On Fermatean Fuzzy Bipolar Soft Topological Space

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I Zainab Zaka, CIIT/FA21-RMT-056/LHR, hereby state that my MS thesis titled “On Fermatean Fuzzy Bipolar Soft Topological Space” is my own work and has not been submitted previously by me for taking any degree from this University ”COMSATS University Islamabad, Lahore Campus” or anywhere else in the country/world. At any time if my statement is found to be incorrect even after my Graduate the university has the right to withdraw my MS degree.

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Certificate

It is certified that Zainab Zaka, CIIT/FA21-RMT-056/LHR has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus and the work fulfills the requirement for award of MS degree.

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DEDICATION

To My Parents and All Family

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Praise to be ALLAH, the Cherisher and Lord of the World, Most gracious and Most Merciful

First and foremost, I would like to thank ALLAH Almighty (the most beneficent and most merciful) for giving me the strength, knowledge, ability and opportunity to undertake this research study and to preserve and complete it satisfactorily. Without countless blessing of ALLAH Almighty, this achievement would not have been possible. May His peace and blessings be upon His messenger Hazrat Muhammad (PBUH), upon his family, companions and whoever follows him. My insightful gratitude to Hazrat Muhammad (PBUH) Who is forever a track of guidance and knowledge for humanity as a whole. In my journey towards this degree, I have found a teacher, an inspiration, a role model and a pillar of support in my life, my kind.

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ABSTRACT

On Fermatean Fuzzy Bipolar Soft Topological Space

In the current thesis, on fermatean fuzzy bipolar soft topological space is investigated. A fermatean fuzzy set is converted by using bipolar soft set in topological space. The soft sets are a family of parameterized sets. The concepts of a soft neighborhood of a point, a soft open set, and a soft closed set are introduced. Soft topological space offers variety of topological spaces that are parameterized.

The **BSS** is made with the two **SS**. One gives negative information while the other one gives us positive information. Two mappings are used to explain the **FBSS**. In **FBS**, one mapping is used to approximate fuzziness relative to the degree of positivity while another mapping is used to approximate fuzziness relative to the degree of negativity in the initial universal set objects. To cope with uncertainty in various real-world condition, a reducing mathematical technique called fermatean fuzzy set is being developed. **FFS** is more flexible than intuitionistic and **PFS**.

The result are evaluated in on **FFBSTS** and **FFBS**. We work on some features such as neighborhood, continuity, and others.

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List of Abbreviation

- BSS Bipolar Soft Set
- BSTS Bipolar Soft Topological Space
- FS Fuzzy Set
- FBSS Fuzzy Bipolar Soft Set
- FBSTS Fuzzy Bipolar Soft Topological Space
- FFS Fermatean Fuzzy Set
- FFTS Fermatean Fuzzy Topological Space
- FFSS Fermatean Fuzzy Soft Set
- FBSS Fuzzy Bipolar Soft Set
- FFBSS Fermatean Fuzzy Bipolar Soft Set
- FFBSTS Fermatean Fuzzy Bipolar Soft Topological Space
- FFBSCS Fermatean Fuzzy Bipolar Soft Close Set
- FFBSP Fermatean Fuzzy Bipolar Soft Point
- FFBSOS Fermatean Fuzzy Bipolar Soft Open Set
- FFBSIP Fermatean Fuzzy Bipolar Soft Interior Point
- FFBSINT Fermatean Fuzzy Bipolar Soft Interior
- FFBSCL Fermatean Fuzzy Bipolar Soft closure
- FFBSE Fermatean Fuzzy Bipolar Soft Exterior
- IFS Intuitionistic Fuzzy Set
- PFS Pythagorean Fuzzy Set
- SS Soft Set

Chapter 1

Introduction

Fuzziness occurs when boundaries of information are not clear. The literal definition of fuzzy is inability to think clearly. In literature the word fuzzy is often used for vagueness. Zadeh [29] invented the fuzzy set theory in 1965. In 1965, the mathematician Zadeh developed a framework for handling uncertainty on fuzzy sets.Zhang developed the notion of **BFSS**, also known as Yin-Yang **BFSS**, in which Yin and Yang stand for a system's positive and negative functions, respectively. A number that has a membership value of 0 indicates that it does not satisfy the related property, whereas a value of 1 suggests that it does. This notion was put forth to address issues with bipolarity.In bipolarity, fuzziness with a hybrid-valued fuzzy variable with negative and positive aspects is encoded as bipolar fuzziness. The theory of **BFSS** unifies fuzziness and polarity into a single model and gives a groundwork for coordination, and bipolar clustering.

A Russian study The first time a **SS** theory was created in 1999 by Molodtsov [20]. By using several instances, He work on various Mathematical operations **AND**, **OR**, **Union**, and **Intersection** of two **SS**. The idea of **SS** has been proposed by mathematics to apply to the algebraic model. The **FSS** notion, Which is solely dependent in models for soft sets and **FSS**.

The concepts of **FSS** and **BSS** [23], The discovery of **FBSS** was Introduced by *M.NAZ* and *M.SHABIR* [21]. It introduced concepts of **FBSS** and produce algebra structure also its application.The mathematical technique that links bipolarity and Soft theory is called Bipolar Soft Set theory. Two soft sets make one Bipolar Soft set, one gives us information that is positive and the other that is negative. According to bipolarity theory, the decisions that we make depend on both positive and negative values, and we favor the more compelling of the two. The **BSS** With by using Two function we can Say $F : S \rightarrow P(v)$ and $G : \neg S \rightarrow P(v)$ Where ¬ S is not set, and G describes opposite or negative approximation. A **BSS** over v The Condition $\digamma(\beth) \cap \daleth(\neg\beth) = \phi \forall \beth$. The disparity occurs because the membership function was not defined clearly, or because there was doubt or confusion. As a rough approximation for the degree of delay, we refer to this doubt or grey region.

