin of galaxies and their cluster formation, these effects enhanced during inflation. The present universe is sketched as an outcome of these effects.

2.4.1 *Inflation as a Resolution of Horizon and Flatness Puzzles*

Guth [2] put forward another era in the time-line of cosmic evolution and coined the term "cosmic inflation" for it. This era is introduced as a resolution to these long-lasting problems. These problems could not explained any logical incompatibility, but to explain the calculated astrophysical data it required some fine-tuned initial conditions. In time-line, this evolution happened before radiation era and after the *Planck* scale. To obtain cosmic inflation, the FRW model must satisfy the following necessary condition

$$\ddot{a} > 0 \quad \Leftrightarrow \quad (\rho + 3P) < 0.$$

It is well-known that density remains a positive quantity, therefore, $P < 0$ during cosmic inflation. An era dominated by ordinary matter and radiation dominated era do not follow such condition. For general physical interpretation of inflation, an alternative expression is given as [2]

$$\text{“}\textbf{inflation} \quad \Leftrightarrow \quad \frac{d(aH)^{-1}}{dt}.\text{”}$$

Throughout cosmic inflation, H^{-1} increases at the rate lesser than al and $a(t)$ develops as $a \propto t^p,\ p > 1$. During inflation, the quantity al is obtained outside H^{-1}. This provokes that the causally connected regions are expanded on scales larger than H^{-1}, as a result the horizon problem is solved. Outside the inflationary era, the physical wavelength expands at a slower rate than *Hubble* radius. The following constraints should be hold to resolve this issue [2].

$$\text{“}\int_{t_\star}^{t_d} \frac{dt}{a(t)} \gg \int_{t_d}^{t_0} \frac{dt}{a(t)},$$

here before the decoupling epoch, cosmic time is represented by $t_\star$", which implies that the covered distance by photons earlier to decoupling needed to be sufficiently larger than after decoupling. It is acquired during inflation with the growth of the universe about e^{70} times.

The quantity $(aH)^2$ increases most rapidly and $\Omega(t)$ converges to unity. The universe expands exponentially during inflationary phase, yielding $\Omega \approx 1$ and also remains of order unity during present era. In this way, solves the *flatness problem*.

2.4.2 *Slowly Rolling Constraints on Inflaton Field:*

The era of cosmic inflation takes place under following constraints [39]

$$\text{“}\frac{\ddot{a}}{a} = H^2 + \dot{H} = H^2(1 - \epsilon) > 0.\text{”}$$

According to the above equation, the necessary and sufficient condition for FRW background to experience cosmic acceleration is

$$\epsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} < 1.$$

The quantity ϵ is a dimensionless standard parameter to measure SR and $dN = Hdt$. At de-Sitter, $\epsilon \sim 0$ because P approached to $-\rho$, thus quantity $V(\phi)$ exceeded to its KE

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi).$$

The phenomenon of accelerating cosmic expansion remains valid for a enough long span of time when the second derivative of ϕ with respect to time is sufficiently small, $\ddot{\phi}^2 \ll 3H\dot{\phi}$. An additional fruitful SR parameter is described as a function of ϵ mentioned below [39]

$$\text{“}\eta = -\frac{\ddot{H}}{2H\dot{H}} = \frac{\epsilon}{2} - \frac{\dot{\epsilon}}{\epsilon H}.\text{”}$$

These parameters $\epsilon,\ \eta \ll 1$ during inflation while inflation terminates at $\epsilon = 1$.

2.4.3 *e-folding Number:*

The extent of expansion of the universe during inflationary epoch is manifested by a parameter N, mathematically defined as

$$N \equiv \ln\left(\frac{a}{a_i}\right) = \int_{t_i}^{t} H(t)dt, \tag{2.3}$$

where a_i and a are the values of $a(t)$ at the start and end of inflation.

2.5 *Perturbations:*

Fortunately, the cosmic inflationary models can be constrained through theory of cosmological perturbations. In early cosmos, tiny fluctuations of quantum type are produced, which are due to inflaton field and metric tensor. Through inflation, the vacuum fluctuations were amplified while some quantum fluctuations are frozen. In basic inflationary model, the perturbations are generally of two types: *Scalar perturbations* (produced in density) and *tensor perturbations* (*i.e.*, gravitational waves (GW)).

2.5.1 *Scalar Perturbations:*

These are originated from quantum fluctuations produced by an inflaton in very early cosmic era. The scalar perturbations in metric are of longer wavelength and assumed to be origin to construct the LSS and contain their effect upon the CMBR spectrum. This influence is useful in constraining the models with inflation. Since the energy density of inflaton field is the only cosmic ingredient, thus the division of the energy density is inhomogeneous due to scalar perturbations. An important parameter to notify the change in these fluctuations is introduced, known as *scalar power spectrum*. The fluctuation's amplitude is calculated by following formula

$$P_s^2(k_0) = \frac{\kappa^2 H^2}{8\pi^2 \epsilon}, \tag{2.4}$$

which is restricted as $P_s^2 = (2.445 \pm 0.096) \times 10^{-9}$ and $k_0 = 0.0002MPc^{-1}$ by current astrophysical data. The quantity P_s^2 diverges insignificantly from scale-invariance. This divergence can be calculated through *scalar spectral index*, which is "logarithmic derivative" of P_s^2 given by

$$1 - n_s = \frac{d \ln P_s^2(k_0)}{d \ln k} = 6\epsilon - 2\eta. \tag{2.5}$$

This parameter is an accurate tool to determine the physical compatibility of the inflationary models as it is constrained via SR parameters. According to *Planck* 2015 data, the value of $n_s = 0.968$.

2.5.2 *Tensor Perturbations:*

The discovery of GW prognosticated by mechanism of inflation is a study of attention these days. The tensor fluctuations analyzed a background of GW disturbing the anisotropy of CMBR. Its amplitude has an imprint on the CMBR temperature perturbations and is directly associated to H, but did not affect the LSS. Mathematically, the amplitude of "tensor power-spectrum" can be written as

$$P_T^2 = \frac{2k^2}{\pi^2} H^2. \tag{2.6}$$

2.5.3 *Tensor-Scalar Ratio:*

The universe is accommodated with contrast anisotropies of the CMBR. Cosmic microwave background is linearly contrasted and can be dissolved within two pseudo-scalar fields that are E (electric) and B (magnetic). Initial E modes are derived by scalar disturbances, but initial B modes are only derived by tensor disturbances composed by GWs. Measurements of fundamental B modes are commonly parameterized by tensor-scalar ratio, described to be the proportion of the power in tensor disturbances to the scalar disturbances at a precise scale.

$$r|_{k_0} \equiv \frac{P_t}{P_s}. \tag{2.7}$$

This parameter is constrained to $r < 0.11$ by recent observation.

2.6 *Exact Inflationary Solutions:*

In order to get the exact inflationary solutions, some standard forms of the scale factor have been proposed in literature. Some of which are defined below:

2.6.1 *Intermediate Inflation Model:*

According to it, the inflationary phase is being characterized by an intermediate model with inflation where $a(t)$ has the succeeding dependency upon time.

$$a(t) = \exp[At^f], \tag{2.8}$$

here $f,\ A$ are arbitrary parameters, satisfying the conditions $0 < f < 1$ and $A > 0.$ Through intermediate phase, the universe enlarge at the amount faster than PL model but slower than the basic de Sitter model $(a = a_0 \exp(H_0 t))$ [46].

2.6.2 *Logamediate Inflation Model:*

In addition, a hypothesized inflation model is given as

$$a(t) = \exp[B \ln(t)^{\lambda}], \tag{2.9}$$

where λ and B are dimensionless uniform parameters so that $\lambda > 1$ and $B > 0.$ It is noted that for special case $B = p$ along with $\lambda = 1$, the model is converted to PL expansion with a potential of exponential form.

2.6.3 *Exponential Inflation Model:*

An exact inflationary solution of H is elaborated instead of parameterizing $a(t)$. Here, H has an exponential dependence on t, defined as

$$H(t) = \alpha \exp[-\beta t], \tag{2.10}$$

in which α represents the value of $H(t)$ for $t \to 0$ and $\beta > 0$ being a constant. In contrast to the models with intermediate/logamediate inflation, this parametrization of $H(t)$ has the novelty of explaining the termination of inflation.

2.7 *Types of Scaler Field:*

In physics and mathematics, a scalar field accomplice a scalar value to everywhere marked in a space conceivably physical space. The scalar may be a physical length or a dimensionless mathematical number. In a physical relation, these fields are prescribed to be autonomous of the preferred of reference frame, meaning that any two observers using the same units at the same absolute point in space or spacetime regardless of their respective points of origin, will agree on the same value of the scalar field.

2.7.1 *Standard Scaler Field:*

The most well motivated model for inflation, based on the general principles of effective field theory, is the standard two-derivative action for gravity with a single scalar field. In this case, we can always exploit field re-definitions of the metric tensor to go to *Einstein* frame and field re-definitions of ϕ to make the kinetic term canonical. The associated energy density (ρ_ϕ) and pressure (P_ϕ) are given as

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad P_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \tag{2.11}$$

2.7.2 *Tachyon Field:*

Even if, the standard scalar field is very important that shows the role of an inflaton, some other fields are also liable for it. Among them, the field which have appropriate interest is the “tachyon field". Because of its condensation close to the top of the effective potential [47], tachyons are considered to be responsible for producing cosmological inflation in the early-time cosmic phase and it could also add some new type of “invisible matter" as well at late-times [48]. The first, who studied the cosmological significance of this “rolling tachyon" was *Gibbons*

[49]. This is absolutely natural to examine some scheme where inflation is directed by a rolling tachyon. The associated ρ_{tac} and P_{tac} are expressed as

$$\begin{aligned}\rho_{tac} &= \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}},\\ P_{tac} &= -V(\phi)\sqrt{1-\dot{\phi}^2}.\end{aligned}$$

2.7.3 *Galileon Field:*

A “non-canonical model", representing by Lagrangian with higher order derivative terms yields a non-trivial sound speed and high/low scale NG. Such specific class of similar models known to be the *Galileon inflationary models*, preserving *Galilean symmetry*. The important thing is that the corresponding FE's are still up to second order, getting rid of ghosts. In this scenario, the Lagrangian density can be written as follows

$$L = K(\phi, X) - G(\phi, X)\phi + f(\phi)R. \tag{2.12}$$

The quantities evaluated from stress-energy tensor are

$$\begin{aligned}\rho_G &= 2K_X - K + 3HG_X\dot{\phi}^3 - 2G_\phi X,\\ P_G &= K - 2(G_\phi + G_X\ddot{\phi})X.\end{aligned} \tag{2.13}$$

Chapter 3

A Study of Galileon Field Inspired Inflation with Generalized Coupling Function during Intermediate, Logamediate and Exponential Regimes

This chapter provides extensive study to explore a model with inflation inspired by GIF. The analytical solutions of the inflaton field and corresponding potential under SR approximation are obtained using specific evolution, i.e., intermediate, logamediate and exponential. Consistency of the predictions of the model with observations is being done by computing the perturbed parameters. We constraint the involved model parameters for which its predictions lie inside the allowed region from *Planck* data.