Zhang [21] introduces the fuzzy bipolar set. Two mappings are used to create **FBSS** one is estimate fuzziness relative degree in positivity or that parameter presence in **FBSS** while the other one is to estimate the fuzziness relative degree in negativity or that parameter

absence in the initial universal set objects. Munazza Naz Bipolarity, fuzziness, and parameterization are three ideas that we have integrated. which the membership function accepts value in 0,1 Taha Yasin A **FB** elements membership function can be defined. The function is an accepted value between 0 and 1 then it is positively connected to the property, meaning that partially satisfies it. On Bipolar Soft subspace topology the notion of Bipolar Soft separation axioms was defined and a consideration of their inheritance followed.
If $(H^+, H^-, k) \in BS(s), \tau$ is the group of the Bipolar Soft subset, the parameter set Is K so, τ is called **BST** Asmaa Fadel Bakhtawar and Shabir introduced the bipolar soft topology [11].
Senapati and Yager (2020) are the ones who first introduced the Fermatean Fuzzy Set is particular instance of qrung orthopair fuzzy set [25]. The claim of Yager's (2017) theory of orthopair, the power of q is non-membership and membership degrees must be ≤ 1. It follows that as q rises, the range of admissible orthopair will as well, and this geometric range gives users more opportunity to make decisions and to demonstrate their preferences, theories, and claims. Yager (2014) renamed the ortho pair fuzzy set as Pythagorean fuzzy set (PFS) by setting $q = 2$ and created fundamental operations on them. In 2020, $q = 3$ referred to this new qROFS as the Fermatean Fuzzy set (FFS) as defined by Yager and Senapati. Because they can express their views on not membership, and membership about a subject's condition, the decision makers have more choice. Bipolar on soft topological space, intelligent **Fs**, **BS**, and Bipolar valued **FS** are all examples.
A theory named fermatean fuzzy set is a reduction method that is being developed to deal with uncertainty in a variety of real-world situations. To make uncertain data from generalized reality decision making situations tractable mathematically Fermatean Fuzzy sets were introduced [25]. A more concise and special case of qROFS is FFS. It offers decision makers a greater representation option than PFs and IFs. Some key characteristics FFBSS model that explored numerical example are subsets, equal FFBSS, relative null, $\cup$ and $\cap$, relative absolute **FFBSS**, OR operation, and AND operation . Fermatean fuzzy topological space is studying some fundamental characteristics of the ideas of image and concerning a function of the preimage of a **FFS**. Nonempty set on X, We create Coarsest **FFT**. We introduced to fermatean fuzzy bipolar soft topology space.

Fuzzy topology is branch of mathematics that makes use of fermatean fuzzy bipolar soft topological spaces. Studying and creating mathematical frameworks for simulating and analyzing ambiguity and vagueness in topological spaces is the goal of this field of study. The construction of fuzzy sets and fuzzy neighbourhoods, which can express ambiguity and uncertainty in the characterization of points and their interactions in a topological space, is made possible by fuzzy topology, an extension of classical topology.

Exploring the characteristics and future applications of Fermatean fuzzy bipolar soft topological spaces in areas like decision-making, pattern recognition, and control systems is the major objective of this field of study [9]. Additionally, researchers want to strengthen the mathematical underpinnings of fuzzy topology and provide algorithms and methods for resolving issues in these areas.In general, research on Fermatean fuzzy bipolar soft topological spaces seeks to provide a mathematical framework for modeling and analyzing uncertainty and vagueness in topological spaces, as well as investigate its potential and applications in diverse domains.

Fuzzy topology makes use of mathematical concepts called Fermatean fuzzy bipolar soft topological spaces. They are a particular kind of fuzzy topological space that incorporates **SS**, **BFS**, and **FFS**SS.

The sort of **FS** is called **FFS** is based on notion of fermatean function, which is a function that gives each element of a set a value between 0 and 1 to indicate how much of the set it is a part of. An example of a mathematical framework that enables the modeling of uncertainty in the definition of sets is soft sets. A unique and complex field of mathematics is fermatean fuzzy bipolar soft topological space. It tries to give topological structures a more adaptable and thorough foundation for dealing with ambiguity, vagueness, and bipolarity.

This concepts soft topological space component offers a flexible idea of set openness and set closeness [18]. Where components are given varying degrees of belongings to a set, as an alternative to depending simply on crisp open and closed sets. This enables the concept of neighborhood and closeness to be treated in a more flexible and progressive manner. Using this model as a starting point, we explore the features, topologies, and operations of Fermatean fuzzy bipolar soft topological spaces. Its goal is to lay a mathematical basis that may be used in a variety of real-world situations where softness, bipolarity, and uncertainty are naturally present.

The key objectives of this research are to create relevant axioms, examine continuity, compactness, and connectedness within this framework, and investigate applications in artificial intelligence. A difficult but promising field of research, understanding and improving the theory of Fermatean fuzzy bipolar soft topological spaces has the potential to progress mathematics and its application to practical issues.

A **FFSS** in X is denoted set and the function denote with the notation of $(\mathbf{F}, \mathbf{G}, \mathbf{A})$. Where $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{A}$ are relative degree membership and nonmembership, and ambiguity. Each element in U is given degree from $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{A}$, indicating whether it belongs, doesn't belong, or is unclear inside a collection. This mathematical formulation offers a framework for investigating topological qualities in the context of Fermatean fuzzy soft sets, including continuity, compactness, connectedness, and convergence. It enables a more adaptable and delicate representation of ambiguity, membership degrees, and uncertainty inside the topological structure.