The patron of this chapter is as follows: Section **3.1** brief the general formalism to get generalized inflationary results, as we are working with most generalized coupling form. In section **3.2**, we evaluated the corresponding analytical solutions by solving the differential equations during intermediate, logamediate and exponential regimes. In each case, these derived analytical solutions lead us to generate perturbed parameters. We constraint the G-inflationary model parameters to attain the allowed region for $n_s = 0.968$ and $r < 0.11$ (*Planck* 2015). In section **3.3** to check the consistency of the considered model, we plot the $r - n_s$ trajectories.

3.1 *G-Inflationary Background Equations*:

This section provides an overview of the dynamics and the cosmological perturbations related to G-inflationary model. To complete the task, the following action is considered as [50, 51]

$$S = \int \sqrt{-g}\left[\frac{R}{2} + \varsigma_\phi\right] d^4x, \tag{3.1}$$

where

$$\varsigma_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \tag{3.2}$$

is a scalar field Lagrangian, g be the determinant of metric tensor. The terms K and G are the general functions of ϕ and kinetic term $X = -\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} =$

$-\frac{\dot{\phi}^2}{2}$. Throughout this work, the *Planck* mass is taken to be unity, *i.e.*, $M_{P1} = (8\pi G)^{-\frac{1}{2}} \equiv 1$ and $\mu,\ \nu$ varies on 0,1,2,3. Varying the action with respect to $g_{\mu\nu}$ and considering flat FRW geometry, where $g_{\mu\nu} = diag(-1, a^2(t), a^2(t), a^2(t))$, we are able to find out the field equations as follows

$$\begin{aligned} 3H^2 &- K_{,X}\dot{\phi}^2 + K + G_{,\phi}\dot{\phi}^2 - 3G_{,X}H\dot{\phi}^3 = 0, \quad (3.3)\\ 2\dot{H} &+ 3H^2 + K - G_{,X}\dot{\phi}^2\ddot{\phi} - G_{,\phi}\dot{\phi}^2 = 0, \quad (3.4) \end{aligned}$$

here, the terms with ", ϕ" and ", X" denote $\frac{\partial}{\partial\phi}$ and $\frac{\partial}{\partial X}$, respectively.

Varying the action of G-inflation given in Eq.(3.1) with respect to ϕ yields the EoM as under

$$\begin{aligned} 3G_{,X}\dot{\phi}^2\dot{H} &+ [3G_{,XX}H\dot{\phi}^3 - 2G_{,\phi} - (G_{,\phi X} - K_{,XX})\dot{\phi}^2 + K_{,X} + 6G_{,X}H\dot{\phi}]\ddot{\phi}\\ &+ 3G_{,\phi X}H\dot{\phi}^3 - K_{,\phi} + [9G_{,X}H^2 + K_{,\phi X} - G_{,\phi\phi}]\dot{\phi}^2 - (2G_{,\phi} - K_{,X})\\ &\times 3H\dot{\phi} = 0. \quad (3.5) \end{aligned}$$

The elimination of the term K from Eqs.(3.3) and (3.4), we get the following equation

$$\varepsilon_1 - \delta_X - 3\delta_{GX} + 2\delta_{G\phi} + \delta_\phi\delta_{GX} = 0, \quad (3.6)$$

where the introduced SR parameters are defined as [52]

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}, \quad \delta_X \equiv \frac{K_{,X}X}{H^2}, \quad \delta_\phi \equiv \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \delta_{GX} \equiv \frac{G_{,X}\dot{\phi}X}{H}, \quad \delta_{G\phi} \equiv \frac{G_{,\phi}X}{H^2}. \quad (3.7)$$

Kobayashi et al. [51, 52] done the pioneer work to analyze the cosmological perturbations in the context of G-inflationary model. Under SR limit, the power spectrum of primitive scalar perturbations at horizon exit reads

$$P_s \simeq \frac{H^2}{8\pi^2 c_s^3 Q_s}|_{c_s k = aH}; \quad Q_s = \frac{\epsilon_s}{c_s^2}, \quad (3.8)$$

where

$$\epsilon_s = \delta_X + 4\delta_{GX} - 2\delta_{G\phi}, \quad c_s^2 = \frac{\epsilon_s}{q_s},$$

the quantity q_s is further described as

$$q_s \equiv \delta_X + 6\delta_{GX} + 6\delta_{GXX},$$

here, $\delta_{GXX} \equiv \frac{G_{XX}\dot{\phi}X^2}{H}$. The Galileon model would be free from ghosts and Laplacian instabilities, if the following constraints hold, $Q_s > 0$ and $c_s^2 > 0$, which further implying the positivity of q_s and ϵ_s.

Another quantity, n_s, can be developed in G-inflation using Eq.(3.8) and the approximation $d\ln k \simeq Hdt$. The n_s, at first-order in terms of SR parameters, can be given as

$$n_s - 1 \simeq -2\epsilon_1 + \eta_{s1} - s_1, \tag{3.9}$$

where the redefined quantities are

$$\eta_{s1} \equiv \frac{\dot{\epsilon}_s}{H\epsilon_s}, \quad s_1 \equiv \frac{\dot{c}_s}{Hc_s}.$$

The second-order quantity, running of n_s is evaluated from Eq.(3.9) as follows

$$\alpha_s = \frac{dn_s}{d\ln k} = -2\epsilon_2\epsilon_1 - \eta_{s2}\eta_{s1} - s_2 s_1, \tag{3.10}$$

here, we characterize the following general SR parameters for $i \geq 1$ as

$$\epsilon_{i+1} \equiv \frac{\dot{\epsilon}_i}{H\epsilon_i}, \quad \eta_{s(i+1)} \equiv \frac{\dot{\eta}_{si}}{H\eta_{si}}, \quad s_{i+1} \equiv \frac{\dot{s}_i}{Hs_i}. \tag{3.11}$$

The tensor power spectrum and associated tensor spectral index are written as

$$P_t \simeq \frac{2H^2}{\pi^2}|_{k=aH}, \quad n_t \equiv \frac{d\ln P_t}{d\ln k}|_{k=aH} = -2\epsilon_1, \tag{3.12}$$

where the last equality in above equation is obtained at the time of horizon crossing specified by $k = aH$. This time is not exactly the same as the time of sound horizon crossing for which $c_s k = aH$, but to lowest order in the SR parameters, the difference is negligible [53]. Another essential observed inflationary parameter of interest is known to be tensor-scalar ratio, which is defined as

$$r = 16c_s\epsilon_s = -8c_s(n_t - 2\delta_{GX}). \tag{3.13}$$

On the fundamental level, G-inflation can be phenomenologically discriminated from standard inflationary model by checking the condition, $c_s^2 = 1$. On contrary, a non-linear parameter denoted by f_{NL} is used to calculate the detected NG, which could distort the degeneracy among a few models with inflation and also enables us to differentiate between inflationary model with a single-field inflation and other alternative phenomenon (a comprehensive study is presented in Refs. [54, 55]). In particular, for a single-field inflationary model with a canonical kinetic term and a smooth inflaton potential, the predicted amount of NG is given as $f_{NL} \ll 1$ [56]-[58]. Moreover, the mentioned characteristics can originate large amount of NG, $\mid f_{NL} \mid \gg 1$, and current observations produce $f_{NL} \lesssim \mathcal{O}(10)$ [59].

Depending upon the shapes of NG, several categories can be determined by using the magnitudes of the wave vectors, $k_1,\ k_2,\ k_3$, in the *Fourier space* satisfying the constraint $k_1 + k_2 + k_3 = 0$ [60]. For example, multi-field inflation [61] and curvaton scenarios [62] give rise a bi-spectrum that has a maximum in squeezed configuration or local shape (for $k_3 \ll k_1 \simeq k_2$) [63, 64]. Specifically, for non-canonical kinetic terms, the NG are well described by the equilateral (*i.e.*, $k_1 = k_2 = k_3$) and orthogonal shapes (*i.e.*, $k_1 = 2k_2 = 2k_3$) [63, 64]. An important linear combination of the equilateral and orthogonal shapes give rise to the so-called enfolded shape and this combination was determined from *Planck* data in [65, 66]. *Felice* and *Tsujikawa* [67] derived the NG of curvature perturbations in the Galileon theories and showed that G-inflationary models generate the equilateral form of NG. Under the SR approximation, the parameter $\mathfrak{f}$ for the models described by the action (3.1) is given in [67]

$$\mathfrak{f} = \frac{85}{324}(1 - \frac{1}{c_s^2}) - \frac{10}{81}\frac{\mu}{\Sigma} + \frac{20}{81\epsilon_s}(\delta_{GX} + \delta_{GXX}) + \frac{65}{162 c_s^2 \epsilon_s}\delta_{GX}, \tag{3.14}$$

where the quantities μ and Σ are given by

$$\begin{aligned}
\mu &\equiv \frac{1}{3}[3X^2K_{,XX} + 2X^3K_{,XXX} + 3H\dot{\phi}(XG_{,X} + 5X^2G_{,XX} + 2X^3G_{,XXX}) \\
&- 2(2X^2G_{,\phi X} + X^3G_{,\phi XXX})], \\
\Sigma &\equiv \frac{8H(1-\delta_{GX}) + 729(H^2(1 - \frac{\delta_X}{3} - \frac{2\delta_{XX}}{3} - 4\delta_{GX} - 2\delta_{GXX} + \frac{2\delta_{G\phi}}{3}))^2}{12}.
\end{aligned}$$