Chapter 2

Fuzzy Set Theory

2.1 Fuzzy set

Definition

A **FS** is a group of objects with various degrees of membership. A **FS** Z is defined by a mapping and ranges from 0 to 1, using the membership function v_Z.

$$v_Z : U \to [0,1]$$

Example:-

$U = \{1,2,3,4,5,6\},\ u \in U$

Now

$$\begin{aligned} &u = 1,\ v_Z(1) = 0.5 \qquad && u = 2,\ v_Z(2) = 0.8 \\ &u = 3,\ v_Z(3) = 1 && u = 4,\ v_Z(4) = 0.8 \\ &u = 5,\ v_Z(5) = 0.5 && u = 6,\ v_Z(6) = 0.1. \end{aligned}$$

$Z = \{(1,0.5),(2,0.8),(3,1),(4,0.8),(5,0.5),(6,0.1)\}$

2.1.1 Soft Set

In order to cope with new ideas defined by uncertain and confusing facts, Molodtsov [20] created set theory in 1982. When examining a few complicated situations, experts might demand right to make wise decision. Results of different parameter settings enable this. The composition of parameters is not taken into account in classical set theory.The notion was inspired by the constraints on parameterization that previous theories encountered while dealing with ambiguous and fuzzy data.

Definition

If E the collection of parameter and U is universe of course, W is nonempty subset in E. Let $P(U)$ is power in U. Over U, pair (Z,W) are referred as **SS**. The mapping is

$$Z : A \to P(U)$$

Example

$U = \{1,2,3,4,5,6\}$

The parameter is $W = \{w_1, w_2, w_3, w_4, w_5\}$, $W \subseteq E$

$Z(w_1) = \{2,4\}, Z(w_2) = \{1,3,5\}, Z(w_3) = \{3,4,5\}$

$(Z,W) = \{(2,4),(1,3,5),(3,4,5)\}$

(Z,W) is the set of ordered pairs.

$(Z,W) = \{w_1, Z(w_1)\}, \{w_2, Z(w_2)\}, \{w_3, Z(w_3)\}$

$(Z,W) = \{w_1,(2,4)\}, \{w_2,(1,3,5)\}, \{w_3,(3,4,5)\}$.

Complement

Complement of **SS** $(\digamma,\beth)$ is $(\digamma,\beth)'$. Where the mapping $\digamma' : \beth \to U$. So, $(\digamma,\beth)' = U - \digamma(s)$, $\forall s \in \beth$.

Proposition 2.1.1. *Let* $(\digamma,\beth)$ *and* $(\daleth,\beth)$ *be the soft set in* U.

$[1]((\digamma,\beth) \cup (\daleth,\beth))' = (\digamma,\beth)' \cap (\daleth,\beth)'$,

$[2]((\digamma,\beth) \cap (\daleth,\beth))' = (\digamma,\beth)' \cup (\daleth,\beth)'$.

Proof. (1) Let $(\digamma,\beth) \cup (\daleth,\beth) = (H,\beth)$, where $H(\beth) = \digamma(\beth) \cup \daleth(\beth)$, $\forall \beth \in \beth$. Then

$$\begin{aligned} H'(\beth) &= (\digamma(\beth) \cup \daleth(\beth))^c \\ &= \digamma(\beth)^c \cap \daleth(\beth)^c \\ &= (\digamma^c(\beth) \cap \daleth^c(\beth)),\ \forall \beth \in \beth. \end{aligned}$$

Thus

$$(H,\beth)' = (R,\beth)' \cap (\daleth,\beth)', i.e. ((\digamma,\beth) \cup (\daleth,\beth))' = (\digamma,\beth)' \cap (\daleth,\beth)'.$$

(2) Let $(\digamma,\beth)\cap(\daleth,\beth)=(H,\beth)$, where $H(\beth)=\digamma(\beth)\cap\daleth(\beth),\ \forall\beth\in\beth$. Then

$$\begin{aligned}
H'(\beth) &= (\digamma(\beth)\cap\daleth(\beth))^c \\
&= (\digamma(\beth))^c\cup(\daleth(\beth))^c \\
&= (F^c(\beth))\cup(G^c(\beth)),\ \forall\beth\in\beth.
\end{aligned}$$

Thus

$$(H,\beth)'=(\digamma,\beth)'\cup(\daleth,\beth)',\ \text{i.e.}\ ((\digamma,\beth)\cap(\daleth,\beth))'=(\digamma,\beth)'\cup(\daleth,\beth)'.$$

□

Soft Topological Space

The soft space in X is (X,τ,S). Then the collections is

$\tau_a=F(a)|(F,S)\in\tau$ for each $a\in S$, define topology in X. Now

$c[1]:\phi,X\in\tau$.

$c[2]$: If $F_i(A)|i\in I$ is the collection of set in τ_a then $(F_i,S)\in\tau$, $\forall\iota\in I$ so,$\bigcup_{\iota\in I}(F_i,S)\in\tau$ then $\bigcup_{\iota\in\mathrm{I}}F_i(A)\in\tau_A$.

$c[3]$: if $(F_a,G_a)\in\tau_a$ for $(F,S)(G,S)\in\tau$ then (F, S)$\cap(G,S)\in\tau_a$.