Planck collaboration has found the current observational constraints at $68\% C.L.$ on primordial NG by combining temperature and polarization data [68]

$$f_{NL}^{local} = 0.8 \pm 5.0, \quad f_{NL}^{equil} = -4 \pm 43, \quad f_{NL}^{ortho} = -26 \pm 21, \quad f_{NL}^{enfold} = 11 \pm 32.$$

The parameter for G-inflation can be evaluated as

$$\mathfrak{f} = \frac{85}{324}\left(1 - \frac{1}{c_s^2}\right) - \frac{10}{81}I_1 + \frac{20}{81}I_2 + \frac{65}{162}I_3,$$

where $I_1,\ I_2$ and I_3 are defined as

$$\begin{aligned}
I_1 &\equiv \frac{4q\phi^{2p}\dot{\phi}^{2q+1}}{2^q M^{6q}}\Big[3H\phi q(1-2q) + \dot{\phi}(q+1)(2p+1)\Big]\Bigg[8\left(H + \frac{q\phi^{2p+1}\dot{\phi}^{2q+1}}{M^{6q}2^q}\right) \\
&+ 729H^4\left(1 + \frac{\dot{\phi}^2 - 4}{6H^2} + \frac{4q\phi^{2p+1}\dot{\phi}^{2q+1}}{2^q H M^{6q}}\left(1 + \frac{q}{\dot{\phi}^2} - \frac{(2p+1)\dot{\phi}^2}{12H\phi}\right)\right)^2\Bigg]^{-1}, \\
I_2 &\equiv -\frac{q\phi^{2p+1}\dot{\phi}^{2q+1}}{H2^q M^{6q}}\left(1 + \frac{2q}{\dot{\phi}^2}\right)\left[\frac{\dot{\phi}^2}{2H^2} + \frac{2\phi^{2p}\dot{\phi}^{2q+1}}{2^q H M^{6q}}\left(\frac{\dot{\phi}(2p+1)}{2H} - 2q\phi\right)\right]^{-1}, \\
I_3 &\equiv -\frac{q\phi^{2p+1}\dot{\phi}^{2q+1}}{2^q H M^{6q}}\left[\frac{\dot{\phi}^2}{2H^2} + \frac{2^{1-q}\phi^{2p}\dot{\phi}^{2q+1}}{HM^{6q}}\left(-2q\phi + \frac{\dot{\phi}(2p+1)}{2H}\right)\right]^{-1} \\
&\times \left[\dot{\phi}\left(2H - \frac{\dot{\phi}(2p+1)}{2q\phi}\right)\left(3H\left(\dot{\phi} - \frac{2q}{\dot{\phi}}\right)\right)^{-1}\right]^{-1}.
\end{aligned}$$

After a brief review of some basic relations governing the general G-inflationary scenarios, let us focus on a specific case and investigate a concrete model. To this end, we consider the potential-driven inflation, choosing following most generalized forms of the functions K and G as [68]

$$K(\phi, X) = X - V(\phi), \quad G(\phi, X) = -\frac{\phi^{2p+1}X^q}{M^{6q}}, \tag{3.15}$$

where $M > 0$ has dimension of mass and $p,\ q > 0$. With the aid of Eq.(3.6) and second expression of Eq.(3.15), we have evaluated an important relation at first order in terms of SR parameters as

$$\epsilon_1 \simeq (1+A)\delta_X, \tag{3.16}$$

here parameter A is found to be

$$A = \frac{q\phi^{2p+1}X^{q-1}}{M^{6q}}\left(3\dot{\phi}H + \frac{2(2p+1)X}{q\phi}\right).$$

According to SR approximation, ignoring the second order terms of $\dot{\phi}$, the evolution equation (3.3) is reduce to $3H^2 \simeq V(\phi)$ and the EoM (3.5) can be modified to

$$3H\dot{\phi}(1-A) - \frac{3\phi^{2p+1}qX^{q-1}\dot{\phi}^2\dot{H}}{M^{6q}} + V_\phi = 0. \tag{3.17}$$

Using expression of A along with Eq.(3.17), we can find the SR parameter δ_X as

$$\delta_X \simeq \frac{\epsilon_v}{(1+A)^2} + \frac{1}{2H^2}\left(\frac{6\phi^{2p+1}X^q q\dot{H}}{3H(1-A)M^{6q}}\right)^2 - \frac{V_\phi X^q q\dot{H}\phi^{2p+1}}{3H^4(1-A)^2M^{6q}}, \tag{3.18}$$

where

$$\epsilon_v \equiv \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2. \tag{3.19}$$

In the presence of Galileon term given in Eq.(3.15), using expressions of ϵ_s and q_s, the squared sound speed is calculated as

$$c_s^2 = \left(2\dot{\phi}H - \frac{X(2p+1)}{q\phi}\right)\left(3\dot{\phi}H(q+1)\right)^{-1}. \tag{3.20}$$

Consider the first limit that Galileon term lags behind the standard kinetic term of inflaton field, *i.e.*, $|\delta_{GX}| \gg \delta_X$. From this condition, the constraint $A >> 1$ is obtained, which leads to reduce the expression of c_s^2 in the following form

$$c_s^2 = \frac{\delta_X + 4\delta_{GX}}{\delta_X + 6\delta_{GX}} = -\frac{\dot{\phi}^2}{3q}. \tag{3.21}$$

The other condition $\delta_X \gg |\delta_{GX}|$, implying $A \to 0$, for which $c_s^2 = 1$. It is worth mentioning here that Eqs.(3.12) and (3.16) yield standard inflation form of the consistency relation (3.13) as $r = -8n_t$.

Using Eqs.(3.18), (3.19) along with the expressions of $\delta_{GX} = (\frac{A}{3})\delta_X$ and $\delta_{GXX} = (n-1)\delta_{GX}$, the P_s (3.8) can be obtained as under

$$\begin{aligned} P_s &= \frac{H^4}{8\pi^2}\Big(2\dot{\phi}H - \frac{X(2p+1)}{q\phi}\Big)^{-\frac{1}{2}}\Big(3\dot{\phi}H(q+1)\Big)^{\frac{1}{2}}\Big(X - 4q\dot{\phi}HX^q\phi^{2p+1}M^{-6q} \\ &+ 2(2p+1)\phi^{2p}X^{q+1}M^{-6q}\Big)^{-1}. \end{aligned} \tag{3.22}$$

Using the above equation, we can find the parameter n_s in the defined Galileon scheme as

$$\begin{aligned} n_s - 1 &= \frac{\dot{H}}{H^2}\Big[4 - H\dot{\phi}\Big(2\dot{\phi}H - \frac{X(2p+1)}{q\phi}\Big)^{-1}\Big] + \Big[\frac{\dot{H}}{2H^2} + \frac{4\dot{\phi}X^q\phi^{2p}}{M^{6q}H}\Big(-q\Big(\dot{H}\phi \\ &+ \dot{\phi}H(2p+1)\Big) + \frac{Xp(2p+1)}{\phi}\Big)\Big(X - 4q\dot{\phi}HX^q\phi^{2p+1}M^{-6q} \\ &+ 2(2p+1)\phi^{2p}X^{q+1}M^{-6q}\Big)^{-1}\Big]. \end{aligned} \tag{3.23}$$

Using previous calculated results, the SR parameter can be written as

$$\begin{aligned} \epsilon_s &= \Big(1 + 9H^2\dot{\phi}^2\Big(\frac{\phi^{2p+1}qX^{q-1}}{M^{6q}}\Big)^2\Big)^{-1}\Big(\epsilon_v + \frac{2X^{2q}}{H^4}\Big(\frac{q\phi^{2p+1}\dot{H}}{M^{6q}}\Big)^2 - \frac{qX^qV_\phi\phi^{2p+1}\dot{H}}{3H^4M^{6q}}\Big) \\ &+ \frac{2(2p+1)\phi^{2p}X^{q+1}}{H^2M^{6q}}. \end{aligned} \tag{3.24}$$

Ultimately, the physical parameter r, defined in Eq.(3.13) is evaluated using Eqs.(3.20) and (3.25) as

$$\begin{aligned} r &= 16\Big[\Big(2\dot{\phi}H - \frac{X(2p+1)}{q\phi}\Big)\Big(3\dot{\phi}H(q+1)\Big)^{-1}\Big]^{\frac{1}{2}}\Big[\Big(1 - \frac{q\phi^{2p+1}X^{q-1}}{M^{6q}}\Big(3\dot{\phi}H \\ &+ \frac{2X(2p+1)}{q\phi}\Big)\Big)^{-2}\Big(\xi_v + \frac{qX^q\dot{H}\phi^{2p+1}}{M^{6q}H^4}\Big(\frac{2qX^q\dot{H}\phi^{2p+1}}{M^{6q}} - \frac{V_\phi}{3}\Big)\Big) - \frac{X^q\phi^{2p}}{M^{6q}H} \\ &\times \Big(4q\phi\dot{\phi} + \frac{2(2p+1)X}{H}\Big)\Big]. \end{aligned} \tag{3.25}$$

Next, we will develop the G-inflationary model during intermediate regime using intermediate law of scale factor.