τ is called soft topology in (X,A)

Example :

Let $U=\{v_1,v_2,v_3\},\ S=\{s_1,s_2\}$ and $\tau=\{\phi,U,(\digamma_1,s),(\digamma_2,s),(\digamma_3,s),(\digamma_4,s)\}$,

where $(\digamma_1,s),\ (\digamma_2,s),\ (\digamma_3,s),\ (\digamma_4,s)$ are soft sets over U, defined as follows

$$\begin{array}{ll}
\digamma_1(s_1)=\{v_2\} & \digamma_1(s_2)=\{v_1\} \\
\digamma_2(s_1)=\{v_2,v_3\} & \digamma_2(s_2)=\{v_1,v_2\} \\
\digamma_3(s_1)=\{v_1,v_2\} & \digamma_3(s_2)=U \\
\digamma_4(s_1)=\{v_1,v_2\} & \digamma_4(s_2)=\{v_1,v_3\}
\end{array}$$

Then τ define **ST** in U. Hence (U,τ,S) is **STS** in U.

$\tau_{s_1}=\{\phi,U,\{v_2\},\{v_2,v_3\},\{v_1,v_2\}\}$.

And

$\tau_{e_2} = \{\phi, U, \{v_1\}, \{v_1, v_3\}, \{v_1, v_2\}\}$

are topologies on U.

Now the two soft topologies may not be union in U.

Example :

Let $U = \{v_1, v_2, v_3\},\ S = \{s_1, s_2\}$ and $\tau_1 = \{\phi, X, (F_1, s),\ (F_2, s),\ (F_3, s),\ (F_4, s)\},\ \tau_2 = \{\phi, X, (\daleth_1, \neg s),\ (\daleth_2, \neg s),\ (\daleth_3, \neg s),\ (\daleth_4, \neg s)\}$ be two soft topologies defined on U where $(F_1, s),\ (F_2, s),\ (F_3, s),\ (F_4, s),\ (\daleth_1, \neg s),\ (\daleth_2, \neg s),\ (\daleth_3, \neg s),\ (\daleth_4, \neg s)$ is soft sets over U, defined as follows

$$F_1(s_1) = \{v_2\} \qquad F_1(s_2) = \{v_1\}$$

$$F_2(s_1) = \{v_2, v_3\} \qquad F_2(s_2) = \{v_1, v_2\}$$

$$F_3(s_1) = \{v_1, v_2\} \qquad F_3(s_2) = U$$

$$F_4(s_1) = \{v_1, v_2\} \qquad F_4(s_2) = \{v_1, v_3\}$$

And

$$\daleth_1(\neg s_1) = \{v_2\} \qquad \daleth_1(\neg s_2) = \{v_1\}$$

$$\daleth_2(\neg s_1) = \{v_2, v_3\} \qquad \daleth_2(\neg s_2) = \{v_1, v_2\}$$

$$\daleth_3(\neg s_1) = \{v_1, v_2\} \qquad \daleth_3(\neg s_2) = \{v_1, v_2\}$$

$$\daleth_4(\neg s_1) = \{v_1, v_2\} \qquad \daleth_4(\neg s_2) = \{v_1, v_3\}$$

Now, $\tau = \tau_1 \cup \tau_2$

$\tau = \{\phi, U, (F_1, s), (F_2, s), (F_3, s), (F_4, s), (\daleth_3, \neg s), (\daleth_4, \neg s)\}$.

if

$(F_2, S) \cup (\daleth_3, \neg S) = (H, S)$

then

$$
\begin{aligned}
H(s_1) &= F_2(s_1) \cup \urcorner_3\neg(s_1) \\
&= \{v_2, v_3\} \cup \{v_1, v_2\} \\
&= U
\end{aligned}
$$

and

$$
\begin{aligned}
H(s_2) &= F_2(s_2) \cup \urcorner_3\neg(s_2) \\
&= \{v_1, v_2\} \cup \{v_1, v_2\} \\
&= \{v_1, v_2\}
\end{aligned}
$$

But $(H, s) \notin \tau$. Thus τ is not a soft topology in X.

2.1.2 Bipolar Soft Set

The term **BSSs** was coined by Munazza to address the issue of bipolarity [23]. Bipolar soft set theory is a branch of mathematics that links bipolarity to soft set theory. Two soft sets serve as its definition; one provides us with knowledge that is positive, the other with information that is negative.

Definition

The triplet $(F, \urcorner, S)$ is **BSS** In U, Where F and T are mapping shows $F : S \to P(U)$ and $\urcorner : \neg S \to P(U)$, then $F(e) \cap \urcorner(\neg e) = \phi$, $\forall e \in S$.

Example:

$U = \{1,2,3,4,5,6\}$

The set of parameters is $O = \{o_1, o_2, o_3, o_4, o_5\}$, $A \subseteq O$. Then $\neg O = \{\neg o_1, \neg o_2, \neg o_3, \neg o_4, \neg o_5\}$

$$F(o_1) = \{2,4\}, F(o_2) = \{1,3,5\}, F(o_3) = \{3,4,5\}$$

$$\urcorner\neg(o_1) = \{3,5\}, \urcorner(\neg o_2) = \{2,4\}, \urcorner(\neg o_3) = \{2,1\}$$

$$(\digamma, A) = \{(2,4),(1,3,5),(3,4,5)\}$$

$$(\daleth, \neg B) = \{(3,5),(2,4),(2,1)\}$$

$(\digamma, A),(G,\neg B)$ is ordered pair.

$$(\digamma, A) = \{o_1, \digamma(o_1)\}, \{o_2, \digamma(o_2)\}, \{o_3, \digamma(o_3)\}$$

$$(\digamma, A) = \{o_1, (2,4)\}, \{o_2, (1,3,5)\}, \{o_3, (3,4,5)\}$$

$$(\daleth, \neg B) = \{o_1, \daleth(\neg o_1)\}, \{e_2, \daleth(\neg o_2\}, \{o_3, \daleth(\neg o_3)\}$$

$$(\daleth, \neg B) = \{o_1, (3,5)\}, \{o_2, (2,4)\}, \{o_3, (2,1)\}$$

Proposition 2.1.2. *Let $(\digamma, \daleth, S)$ and $(\digamma_1, \daleth_1, E)$ be two* ***BSSs*** *in universe U. The following is :*
$(1)((\digamma, \daleth, S) \cup (\digamma_1, \daleth_1, E))^c = (\digamma, \daleth, S)^c \cap (\digamma_1, \daleth_1, E)^c$,
$(2)((\digamma, \daleth, S) \cap (\digamma_1, \daleth_1, E))^c = (\digamma, \daleth, S)^c \cup (\digamma_1, \daleth_1, E)^c$,

Proof. (1) Let $\gimel \in S \cup E$. The three cases:
(i) If $\gimel \in S - E$, Then

$$(\digamma \cup \digamma_1)^c(\gimel) = (\digamma(\gimel))^c = (\digamma^c \cap (\digamma_1)^c)(\gimel)$$

$$(\daleth \cup \daleth_1)^c(\gimel) = (\daleth(\gimel))^c = (\daleth^c \cap (\daleth_1)^c)(\gimel)$$

(ii) If $\gimel \in E - S$, then

$$(\digamma \cup \digamma_1)^c(\gimel) = (\digamma_1(\gimel))^c = (\digamma^c \cap (\digamma_1)^c)(\gimel)$$

$$(\daleth \cup \daleth_1)^c(\gimel) = (\daleth_1(\gimel))^c = (\daleth^c \cap (\daleth_1)^c)(\gimel)$$

(iii) if $\gimel \in S \cap E$, then

$$(\digamma \cup \digamma_1)^c(\gimel) = (\digamma(\gimel) \cup \digamma_1(\gimel))^c = (\digamma(\gimel)^c \cap (\digamma_1)(\gimel)^c)$$

$$(\daleth \cup \daleth_1)^c(\gimel) = (\daleth(\gimel) \cap \daleth_1(\gimel))^c = (\daleth(\gimel)^c \cup (\daleth_1)(\gimel)^c)$$

$$(\digamma \cap \digamma_1)^c(\gimel) = (\digamma(\gimel)^c \cap (\digamma_1)(\gimel)^c)$$

$$(\daleth \cap \daleth_1)^c(\gimel) = \daleth(\gimel)^c \cup (\daleth_1)(\gimel)^c).$$

Therefore, in all three cases are equal.

$((\digamma, \daleth, S) \cup (\digamma_1, \daleth_1, E))^c = (\digamma, \daleth, S)^c \cap (\digamma_1, \daleth_1, E)^c$.

(2) Let $\gimel \in S \cup E$. Three cases are:

(i) If $\gimel \in S - E$, then

$$(\digamma \cap \digamma_1)^c(\gimel) = (\digamma(\gimel))^c = (\digamma^c \cup (\digamma_1)^c)(\gimel)$$

$$(\daleth \cap \daleth_1)^c(\gimel) = (\daleth(\gimel))^c = (\daleth^c \cup (\daleth_1)^c)(\gimel)$$

(ii) If $\gimel \in E - S$, then

$$(\digamma \cap \digamma_1)^c(\gimel) = (\digamma_1 c(\gimel) = (\digamma^c \cup (\digamma_1)^c(\gimel)$$

$$(\daleth \cap \daleth_1)^c(\gimel) = (\daleth_1(\gimel))^c = (\daleth^c \cup (\daleth_1)^c(\gimel)$$

(iii) If $\gimel \in S \cup E$, then

$$(\digamma \cap \digamma_1)^c(\gimel) = (\digamma(\gimel) \cap \digamma_1(\gimel))^c = (\digamma(\gimel)^c \cup (\digamma_1)(\gimel)^c)$$

$$(\daleth \cap \daleth_1)^c(\gimel) = (\daleth(\gimel) \cup \daleth_1(\gimel))^c = (\daleth(\gimel)^c \cap (\daleth_1)(\gimel)^c)$$

$$(\digamma \cup \digamma_1)^c(\gimel) = (\digamma(\gimel)^c \cup (\digamma_1)(\gimel)^c)$$

$$(\daleth \cup \daleth_1)^c(\gimel) = (\daleth(\gimel)^c \cap (\daleth_1)(\gimel)^c)$$

These three cases are equality.

$((\digamma, \daleth, S) \cup (\digamma_1, \daleth_1, E))^c = (\digamma, \daleth, S)^c \cap (\digamma_1, \daleth_1, E)^c$. □

2.1.3 Bipolar Soft Topological space

The first researchers to study the topological structure of **BSS** were Shabir and Bakhtawar [11]. They defined the **BSTS** on a universal set. On **BSS**, **BSTS**, there is variation of concepts, was introduced in It also shows a few topological ideas and their characteristics.

Definition

A triplet $(\digamma, \urcorner, S)$ is a **BSS** and τ is the collection of **BS** Subset from $(\digamma, \urcorner, S)$.S is the parameter. τ are called **BSTS** in (U,A).

$c[1] : (\phi, U, S), (U, \phi, S) \in \tau$.

$c[2] : If(\digamma_1, \urcorner_1, s_1), (\digamma_2, \urcorner_2, s_2) \in \tau$ then $(\digamma_1, \urcorner_1, s_1) \cap (\digamma_2, \urcorner_2, s_2) \in \tau$.