3.2 *G-inflation during Intermediate Era*:

An intermediate model of inflation is described in Eq.(2.8), which yields the parameter H in the following form

$$H(t) = \frac{Af}{t^{1-f}}. \tag{3.26}$$

Inserting Eq.(3.26) into expression of A (calculated in the previous section) along with $X = -\frac{\dot{\phi}^2}{2}$, we get following result

$$\mathcal{A} = \frac{q\dot{\phi}^{2(q-1)}\phi^{2p+1}}{2^{q-1}M^{6q}}\left(\frac{3Af\dot{\phi}}{t^{1-f}} + \frac{(2p+1)\dot{\phi}^2}{q\phi}\right). \tag{3.27}$$

Now, Eq.(3.17) provided us the following differential equation

$$\frac{3Af}{t^{1-f}}\left[\dot{\phi} - \frac{\dot{\phi}qX^q\phi^{2p+1}}{M^{6q}}\left(3H\dot{\phi} + \frac{2(2p+1)X}{q\phi} + \frac{\dot{\phi}(1-f)}{t}\right) - \frac{2Af(1-f)}{\dot{\phi}t^{2-f}}\right] = 0, \tag{3.28}$$

whose solution is

$$\dot{\phi} = \left[\sqrt{2}Af(1-f)\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-2}\right]^{\frac{1}{2}}.$$

The value of t, which is inserted in the above equation is calculated through the following procedure

$$N \equiv \int_t^{t_e} H dt \quad \Rightarrow \quad t = t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}, \tag{3.29}$$

where H is given in Eq.(3.26). Integrating the above quantity, we get inflaton in terms of N as

$$\phi = 2^{\frac{3}{2}}\left(\frac{A(1-f)}{f}\right)^{\frac{1}{2}}\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{f}{2}}, \tag{3.30}$$

without any loss of generality, the constant of integration is set to be zero. Driving the intermediate inflation in the standard inflation model the potential has an inverse power law form as $V(\phi) \propto \phi^{-\tilde{p}}$, here $\tilde{p} = \frac{4(1-f)}{f}$. The first

Friedmann Eq.(3.3) established the following form of the inflationary potential under slow-roll approximation

$$V_0 = 3(Af)^2 \left[\frac{2^{\frac{3}{2}}(Af(1-f))^{\frac{1}{2}}}{f}\right]^{\frac{4(1-f)}{f}}. \tag{3.31}$$

Substituting the value of t, we are able to express H as a function of N as under

$$H(N) = Af\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-1}. \tag{3.32}$$

The standard SR parameter ϵ is modified to

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{(1-f)}{Af}\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{-f}. \tag{3.33}$$

This relationship of ϵ and t_e has a specific worth because it helps us to calculate the exact value of time at the end of inflation for enough e-folding number by taking unit value of ϵ as

$$\epsilon = 1 \quad \Rightarrow \quad t_e = \left(\frac{1-f}{Af}\right)^{\frac{1}{f}} + \left(\frac{N}{A}\right)^{\frac{1}{f}}. \tag{3.34}$$

Making use of the solution ϕ and its derivative $\dot{\phi}$ into Eq.(3.22), we achieved the quantity P_s as

$$\begin{aligned}
P_s &= \frac{3^{\frac{1}{2}}}{8\pi^2}(Af)^{\frac{9}{2}}\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{9(f-1)}{2}}(q+1)^{\frac{1}{2}}\left[2Af\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-1}\right. \\
&- \left.\frac{f(2p+1)}{4q}\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{-1}\right]^{\frac{-1}{2}}\left[2^{\frac{-1}{2}}Af(1-f)\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-2}\right. \\
&- 4qM^{-6q}A^{p+q+2}f^{-p+q+1}\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{4f-4+2pf+2qf-4q}{2}}2^{\frac{5p-q+3}{2}}(1-f)^{p+q+1} \\
&+ \left.(2p+1)M^{-6q}(A(1-f))^{p+q+1}f^{-p+q+1}2^{\frac{5p-q+1}{2}}\left(t_e - \left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{pf+(f-2)(q+1)}\right]^{-1}.
\end{aligned}$$

Equation (3.23) yields the following form of n_s

$$
\begin{aligned}
n_s &= 1-(Af)^{-1}(f-1)\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{-f}\left[\frac{9}{2}-Af\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-1}\left(2Af\right.\right.\\
&\times \left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-1}-\frac{f(2p+1)}{4q}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{-1}\Bigg)^{-1}\Bigg]+4.2^{\frac{1+10p-2q}{4}}M^{-6q}\\
&\times (Af)^{\frac{2q+2p-1}{2}}(1-f)^{\frac{2p+2q+1}{2}}(\frac{1}{f})^{2p}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{2qf-f-4q+2pf}{2}}\Bigg[qA^{\frac{3}{2}}f^{\frac{1}{2}}(1-f)^{\frac{3}{2}}2^{\frac{5}{4}}\\
&\times \left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{3f-4}{2}}-2^{\frac{1}{4}}q(2p+1)(Af)^{\frac{3}{2}}(1-f)^{\frac{1}{2}}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{3f-4}{2}}\\
&+ 2^{-\frac{7}{4}}pf(2p+1)(Af(1-f))^{\frac{1}{2}}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{f-4}{2}}\Bigg]\Bigg[\frac{Af(1-f)}{\sqrt{2}}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-2}\\
&- 4qM^{-6q}2^{\frac{5p-q+3}{2}}(Af)^{2+p+q}(1-f)^{p+q+1}(\frac{1}{f})^{2p+1}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{2f(2+p+q)-4(q+1)}{2}}\\
&+ (2p+1)2^{\frac{5p-q+1}{2}}M^{-6q}(Af(1-f))^{p+q+1}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{pf+(q+1)(f-2)}\Bigg]^{-1}.
\end{aligned}
$$

Ultimately, we have tensor-scalar ratio as under

$$
\begin{aligned}
r &= 16\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{-\frac{f}{2}}(3Af(q+1))^{-\frac{1}{2}}\left[2Af\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f}-\frac{f(2p+1)}{4q}\right]^{\frac{1}{2}}\\
&\times \Bigg[\Bigg(1-2^{\frac{5p-q+4}{2}}qM^{-6q}(Af(1-f))^{p+q}(\frac{1}{f})^{2p+1}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f(p+q)-2q}\\
&\times \left(3Af\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f}+\frac{f(2p+1)}{2q}\right)\Bigg)^{-2}\Bigg(3\sqrt{2}Af(1-f)\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{f-2}\\
&- 2^{\frac{5(2p+1)-2q}{4}}qM^{-6q}(\frac{1}{f})^{2p+1}(Af)^{\frac{2(p+q+1)-7}{2}}(1-f)^{\frac{2(p+q+1)+1}{2}}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{f(2p+2q-5)-4(q-1)}{2}}\\
&\times \Bigg(-qM^{-6q}2^{\frac{10p+9-2q}{4}}(\frac{1}{f})^{2p+1}(Af(1-f))^{\frac{2p+2q+3}{2}}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{f(2p+2q+3)-4(q+1)}{2}}\\
&+ 2^{\frac{3}{4}}((Af)^3(1-f))^{\frac{1}{2}}\Bigg)\Bigg)-2^{\frac{10p-2q+1}{4}}M^{-6q}(Af)^{\frac{2q+2p-1}{2}}(1-f)^{\frac{2q+2p+1}{2}}(\frac{1}{f})^{2p}\\
&\times \left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{f(2p+2q-1)-4q}{2}}\Bigg(4q2^{\frac{5}{4}}(Af(1-f))^{\frac{1}{2}}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{\frac{f}{2}}(\frac{1}{f})+2^{\frac{1}{4}}(2p+1)\\
&\times ((Af)^{-1}(1-f))^{\frac{1}{2}}\left(t_e-\left(\frac{N}{A}\right)^{\frac{1}{f}}\right)^{-\frac{f}{2}}\Bigg)\Bigg].
\end{aligned}
$$

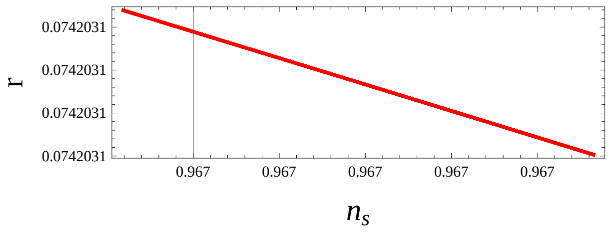

Figure 3.1: $r - n_s$ trajectory for $f = 0.35;\ p = 3;\ q = 2;\ A = 12 \times 10^{-1};\ M = 3.5 \times 10^3 M_p$.

f	p	q	A	M	n_s	r
0.35	3	2	12×10^{-1}	3.5×10^3	0.967	0.074
0.5	7	5	14×10^{-2}	3.5×10^3	0.958	0.030
0.65	11	9	16.8×10^{-3}	3.5×10^3	0.977	0.002
0.9	11	9	29×10^{-4}	3.5×10^3	0.969	0.003

Table 3.1: Constraints on model parameters to find physical acceptable range of n_s and r for $N \leq 70$.

The G-inflationary intermediate model parameters are constrained in Table **3.1**. For four above mentioned values of f, we constraint the other variables like $p,\ q,\ A$ to attain the physical allowed region for $r - n_s$ trajectories. The $r - n_s$ trajectory corresponding to first set of values is shown in Fig. **3.1**, which represent the consistency of the Galileon inspired inflationary model with recent *Planck* scientific data. Generally, the model is very sensitive for the model parameter A. We are not able to find a specific range (maxima and minima) of A for constant value of f and by fixing $p,\ q$ as well. Further, it is clear from the table that there is an inverse relationship between f and A to meet the requirements. The exponent $p,\ q$ of generalized coupling function leads to produce physical results of n_s and r for all positive values.

In the next section, we will constraint the model for logamediate regime.

3.3 *G-inflation during Logamediate Era:*

Here, we consider the scenario in which $a(t)$ evolves according to logamediate law given in Eq.(2.9), which modify the H as

$$H(t) = \frac{A\lambda(\ln t)^{\lambda-1}}{t}. \tag{3.35}$$

In the similar manner as implemented during intermediate era, the quantity $\mathcal{A}$ is expressed as

$$\mathcal{A} = \frac{q\phi^{2p+1}\dot{\phi}^{2q-1}}{2^{q-1}M^{6q}}\left[\frac{3A\lambda(\ln t)^{\lambda-1}}{t} + \frac{(2p+1)\dot{\phi}}{q\phi}\right]. \tag{3.36}$$

In this era, the field velocity $\dot{\phi}$ is calculated as

$$\begin{aligned}\dot{\phi} &= \left(\frac{2^q M^{6q} A\lambda(\lambda-1)}{q(2p+1)}\right)^{\frac{1}{2(p+q+1)}}\left(\frac{1}{(2-\lambda)t\ln t}\right)^{\frac{1}{2(q+1)}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{-p(-\lambda-2q)}{2(q+1)(p+q+1)}}\\ &\times \left(-\Gamma[\frac{\lambda+2q}{2q+2}, \frac{(-3-2q)\ln t}{2q+2}]\right)^{\frac{-p}{p+q+1}},\end{aligned} \tag{3.37}$$

where Γ is the mathematical Gamma function. On integration, we arrived at

$$\phi(t) = -\left(\frac{2^q M^{6q} A\lambda(\lambda-1)}{q(2p+1)}\right)^{\frac{1}{2q+2}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{-\lambda-2q}{2q+2}}\Gamma[\frac{\lambda+2q}{2q+2}, \frac{(-3-2q)\ln t}{2q+2}]. \tag{3.38}$$