$c[3]$: If $(\digamma_i, \urcorner_i, s_i) \in \tau$ for all $\iota \in I$ then $\bigcup_{\iota \in I} (\digamma_i, \urcorner_i, s_i) \in \tau$.

τ is called **BSTS** in (U,A).

Example :-

Let $U = \{z_1, z_2, z_3\}$, $S = \{s_1, s_2\}$ and $\tau = \{(U, \phi, S), (\phi, U, S), (\digamma_1, \urcorner_1, s), (\digamma_2, \urcorner_2, s), (\digamma_3, \urcorner_3, s), (\digamma_4, \urcorner_4, s)$

Where $(\digamma_1, \urcorner_1, s), (\digamma_2, \urcorner_2, s), (\digamma_3, \urcorner_3, s), (\digamma_4, \urcorner_4, s)$ are the bipolar soft set over U.

$$\digamma_1(s_1) = \{z_2\} \qquad \urcorner_1(\neg s_1) = \{z_1\}$$

$$\digamma_2(s_1) = \{z_2, z_3\} \qquad \urcorner_2(\neg s_1) = \{z_1, z_2\}$$

$$\digamma_3(s_2) = \{z_1, z_2\} \qquad \urcorner_3(\neg s_2) = \{U\}$$

$$\digamma_4(s_2) = \{z_1, z_2\} \qquad \urcorner_4(\neg s_2) = \{z_1, z_3\}$$

Then τ is a **BSTS** in U and Hence (U, ϕ, S) is **BSTS** bipolar soft topological space in U.

$\tau = \{(U, \phi, S), (\phi, U, S), \{z_2\}, \{z_2, z_3\}, \{z_1, z_2\}\}$

And

$\tau = \{(U, \phi, S), (\phi, U, S), \{z_1\}, \{z_1, z_3\}, \{z_1, z_2\}\}$ are topologies on U.

2.1.4 Fuzzy Bipolar Soft Set

In **FBSS**, **Naz** and **Shabir** discovered the **FBSSs**, also discovered the parameters of bipolarity [21]. The concepts **SSs** is the information of given data. If the membership function takes values in the range 0 to 1, the element is highly related to the property or partially meets the requirements for membership. When the membership function takes values in the range $[0,1]$, the element is either partly or negatively related to the property.

Definition

A **FBSS** in (U,A) is triplet of $(F,\urcorner,S)$, F and $\urcorner$ are mapping, F : S → P(U) and $\urcorner$ $:\neg S \rightarrow P(U)$.

$$0 \leq F(a)(x) + \urcorner(\neg a)(x) \leq 1$$

$\forall a \in S$.

$$0 \leq F(a)(x) + \urcorner(\neg a)(x) \leq 1$$

$\forall A \in S$ is applied as a consistency restriction.

Example :

Let U is set, and E be set of parameter, $U = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5\}$, $O = \{o_1, o_2, o_3, o_4, o_5\}$.
Let $\neg O = \{\neg o_1, \neg o_2, \neg o_3, \neg o_4, \neg o_5\}$.

Suppose that $A = \{o_1, o_2, o_3\}$, $B = \{o_2, o_3, o_4\}$ and $C = \{o_3, o_4, o_5\}$. The fuzzy bipolar soft sets $(F,\urcorner,A)$, $(F_1,\urcorner_1,B)$ and $(F_2,\urcorner_2,C)$ suppose that

$$F(e_1) = \{\wp_{0.4}, \wp_{0.2}, \wp_{0.5}, \wp_{0.2}, \wp_{0.8}\}$$
$$F(e_2) = \{\wp_{0.3}, \wp_{0.7}, \wp_{0.1}, \wp_{0.5}, \wp_{0.6}\}$$
$$F(e_3) = \{\wp_{0.2}, \wp_{0.4}, \wp_{0.2}, \wp_{0.1}, \wp_{0.7}\}$$
$$\urcorner(\neg e_1) = \{\wp_{0.4}, \wp_{0.7}, \wp_{0.6}, \wp_{0.3}, \wp_{0.1}\}$$
$$\urcorner(\neg e_2) = \{\wp_{0.9}, \wp_{0.1}, \wp_{0.4}, \wp_{0.2}, \wp_{0.7}\}$$
$$\urcorner(\neg e_3) = \{\wp_{0.4}, \wp_{0.2}, \wp_{0.1}, \wp_{0.3}, \wp_{0.8}\}$$

and

$$
\begin{aligned}
\digamma_1(e_2) &= \{\wp_{0.2}, \wp_{0.9}, \wp_{0}, \wp_{0.7}, \wp_{0.4}\},\\
\digamma_1(e_3) &= \{\wp_{0.2}, \wp_{0.8}, \wp_{0.6}, \wp_{0.3}, \wp_{0.1}\},\\
\digamma_1(e_4) &= \{\wp_{0.3}, \wp_{0.5}, \wp_{0.7}, \wp_{0.8}, \wp_{0.6}\}\\
\daleth_1(\neg e_2) &= \{\wp_{0.2}, \wp_{0.4}, \wp_{0.1}, \wp_{0.5}, \wp_{0.8}\},\\
\daleth_1(\neg e_3) &= \{\wp_{0}, \wp_{0.1}, \wp_{0.2}, \wp_{0.3}, \wp_{0.7}\},\\
\daleth_1(\neg e_4) &= \{\wp_{0.8}, \wp_{0.6}, \wp_{0.4}, \wp_{0.2}, \wp_{0.2}\}.
\end{aligned}
$$

and

$$
\begin{aligned}
\digamma_2(e_3) &= \{\wp_{0.2}, \wp_{0.1}, \wp_{0.4}, \wp_{0.5}, \wp_{0.9}\},\\
\digamma_2(e_4) &= \{\wp_{0.2}, \wp_{0.8}, \wp_{0.7}, \wp_{0.1}, \wp_{0.4}\},\\
\digamma_2(e_5) &= \{\wp_{0.1}, \wp_{0.2}, \wp_{0.4}, \wp_{0.6}, \wp_{0.9}\},\\
\daleth_2(\neg e_3) &= \{\wp_{0.2}, \wp_{0.4}, \wp_{0.6}, \wp_{0.8}, \wp_{0}\},\\
\daleth_2(\neg e_4) &= \{\wp_{0.9}, \wp_{0.7}, \wp_{0.1}, \wp_{0.3}, \wp_{0.5}\},\\
\daleth_2(\neg e_5) &= \{\wp_{0.2}, \wp_{0.1}, \wp_{0.5}, \wp_{0.7}, \wp_{0.3}\}.
\end{aligned}
$$

let $(\digamma, \daleth, \text{A̧}) \cup ((\digamma_1, \daleth_1, \text{B̧}) \cap (\digamma_2, \daleth_2, \text{Ç})) = (H_1, I_1, \text{A̧} \cup \text{B̧} \cup \text{Ç})$
and
$((\digamma, \digamma, \text{A̧}) \cup (\digamma_1, \daleth_1, \text{B̧})) \cap ((\digamma, \daleth, \text{A̧}) \cup (\digamma_2, \daleth_2, \text{Ç}))$
$= (H_2, I_2, \text{A̧} \cup \text{B̧} \cup \text{Ç})$.

$((\digamma, \digamma, \text{A̧}) \cup (\digamma_1, \daleth_1, \text{B̧})) \cap ((\digamma, \daleth, \text{A̧}) \cup (\digamma_2, \daleth_2, \text{Ç}))$
$= (H_2, I_2, \text{A̧} \cup \text{B̧} \cup \text{Ç})$.
The collection of **FBSS** in U is now being considered. It is denoted by the letters **FBSS(U)**^{E}, and its subset, which includes all **FBSS** in U with a defined Set of parameters, is denoted with the letters **FBSS(U)**^{A}.

2.1.5 Fuzzy Bipolar Soft Topological Space

In the Addition, **Naz** and **Shabir** initially developed **FBSS** in 2013 [9], Which is new class of hybrid models for vagueness. They took into consideration the **FBSS** algebraic structures study.The basic characteristics of this new topological structure are presented, together with the fuzzy bipolar soft topological spaces. We look at the connections between soft topology, fuzzy topology, and fuzzy bipolar topology.

Definition

A triplet $(\digamma, \daleth, S)$ is a **BSS** and τ be the collections of **FBSS** from $(\digamma, \daleth, S)$. The s is the parameter in S. τ are called **FBST** in (U,A).

$c[1] : (\phi, U, S), (U, \phi, S) \in \tau.$

$c[2]$: If $(\digamma_1, \daleth_1, s_1), (\digamma_2, \daleth_2, s_2) \in \tau$ then $(\digamma_1, \daleth_1, s_1) \cap (\digamma_2, \daleth_2, s_2) \in \tau$.

$c[3]$: If $(\digamma_i, \daleth_i, s_i) \in \tau$ for all $\iota \in I$ Then $\bigcup_{\iota \in I} (\digamma_i, \daleth_i, s_i) \in \tau$.

τ are called **FBSTS** in (U,A).

Example:-

Let $U = \{x,y,z\}$, $S = \{s_1, s_2\}$ and $\tau = \{(U,\phi,S)$, (ϕ,U,S), $(\digamma_1, \daleth_1, S)$, $(\digamma_2, \daleth_2, S)$, $(\digamma_3, \daleth_3, S)$, $(\digamma_4, \daleth_4, S$

Where $(\digamma_1, \daleth_1, S)$, $(\digamma_2, \daleth_2, S)$, $(\digamma_3, \daleth_3, S)$, $(\digamma_4, \daleth_4, S)$ are the bipolar soft set over U.

$$\digamma_1(e_1) = \{x_{0.4}, y_{0.5}, z_{0.2}\}, \quad \daleth_1(\neg e_1) = \{x_{0.5}, y_{0.3}, z_{0.4}\},$$

$$\digamma_1(e_2) = \{x_{0.6}, y_{0.3}, z_{0.1}\}, \quad \daleth_1(\neg e_2) = \{x_{0.3}, y_{0.6}, z_{0.8}\},$$

$$\digamma_2(e_1) = \{x_{0.3}, y_{0.4}, z_{0.6}\}, \quad \daleth_2(\neg e_1) = \{x_{0.4}, y_{0.5}, z_{0.3}\},$$

$$\digamma_2(e_3) = \{x_{0.2}, y_{0.9}, z_{0.1}\}, \quad \daleth_2(\neg e_3) = \{x_{0.6}, y_{0.5}, z_{0.2}\},$$

$$\digamma_3(e_1) = \{x_{0.7}, y_{0.1}, z_{0.2}\}, \quad \daleth_3(\neg e_1) = \{x_{0.4}, y_{0.3}, z_{0.3}\},$$

$$\digamma_3(e_2) = \{x_{0.3}, y_0, z_{0.8}\}, \quad \daleth_3(\neg e_2) = \{x_{0.3}, y_{0.6}, z_{0.8}\},$$

$$\digamma_3(e_3) = \{x_{0.2}, y_{0.3}, z_{0.4}\}, \quad \daleth_3(\neg e_3) = \{x_{0.6}, y_{0.5}, z_{0.2}\},$$

$$\digamma_4(e_1) = \{x_{0.3}, y_{0.4}, z_{0.2}\}, \quad \daleth_4(\neg e_1) = \{x_{0.5}, y_{0.5}, z_{0.4}\},$$

$$\digamma_4(e_2) = \{x_{0.6}, y_{0.3}, z_{0.1}\}, \quad \daleth_4(\neg e_2) = \{x_{0.3}, y_{0.6}, z_{0.8}\},$$

$$\digamma_4(e_3) = \{x_{0.2}, y_{0.3}, z_{0.4}\}, \quad \daleth_4(\neg e_3) = \{x_{0.6}, y_{0.5}, z_{0.2}\}.$$