Using the first Friedmann equation (3.3) and the Hubble parameter (3.26), the SR parameter (3.19) is given as

$$\epsilon_v = \frac{1}{2}\left[\frac{2\left((1-\lambda)\ln t - t^{-1}\right)}{\dot{\phi}}\right]^2. \tag{3.39}$$

The inflationary potential in the SR approximation is found to be

$$V(N) = 6(A\lambda)^2(\lambda-1)^2\ln\left(t_e(\exp[N])^{-\frac{1}{Af}}\right)\left(t_e(\exp[N])^{-\frac{1}{Af}}\right)^{-2}. \tag{3.40}$$

We get the following expression for t_e

$$t_e = \exp[\frac{A\lambda}{1-\lambda}]^{-\frac{1}{\lambda}}. \tag{3.41}$$

It is useful to find the cosmic time t in terms of N, this can be easily achieved by using Eqs.(3.29) and (3.35) as

$$t = t_e(\exp[N])^{-\frac{1}{Af}}. \tag{3.42}$$

The power spectrum of the scalar perturbations and corresponding spectral index can be written as

$$\begin{aligned}
P_s &= \frac{3^{\frac{3}{2}}(q+1)^{\frac{1}{2}}}{8\pi^2}\ln(t_e(\exp[N])^{\frac{-1}{A\lambda}})\left(A\lambda(\lambda-1)(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{-1}\right)^{\frac{9}{2}} \\
&\times \Bigg[2A\lambda(\lambda-1)(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{-1} - \left(\frac{1}{2q+2}\right)(M^{6q}A\lambda(\lambda-1))^{-\frac{p}{2(q+1)(p+q+1)}} \\
&\times ((2-\lambda)(t_e(\exp[N])^{\frac{-1}{A\lambda}}))^{-\frac{1}{2q+2}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{(\lambda+2q)(2p+q+1)}{2(q+1)(p+q+1)}} q^{\frac{p-2(q+1)(p+q+1)}{2(q+1)(p+q+1)}} \\
&\times (2p+1)^{\frac{2(q+1)(p+q+1)+p}{2(q+1)(p+q+1)}} 2^{\frac{-qp-2(q+1)(p+q+1)}{2(q+1)(p+q+1)}}(-\Gamma(\alpha))^{-1}\Bigg]^{-\frac{1}{2}}\Bigg[\left(\frac{-1}{q+1}\right)((2-\lambda) \\
&\times (t_e(\exp[N])^{\frac{-1}{A\lambda}}))^{\frac{-1}{q+1}} 2^{\frac{-p-1}{p+q+1}}\left(\frac{M^{6q}A\lambda(\lambda-1)}{q(2p+1)}\right)^{\frac{1}{p+q+1}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{p(\lambda+2q)}{(q+1)(p+q+1)}} \\
&- 4q^{\frac{(p+q+1)(2q-2p+1)-(2q+1)(q+1)}{2(q+1)(p+q+1)}}(2^q M^{6q})^{\frac{(2p+1)(p+q+1)+(q+1)(-1-2p)}{2(q+1)(p+q+1)}} \\
&\times (A\lambda(\lambda-1))^{\frac{2p(p+2q)+5p+4q(2+q)+4}{2(q+1)(p+q+1)}}(2p+1)^{\frac{-(2p+1)(p+q+1)-(2q+1)(q+1)}{2(q+1)(p+q+1)}}(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{-4q-3}{2q+2}} \\
&\times \left(\frac{1}{2q+2}\right)(2-\lambda)^{\frac{-2q-1}{2q+2}}(-\Gamma(\alpha))^{2p+1}\left(\frac{-3-2q}{2q+2}\right)^{\frac{(-\lambda-2q)(2p(p+1)+q+1)}{2(q+1)(p+q+1)}} \\
&- (2p+1)^{\frac{-p^2}{(q+1)(p+q+1)}}(2^q M^{6q})^{\frac{p^2}{(q+1)(p+q+1)}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{p^2(-\lambda-2q)}{(q+1)(p+q+1)}} \\
&\times (A\lambda(\lambda-1)q^{-1})^{\frac{p(p+q+1)+(q+1)^2}{(q+1)(p+q+1)}}((2-\lambda)(t_e(\exp[N])^{\frac{-1}{A\lambda}}))^{-1}\Bigg]^{-1},
\end{aligned}$$

$$
\begin{aligned}
n_s &= 1-(\lambda-3)(A\lambda(\lambda-1))^{-1}\ln(t_e(\exp[N])^{\frac{-1}{A\lambda}})\Bigg[\frac{9}{2}-A\lambda(\lambda-1)(t_e(\exp[N])^{-\frac{1}{A\lambda}})^{-1}\\
&\times\ \Bigg(2A\lambda(\lambda-1)(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{-1}+2^{\frac{-qp-2(q+1)(p+q+1)}{2(q+1)(p+q+1)}}(2p+1)^{\frac{p+2(q+1)(p+q+1)}{2(q+1)(p+q+1)}}\\
&\times\ q^{\frac{p-2(q+1)(p+q+1)}{2(q+1)(p+q+1)}}(M^{6q}A\lambda(\lambda-1))^{\frac{-p}{2(q+1)(p+q+1)}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{(\lambda+2q)(2p+q+1)}{2(q+1)(p+q+1)}}\\
&\times\ ((2-\lambda)((t_e(\exp[N])^{\frac{-1}{A\lambda}})))^{\frac{-1}{2q+2}}(2q+2)^{-1}(-\Gamma(\alpha))^{\frac{-2p-q-1}{p+q+1}}\Bigg)^{-1}\Bigg]-4(q+1)^{-1}\\
&\times\ (t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{1}{2q+2}}(2^qM^{6q}A\lambda(\lambda-1))^{\frac{2p^2-q-1}{2(q+1)(p+q+1)}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{pq(\lambda+2q)}{(q+1)(p+q+1)}}\\
&\times\ (2-\lambda)^{\frac{-2q-1}{2q+2}}(q(2p+1))^{\frac{-2q(p+q)-2p(p+1)-3q-1}{2(q+1)(p+q+1)}}(-\Gamma(\alpha))^{\frac{p(1+2p)}{p+q+1}}\Bigg[(\lambda-3)q^{\frac{2q+1}{2q+2}}\\
&\times\ (t_e(\exp[N])^{\frac{-1}{A\lambda}})^{-2}(A\lambda(\lambda-1))^{\frac{3+2q}{2q+2}}(2^qM^{6q}(2p+1)^{-1})^{\frac{1}{2q+2}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{-\lambda-2q}{2q+2}}\\
&\times\ (\Gamma(\alpha))-(A\lambda(\lambda-1))^{\frac{2(p+q)+3}{2(p+q+1)}}(q(2p+1))^{\frac{2(p+q)+1}{2(p+q+1)}}\left(\frac{2q+1}{2q+2}\right)(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{-2q-3}{2q+2}}\\
&\times\ (2^qM^{6q})^{\frac{1}{2(p+q+1)}}(2-\lambda)^{\frac{-1}{2q+2}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{p(\lambda+2q)}{2(q+1)(p+q+1)}}(-\Gamma(\alpha))^{\frac{-p}{p+q+1}}-p\\
&\times\ (2p+1)^{\frac{(p+q)(2q+3)+1}{2(q+1)(p+q+1)}}2^{\frac{2q(-p-q)-3(p+q)-1}{2(q+1)(p+q+1)}}(q^{-1}M^{6q}A\lambda(\lambda-1))^{\frac{q-p+1}{2(q+1)(p+q+1)}}\\
&\times\ ((2-\lambda)(t_e(\exp[N])^{\frac{-1}{A\lambda}}))^{\frac{-1}{q+1}}(q+1)^{-1}\left(\frac{-3-2q}{2q+2}\right)^{\frac{(\lambda+2q)(3p+q+1)}{2(q+1)(p+q+1)}}(-\Gamma(\alpha))^{\frac{-3p-q-1}{p+q+1}}\Bigg]\\
&\times\ \Bigg[(M^{6q}q^{-1}(2p+1)^{-1}A\lambda(\lambda-1))^{\frac{1}{p+q+1}}\left(\frac{-1}{q+1}\right)((2-\lambda)(t_e(\exp[N])^{\frac{-1}{A\lambda}}))^{\frac{-1}{q+1}}2^{\frac{-p-1}{p+q+1}}\\
&\times\ \left(\frac{-3-2q}{2q+2}\right)^{\frac{p(\lambda+2q)}{(q+1)(p+q+1)}}(-\Gamma(\alpha))^{\frac{-2p}{p+q+1}}-2(q+1)^{-1}q^{\frac{p(-2p-3)}{2(q+1)(p+q+1)}}(2^qM^{6q})^{\frac{2p(p-1)+3p}{2(q+1)(p+q+1)}}\\
&\times\ (A\lambda(\lambda-1))^{\frac{2p(p+2q)+4q(q+2)+5p+4}{2(q+1)(p+q+1)}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{(\lambda+2q)(-p-1+2q)}{2(q+1)(p+q+1)}}(2-\lambda)^{\frac{-2q-1}{2q+2}}\\
&\times\ (t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{-4q-3}{2q+2}}(-\Gamma(\alpha))^{\frac{2p(p+1)+q+1}{p+q+1}}-((2p+1)^{-1}2^qM^{6q})^{\frac{p^2}{(q+1)(p+q+1)}}\\
&\times\ (A\lambda(\lambda-1)q^{-1})^{\frac{p(p+q+1)+(q+1)^2}{(q+1)(p+q+1)}}\left(\frac{-3-2q}{2q+2}\right)^{\frac{-p^2(\lambda+2q)}{(q+1)(p+q+1)}}((t_e(\exp[N])^{\frac{-1}{A\lambda}})(2-\lambda))^{-1}\\
&\times\ (-\Gamma(\alpha))^{\frac{2p^2}{p+q+1}}\Bigg]^{-1}.
\end{aligned}
$$

The ratio can be written as

$$
\begin{array}{rcl}
r &=& 16\bigg[3(q+1)A\lambda(\lambda-1)(t_e(\exp[N])^{-\frac{1}{A\lambda}})^{-1}\bigg]^{\frac{-1}{2}}\bigg[2A\lambda(\lambda-1)(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{-1} \\
&\times& \ln(t_e(\exp[N])^{\frac{-1}{A\lambda}}) + 2^{\frac{-qp-2(q+1)(p+q+1)}{2(q+1)(p+q+1)}}(2p+1)^{\frac{2(q+1)(p+q+1)+p}{2(q+1)(p+q+1)}}q^{\frac{p-2(q+1)(p+q+1)}{2(q+1)(p+q+1)}} \\
&\times& (M^{6q}A\lambda(\lambda-1))^{\frac{-p}{2(q+1)(p+q+1)}}((t_e(\exp[N])^{\frac{-1}{A\lambda}})(2-\lambda))^{\frac{-1}{2q+2}}(2q+2)^{-1} \\
&\times& \bigg(\dfrac{-3-2q}{2q+2}\bigg)^{\frac{(\lambda+2q)(2p+q+1)}{2(q+1)(p+q+1)}}(-\Gamma(\alpha))^{\frac{-2p-q-1}{p+q+1}}\bigg]^{\frac{1}{2}}\bigg[\bigg(1-2^{\frac{2p(pq+2)+5pq+2q(q+3)+4}{2(q+1)(p+q+1)}}q^{\frac{2(q+1)-p(1+2p)}{2(q+1)(p+q+1)}} \\
&\times& (M^{6q})^{\frac{2p(p+1)-2(q+1)}{2(q+1)(p+q+1)}}(A\lambda(\lambda-1)(2p+1)^{-1})^{\frac{2p(p+q)+3p+2q(q+1)}{2(q+1)(p+q+1)}} \\
&\times& \bigg(\dfrac{-3-2q}{2q+2}\bigg)^{\frac{(\lambda+2q)(-2p^2-4p-q-1)}{2(q+1)(p+q+1)}}((t_e(\exp[N])^{\frac{-1}{A\lambda}})(2-\lambda))^{\frac{1-2q}{2q+2}}\bigg(\dfrac{1-2q}{2q+2}\bigg) \\
&\times& \ln(t_e(\exp[N])^{\frac{-1}{A\lambda}})(-\Gamma(\alpha))^{\frac{2p(p+2)+q+1}{p+q+1}}\bigg(3A\lambda(\lambda-1)(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{-1} \\
&-& (2p+1)^{\frac{p+2(q+1)(p+q+1)}{2(q+1)(p+q+1)}}q^{\frac{p-2(q+1)(p+q+1)}{2(q+1)(p+q+1)}}(2^qM^{6q}A\lambda(\lambda-1))^{\frac{-p}{2(q+1)(p+q+1)}} \\
&\times& \bigg(\dfrac{-3-2q}{2q+2}\bigg)^{\frac{(\lambda+2q)(2p+q+1)}{2(q+1)(p+q+1)}}((t_e(\exp[N])^{\frac{-1}{A\lambda}})(2-\lambda))^{\frac{-1}{2q+2}}(2q+2)^{-1}(-\Gamma(\alpha))^{\frac{-2p-q-1}{p+q+1}}\bigg)\bigg)^{-2} \\
&\times& \bigg(2(\lambda-3)^2(2-\lambda)^{\frac{1}{q+1}}\bigg(\dfrac{2q+3}{2q+2}\bigg)^2(2^qM^{6q}q^{-1}(2p+1)^{-1})^{\frac{-1}{p+q+1}}(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{-2q-1}{q+1}} \\
&\times& (A\lambda(\lambda-1))^{\frac{-1}{p+q+1}}\bigg(\dfrac{-3-2q}{2q+2}\bigg)^{\frac{p(-\lambda-2q)}{(q+1)(p+q+1)}}(-\Gamma(\alpha))^{\frac{2p}{p+q+1}} - q^{\frac{(p+q+1)(4q-2p+3)-2q(q+1)}{2(q+1)(p+q+1)}} \\
&\times& (2^qM^{6q})^{\frac{(p+q+1)(2p-2q-1)+2q(q+1)}{2(q+1)(p+q+1)}}(A\lambda(\lambda-1))^{\frac{p(2p-3)-4pq-q(9+4q)-5}{2(q+1)(p+q+1)}} \\
&\times& (2p+1)^{\frac{-(2p+1)(p+q+1)-2q(q+1)}{2(q+1)(p+q+1)}}\bigg(\dfrac{-3-2q}{2q+2}\bigg)^{\frac{(\lambda+2q)(-2p^2-3p-q-1)}{2(q+1)(p+q+1)}}\bigg(\dfrac{(2-\lambda)^{\frac{-q}{q+1}}(\lambda-3)}{2(q+1)}\bigg) \\
&\times& (t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{q+2}{q+1}}\ln(t_e(\exp[N])^{\frac{-1}{A\lambda}})\bigg(2^{\frac{(p+q+1)(2(pq+1)+q(1-2q))+2q^2(q+1)}{2(q+1)(p+q+1)}} \\
&\times& q^{\frac{(p+q+1)(2q-2p+1)-2q(q+1)}{2(q+1)(p+q+1)}}(M^{6q})^{\frac{(p+q+1)(2p-2q-1)+2q(q+1)}{2(q+1)(p+q+1)}}\bigg(\dfrac{\lambda-3}{q+1}\bigg) \\
&\times& (A\lambda(\lambda-1))^{\frac{(p+q+1)(2p+2q+3)+2q(q+1)}{2(q+1)(p+q+1)}}(2p+1)^{\frac{-(2p+1)(p+q+1)-2q(q+1)}{2(q+1)(p+q+1)}}(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{-3q-2}{q+1}} \\
&\times& \bigg(\dfrac{-3-2q}{2q+2}\bigg)^{\frac{(\lambda+2q)(-2p^2-3p-q-1)}{2(q+1)(p+q+1)}}(-\Gamma(\alpha))^{\frac{p(2p+3)+q+1}{p+q+1}} - 4(\lambda-3)(2-\lambda)^{\frac{1}{2q+2}} \\
&\times& (A\lambda(\lambda-1))^{\frac{4(p+q)+3}{2(p+q+1)}}(t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{-6q-5}{2q+2}}\bigg(\dfrac{2q+3}{2q+2}\bigg)(2^qM^{6q}q^{-1}(2p+1)^{-1})^{\frac{-1}{2(p+q+1)}}
\end{array}
$$

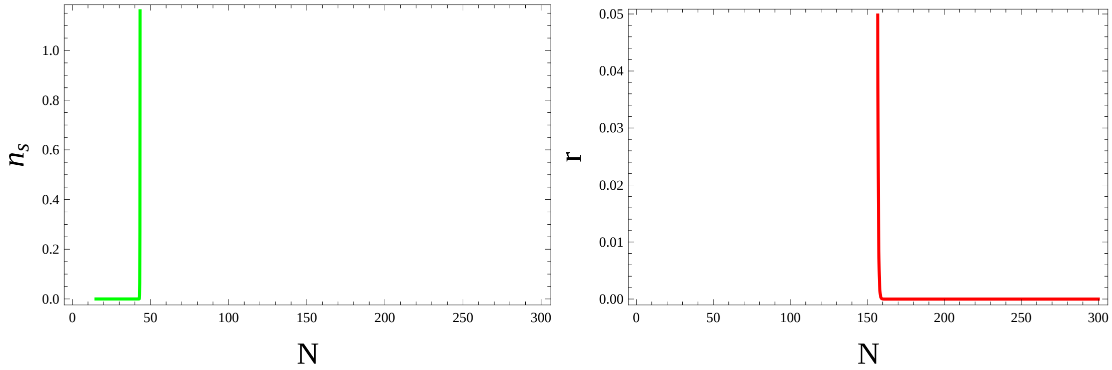

Figure 3.2: Left: $n_s - N$ trajectory; Right: $r - N$ trajectory for $\lambda = 4;\ p = 3;\ q = 2;\ A = 10^{-1};\ M = 10^2 M_p$.

$$\begin{aligned}
&\times \quad \left(\frac{-3-2q}{2q+2}\right)^{\frac{p(-\lambda-2q)}{2(q+1)(p+q+1)}} (-\Gamma(\alpha))^{\frac{p}{p+q+1}} \Big)\Big) - (2^q M^{6q} A\lambda(\lambda-1))^{\frac{-q-1+2p^2}{2(q+1)(p+q+1)}} \\
&\times \quad (t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{1}{2q+2}} \left(\frac{-4q-3}{2q+2}\right) \ln(t_e(\exp[N])^{\frac{-1}{A\lambda}})(2p+1)^{\frac{-2q(q+p)-3q-1-2p(p+1)}{2(q+1)(p+q+1)}} \\
&\times \quad \left(\frac{-3-2q}{2q+2}\right)^{\frac{(\lambda+2q)(p-2p(p+1))}{2(q+1)(p+q+1)}} (-\Gamma(\alpha))^{\frac{p(2p+1)}{p+q+1}} \Big(4q^{\frac{2q+1}{2q+2}} (2^q M^{6q}(2p+1)^{-1} A\lambda(\lambda-1))^{\frac{1}{2q+2}} \\
&\times \quad \left(\frac{-3-2q}{2q+2}\right)^{\frac{-\lambda-2q}{2q+2}} (-\Gamma(\alpha)) + (2p+1)^{\frac{1+2(p+q)}{2(p+q+1)}} (2-\lambda)^{\frac{-1}{2q+2}} (2^q M^{6q} q^{-1})^{\frac{1}{2(p+q+1)}} \\
&\times \quad (A\lambda(\lambda-1))^{\frac{-1-2(p+q)}{2(p+q+1)}} (t_e(\exp[N])^{\frac{-1}{A\lambda}})^{\frac{2q+1}{2q+2}} \left(\frac{-3-2q}{2q+2}\right)^{\frac{p(\lambda+2q)+2(q+1)(p+q+1)}{2(q+1)(p+q+1)}} \\
&\times \quad (-\Gamma(\alpha))^{\frac{-p}{p+q+1}} \Big) \ln(t_e(\exp[N])^{\frac{-1}{A\lambda}}) \Big].
\end{aligned}$$

Firstly, we would like to mention here that the parametric representation of $r - n_s$ plot is not possible in logamediate case because of different range of e-folding number for both graphs of Fig. **3.2**. The left $n_s - N$ and right $r - N$ trajectories are attained for constraint values of the model parameters given in Table **3.2**. For first set of values, the left trajectory falls in the allowed region of *Planck* data as $n_s = 0.968 < 1$ for number of e-folds less than 50. So, the $n_s - N$ trajectory of logamediate G-inflationary model suffers with the problem of less e-folding number as it does not lie in the standard region $1 \leq N \leq 70$.

The right plot of Fig. **3.2** shows that enough number of e-folds such that greater than 150 are required to locate r in the physical acceptable range as $r < 0.11$. It is observed that as logamediate model parameter λ increases in n_s expression, the e-folding number are approaches to zero. Moreover, the logamediate model is less sensitive for exponents p and q, as both plots of Fig. **3.2** lies in the physical acceptable region for all values of $p,\ q$ by fixing other parameters given in following table.

λ	p	q	A	M
4	3	2	1×10^{-1}	1×10^{2}
6.5	7	5	3×10^{-2}	1×10^{2}
9	9	9	7×10^{-2}	1×10^{2}
15	13	11	12×10^{-3}	1×10^{2}

Table 3.2: Constraints on model parameters to find physical acceptable range of n_s and r for $1 \leq N \leq 300$.

Now, we will implement the same procedure using the parametrization of *Hubble* parameter.

3.4 *G-inflation during Exponential Era:*

In this case, we used the following parametrization given in Eq.(2.10) we will achieve the DE as under

$$\dot{\phi}^2 + \frac{q\beta}{2^{q-1}M^{6q}}\phi^{2p+1}\dot{\phi}^{2q+1} = 2\beta\alpha e^{-\beta t}, \tag{3.43}$$

which leads us to field velocity as

$$\dot{\phi} = \left(\frac{Be^{-\beta t}}{D}\right)^{\frac{1}{2(p+q+1)}}\left(\frac{-2q-1}{\beta}\right)^{\frac{-2p-1}{2(p+q+1)}}. \tag{3.44}$$

The above equation is integrated to obtain solution of inflaton

$$\phi = \left(\frac{-2q-1}{\beta}\right)^{\frac{2q+1}{2(p+q+1)}} \left(\frac{Be^{-\beta t}}{D}\right)^{\frac{1}{2(p+q+1)}}, \tag{3.45}$$

here $D = \frac{q\beta}{2^{q-1}M^{6q}}$ and $B = 2\beta\alpha$.

Here, the perturbed parameters are evaluated as

$$\begin{aligned}
P_s &= \frac{1}{8\pi^2}(\alpha e^{-\beta t})^{\frac{9}{2}}\left[2\alpha e^{-\beta t} - \left(\frac{2p+1}{2q}\right)\left(\frac{-2q-1}{\beta}\right)^{-1}\right]^{-\frac{1}{2}}(3(q+1))^{\frac{1}{2}} \\
&\times \left[\frac{1}{2}\left(\frac{Be^{-\beta t}}{D}\right)^{\frac{1}{p+q+1}}\left(\frac{-2q-1}{\beta}\right)^{\frac{-2p-1}{p+q+1}} - 4q\alpha 2^{-q}M^{-6q}\left(\frac{B}{D}\right)e^{-2\beta t}\right. \\
&+ \left.(2p+1)2^{-q}M^{-6q}\left(\frac{-2q-1}{\beta}\right)^{-1}\left(\frac{Be^{-\beta t}}{D}\right)\right]^{-1}, \\
n_s &= 1 - \frac{\beta e^{\beta t}}{\alpha}\left[\frac{9}{2} - \alpha e^{-\beta t}\left(2\alpha e^{-\beta t} - \left(\frac{2p+1}{2q}\right)\left(\frac{-2q-1}{\beta}\right)^{-1}\right)^{-1}\right] + 4\alpha^{-1}2^{-q}M^{-6q} \\
&\times (e^{-\beta t})^{\frac{-1}{2(p+q+1)}}\left(\frac{B}{D}\right)\left(\frac{-2q-1}{\beta}\right)^{\frac{-2q-1}{2(p+q+1)}}\left[\alpha q\beta^{\frac{2p+1}{2(p+q+1)}}(e^{-\beta t})^{\frac{2(p+q)+3}{2(p+q+1)}}(-2q-1)^{\frac{2q+1}{2(p+q+1)}}\right. \\
&- q\alpha(2p+1)\left(\frac{-2q-1}{\beta}\right)^{\frac{-2p-1}{2(p+q+1)}}(e^{-\beta t})^{\frac{2(p+q)+3}{2(p+q+1)}} + \frac{p(2p+1)}{2}(e^{-\beta t})^{\frac{1}{p+q+1}} \\
&\times \left.\left(\frac{-2q-1}{\beta}\right)^{\frac{-4p-2q-3}{2(p+q+1)}}\right]\left[\frac{1}{2}\left(\frac{Be^{-\beta t}}{D}\right)^{\frac{1}{p+q+1}}\left(\frac{-2q-1}{\beta}\right)^{\frac{-2p-1}{p+q+1}} - 4q\alpha 2^{-q}M^{-6q}e^{-2\beta t}\right. \\
&\times \left.\left(\frac{B}{D}\right) + (2p+1)2^{-q}M^{-6q}\left(\frac{-2q-1}{\beta}\right)^{-1}\left(\frac{Be^{-\beta t}}{D}\right)\right]^{-1}.
\end{aligned}$$

Finally, we get r as follows

$$\begin{aligned}
r &= 16\left[\left(3\alpha e^{-\beta t}(q+1)\right)^{-1}\left(2\alpha e^{-\beta t} - \frac{2p+1}{2q}\left(\frac{-2q-1}{\beta}\right)^{-1}\right)\right]^{\frac{1}{2}}\left[\left(1 - q2^{1-q}M^{-6q}\right.\right. \\
&\times \left.\left(\frac{-2q-1}{\beta}\right)^{\frac{2p+1}{p+q+1}}\left(\frac{Be^{-\beta t}}{D}\right)^{\frac{P+q}{p+q+1}}\left(3\alpha e^{-\beta t} + \frac{2p+1}{q}\left(\frac{-2q-1}{\beta}\right)^{-1}\right)\right)^{-2} \\
&\times \left(2(-2q-1)^{\frac{2p+1}{p+q+1}}\beta^{\frac{2q+1}{p+q+1}}\left(\frac{Be^{-\beta t}}{D}\right)^{\frac{-1}{p+q+1}} - q\alpha^{-3}2^{-q}M^{-6q}\alpha^{-3}\beta^{\frac{q-p}{p+q+1}}(e^{-\beta t})^{\frac{8(p+q)+2}{p+q+1}}\right. \\
&\times \left.(-2q-1)^{\frac{2p+1}{p+q+1}}\left(\frac{B}{D}\right)^{\frac{2p+2q+1}{2(p+q+1)}}\left(-q\alpha 2^{1-q}M^{-6q}\left(\frac{B}{D}\right)^{\frac{2p+2q+1}{2(p+q+1)}} + 2\alpha^2\left(\frac{B}{D}\right)^{\frac{-1}{2(p+q+1)}}\right)\right) \\
&- \alpha^{-1}2^{-q}M^{-6q}(e^{-\beta t})^{\frac{-1}{2(p+q+1)}}\left(\frac{B}{D}\right)\left(\frac{-2q-1}{\beta}\right)^{\frac{-2q-1}{2(p+q+1)}}\left(4q(e^{-\beta t})^{\frac{1}{2(p+q+1)}}\right.
\end{aligned}$$

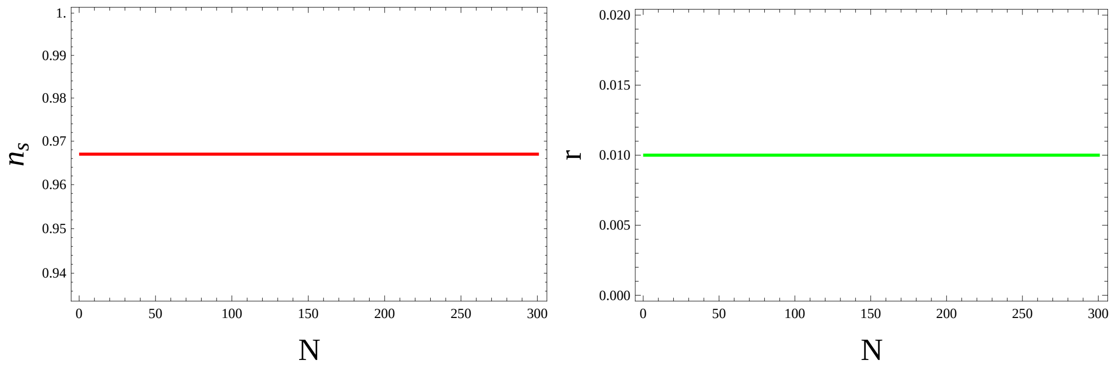

Figure 3.3: Left: $r - N$ trajectory; Right: $r - n_s$ trajectory for $\alpha = 0.5;\ \beta = 1.8;\ p = 3;\ q = 2;\ A = 10^{-1};\ M = 10^2 M_p$.

$$\times \left(\frac{-2q-1}{\beta}\right)^{\frac{2q+1}{2(p+q+1)}} + (2p+1)\alpha^{-1}\left(\frac{-2q-1}{\beta}\right)^{\frac{-2p-1}{2(p+q+1)}} (e^{-\beta t})^{\frac{-2p-2q-1}{2(p+q+1)}} \Bigg)\Bigg].$$

The spectral index versus e-folding number is plotted in left plot of Fig. **3.3** and tensor to scalar ratio is being plotted versus N in right plot of Fig. **3.3**. We have constraint the model parameters using this parametrization of Hubble parameter. It is quite clear from both graphs that chosen parametric set of values leads to generate standard results as $n_s = 0.968$ and $r = 0.010 < 0.11$. Further, it is noted that the parameter $\alpha \leq 7$ for best fit values of n_s and r. The values of other parameters is not bounded and can vary over the whole domain.

Chapter 4

Concluding Remarks

This thesis is devoted to explore a model with inflation inspired by GIF and how this model is being modified by introducing a generalized coupling form $G(\phi, X) = \frac{\phi^{2p+1}X^q}{M^{6q}}$, in the action. The analytical solutions of the inflaton field and corresponding potential under SR approximation are obtained using specific evolution of the isotropic scale factor, *i.e.*, intermediate, logamediate and exponential. Consistency of the predictions of the model with observations is being done by computing the scalar-tensor power spectra, the scalar spectral index as well as the tensor-scalar ratio. For all the three models with inflation, we constraint the involved model parameters for which its predictions lie inside the allowed region from *Planck* data.

To check the consistency of the considered model, we plot the trajectories for perturbed parameters. In Fig. **3.1**, $r-n_s$ trajectory is plotted for constraint model parameters. This trajectory lies in the allowed region of *Planck* data, which ultimately verifies that intermediate form of scale factor is fitted well with observation data. The other sets of constraint values are given in Table **3.1**, which also provide compatible results. Logamediate model with inflation is constrained for good compatibility with *Planck* 2015 data and represented in left and right plots of Fig. **3.2**. In this case, n_s and r are plotted versus e-folding number. It is clear from both of the graphs that n_s shows compatible results for less e-folding number while r lies in compatible region for enough e-folding number. Due to aforementioned reason, in this case, we are unable to draw a parametric plot $r - n_s$. The corresponding model parameters are constraint in Table **3.2**. In the last section, the process is repeated using parametrization of *Hubble* parameter. In this situation, we get linear graph in left plot of Fig. **3.3**, which gives exact value of $n_s = 0.967$ throughout the domain of N. Whereas the right plot $r - N$ of Fig. **3.3** generates a fixed value of $r = 0.010$ for all N. The trajectory remains the same for all other values of the model parameters.

It is worth mentioning here that our model is the generalization of all the previous work. It can recovered already existed results on G-inflationary model for specific values of p and q. For example, if $p = -\frac{1}{2}$ [38], then coupling parameter becomes function of X, only while for $q = 0$ [37], it reduced to function of inflaton field; for $q = 0;\ p = 0$ [39], the coupling G becomes a linear function of inflaton.

The paper is submitted for publication in an international journal. Moreover, as a future work, the reconstruction of these models is under process.

Chapter 5

References

Bibliography

[1] de Haro, J., Pan, S. and Areste Salo, L.: arXiv:1903.01181v1.

[2] Guth, A.H.: Phys. Rev. D **23**(1981)347.

[3] Linde, A.D.: Phys. Lett. B **108**(1982)389.

[4] Burd, A.B. and Barrow, J.D.: Nucl. Phys. B **308**(1988)929.

[5] Barrow, J.D.: Phys. Lett. B **235**(1990)40.

[6] Barrow, J.D.: Phys. Rev. D **49**(1994)3055.

[7] Chibisov, G. and Mukhanov, V.: Mon. Not. Roy. Astron. Soc. **200**(1982)535.

[8] Starobinsky, A.A.: Phys. Lett. B **117**(1982)175.

[9] Guth, A.H. and Pi, S.Y.: Phys. Rev. Lett. **49**(1982)1110.

[10] Bardeen, J.M., Steinhardt, P.J. and Turner, M.S.: Phys. Rev. D **28**(1983)679.

[11] Linde, A.D.: Phys. Lett. B **116**(1982)335.

[12] Mukhanov, V.: *Physical Foundations of Cosmology* (Cambridge University Press, 2005); Lyth, D. and Liddle, A.: *Primordial Density Perturbations* (Cambridge University Press, 2009).

[13] Kolb, E.W. and Turner, M.S.: *The Early Universe* (Addison-Wesley, 1990).

[14] Sanyal, A.K.: Phys. Lett. B **645**(2007)1.

[15] Lucchin, F. and Matarrese, S.: Phys. Rev. D **32**(1985)1316.

[16] Myrzakulov, R. and Sebastiani, L.: Astrophys. Space Sci. **357**(2015)5.

[17] Nicolis, A., Rattazzi, R. and Trincherini, E.: Phys. Rev. D **79**(2009)064036.

[18] Defayet, C., Esposito-Farese, G. and Vikman, A.: Phys. Rev. D **79**(2009)084003.

[19] Defayet, C., Deser, S. and Esposito-Farese, G.: Phys. Rev. D **80**(2009)064015.

[20] Choudhury, S. and Pal, S.: Eur. Phys. J. C **75**(2015)241; Kumar, K.S. et al.: JCAP **02**(2016)063; Bazrafshan Moghaddam, H., Brandenberger, R. and Yokoyama, J.: Phys. Rev. D **95**(2017)063529; Motaharfar, M., Massaeli, E. and Sepangi, H.R.: Phys. Rev. D **96**(2017)103541; Banerjee, R., Chakraborty, S., Mitra, A. and Mukherjee, P.: Phys. Rev. D **96**(2017)064023; Banerjee, R. and Mukherjee, P.: Class. Quant. Grav. **34**(2017)235005.

[21] Horndeski, G.W.: Int. J. Theor. Phys. **10**(1974)363.

[22] Bassett, B.A., Tsujikawa, S. and Wands, D.: Rev. Mod. Phys. **78**(2006)537.

[23] Setare, M.R., Kamali, V.: Phys. Lett. B **726**(2013)56.

[24] Sharif, M. and Saleem, R.: Eur. Phys. J. C **74**(2014)2738; AstroPart. Phys. **361**(2016)107.

[25] Sharif, M. and Saleem, R.: Eur. Phys. J. C **74**(2014)2943.

[26] Sharif, M. and Saleem, R.: J. Cosmol. Astropart. Phys. **12**(2014)038; Astropart. Phys. **62**(2015)100; Mon. Not. Roy. Astro. Soc. **450**(2015)3802.

[27] Jawad, A., Hussain, S., Rani, S. and Videla, N.: Eur. Phys. J. C **77**(2017)808.

[28] Setare, M., Ravanpak, A. and Farajollahi, H.: Phys. Rev. **24**(2018)52.

[29] Chen, J., Wenjie, H., Hou, D. and Qiu, T.: **42**(2018)045102.

[30] Sadjadi, H.M. and Goodarzi, P.: Phys. Lett. B **732**(2014)278.

[31] Damour, T. and Mukhanov, V.F.: Phys. Rev. Lett. **80**(1998)3440.

[32] Saleem, R.: Astrophys. Space Sci. **363**(2018)136.

[33] Qiu, T., Phys. Rev. D **93**(2016)123515.

[34] Horndeski, G.W.: Int. J. Theor. Phys. **10**(1974)363.

[35] Charmousis, C., Copeland, E.J., Padilla, A. and Saffin, P.M.: Phys. Rev. Lett. **108**(2012)051101.

[36] Kousar, F., Saleem, R. and Zubair, M.: Adv. High Ener. Phys. **4**(2018)1.

[37] Teimoori, Z. and Karami, K.: Astrophys. J. **864**(2018)41.

[38] Ohashi, J. and Tsujikawa, S.: JCAP **1210**(2012)035.

[39] Herrera, R.: Phys. Rev. D **98**(2018)023542.

[40] Motaharfar, M., Massaeli, E. and Sepangi, H.R.: Phys. Rev. D **96**(2017)103541.

[41] Vieira, J. P.: arXiv:1903.11472.

[42] Bintruy, P. and Dvali, G.R.: Phys. Lett. B **388**(1996)241.

[43] Barnaby, N. and Cline, J.M.: Phys. Rev. D **70**(2004)023506.

[44] Koivisto, T.S., Mota, D.F. and Zumalacarregui, M.: Phys. Rev. Lett. **109**(2012)241102.

[45] Martin, J.: Universe **5**(2019)92.

[46] Yokoyama, J. and Maeda, K.: Phys. Lett. B **207**(1988)31.

[47] Sen, A.: Mod. Phys. Lett. A **17**(2002)1797.

[48] Sami, M., Chingangbam, P. and Qureshi, T.: Phys. Rev. D **66**(2002)043530.

[49] Gibbons, G.W.: Phys. Lett. B **537**(2002)1.

[50] Defayet, C., Gao, X., Steer, D.A. and Zahariade, G.: Phys. Rev. D **84**(2011)064039.

[51] Kobayashi, T., Yamaguchi, M. and Yokoyama, J.: Phys. Rev. Lett. **105**(2010)231302.

[52] Kobayashi, T., Yamaguchi, M. and Yokoyama, J.: Phys. Rev. D **83**(2011)103524.

[53] Garriga, J. and Mukhanov, V.F.: Phys. Lett. B **458**(1999)219.

[54] Bartolo, N., Komatsu, E., Matarrese, S. and Riotto, A.: Phys. Rept. **402**(2004)103.

[55] Renaux-Petel, S.: Comptes Rendus Phys. **16**(2015)969.

[56] Gangui, A., Lucchin, F., Matarrese, S. and Mollerach, S.: Astrophys. J. **430**(1994)447.

[57] Acquaviva, V., Bartolo, N., Matarrese, S. and Riotto, A.: Nucl. Phys. B **667**(2003)119.

[58] Maldacena, J.M.: J. High Ener. Phys. **0305**(2003)013.

[59] Ade, P.A.R. et al.: [Planck Collaboration], Astron. Astrophys. **594**(2016)A20.

[60] Babich, D., Creminelli, P. and Zaldarriaga, M.: J. Cosmol. Astropart. Phys. **0408**(2004)009.

[61] Seery, D. and Lidsey, J.E.: J. Cosmol. Astropart. Phys. **0509**(2005)011.

[62] Sasaki, M., Valiviita, J. and Wands, D.: Phys. Rev. D **74**(2006)103003.

[63] Wang, L.M. and Kamionkowski, M.: Phys. Rev. D **61**(2000)063504.

[64] Verde, L., Wang, L.M., Heavens, A. and Kamionkowski, M.: Mon. Not. Roy. Astron. Soc. **313**(2000)L141.

[65] Chen, X., Huang, M.X., Kachru, S. and Shiu, G.: JCAP **0701**(2007)002.

[66] Senatore, L., Smith, K.M. and Zaldarriaga, M.: JCAP **1001**(2010)028.

[67] De Felice, A. and Tsujikawa, S.: JCAP **1303**(2013)030.

[68] Motaharfar, M., Massaeli, E. and Sepangi, H.R.: JCAP **1810**(2018)002.