Chapter 3

Fermatean Fuzzy Set

3.1 Fermatean Fuzzy Set

Definition

[25] A **FFS** is $\mathscr{R}$ in W is a form that has the property

$$\mathscr{R} = \{< w, \alpha_R(w), \beta_R(w) > | w \in W\}$$

Where:-

$$\alpha_R(w) = W \rightarrow [0,1]$$

$$\beta_R(w) = W \rightarrow [0,1]$$

Main state of fermatean fuzzy set is

$$0 \leq (\alpha_R(w))^3 + (\beta_R(w))^3 \leq 1$$

for all $w \in W$. The symbols $\alpha_R(w)$ and $\beta_R(w)$ denote the degree of membership, and non membership of element w in set $\mathscr{R}$ respectively.

Example

Let $\alpha_R(w) = 0.61,\ \beta_R(w) = 0.47$. Then we get

$$(0.61)^3 + (0.47)^3 = 0.32.$$

3.1.1 Intuitionistic Fuzzy Set

The objects provided in nonempty set W for **IFS** is $\mathscr{R} = \{< w, \alpha_R(w), \beta_R(w) > | w \in W\}$, where the function is,

$$\alpha_R(w) = W \rightarrow [0,1]$$

$$\beta_R(w) = W \rightarrow [0,1]$$

The degree of membership and degree of non-membership to the set $\mathscr{R}$ are indicated by each element $w \in W$ [4].

$$0 \leq (\alpha_R(w)) + (\beta_R(w)) \leq 1$$

Example

Let $\alpha_R(w) = 0.4,\ \beta_R(w) = 0.3$. Then we get

$$(0.4) + (0.3) = 0.7.$$

3.1.2 Pythagorean Fuzzy Set

[27] The objects on **PFS** defined in nonempty set W are of the type $\mathscr{R} = \{< w, \alpha_R(w), \beta_R(w) > |w \in W\}$, Where the function is,‘

$$\alpha_R(w) = W \to [0,1]$$

$$\beta_R(w) = W \to [0,1]$$

The each element $w \in W$ denotes degree of membership, nonmembership $\mathscr{R}$.

$$0 \leq (\alpha_R(w))^2 + (\beta_R(w))^2 \leq 1$$

Example

Let $\alpha_R(w) = 0.2,\ \beta_R(w) = 0.6$. Then we get

$$(0.2)^2 + (0.6)^2 = 0.64.$$

Fermatean fuzzy set is better because it easily hold the value. For example in intuitionistic is $0.8 + 0.7 = 1.5 > 1$ which is not correct in pythagorean is $0.8^2 + 0.7^2 = 1.13 > 1$ which is not correct. in fermatean is $0.8^3 + 0.7^3 = 0.8 < 1$ which is correct. thats why is better than others.

Set operations of fermatean fuzzy set

Let $R = (\alpha_r, \beta_r), R_1 = (\alpha_{r_1}, \beta_{r_1})$ and $R_2 = (\alpha_{r_2}, \beta_{r_2})$ be **FFS**, Their condition are define.
(1)$R \cap R$ = $(min\{\alpha r_1, \alpha r_2\}, max\{\beta r_1, \beta r_2\})$
(2)$R \cup R$ = $(max\{\alpha r_1, \alpha r_2\}, min\{\beta_1, \beta_2\})$
(3)$R^c = (\beta_r, \alpha_r)$
Let $R = (\alpha_r, \alpha_r),\ R_1 = (\alpha_{r_1}),\ R_2 = (\alpha_{r_2})$ be three fermatean fuzzy set.

Theorem 3.1.1. *For three **FFSs** $\mathcal{F} = (\alpha_r, \alpha_r), \mathcal{F}_1 = (\alpha_{r_1}), \mathcal{F}_2 = (\alpha_{r_2})$ are defined as follow*
[1]$\mathcal{F}_1 \cap \mathcal{F}_2 = \mathcal{F}_2 \cap \mathcal{F}_1$
[2]$\mathcal{F}_1 \cup \mathcal{F}_2 = \mathcal{F}_2 \cup \mathcal{F}_1$

$[3] F_1 \cap (F_2 \cap F_3) = (F_1 \cap F_2) \cap F_3$

$[4] F_1 \cup (F_2 \cup F_3) = (F_1 \cup F_2) \cup F_3$

Proof.

$[1] F_1 \cap F_2$

$(min\{\alpha r_1, \alpha r_2\}, max\{\beta r_1, \beta r_2\})$

$(min\{\alpha r_2, \alpha r_1\}, max\{\beta r_2, \beta r_1\})$

$F_2 \cap F_1$

$(2) F_1 \cup F_2$

$(max\{\alpha r_1, \alpha r_2\}, min\{\beta r_1, \beta r_2\})$

$(max\{\alpha r_2, \alpha r_1\}, min\{\beta r_2, \beta r_1\})$

$F_2 \cap F_1$

$(3) F_1 \cap (F_2 \cap F_3)$

$(\alpha r_1, \beta r_1) = (min\{\alpha r_2, \alpha r_3\}, max\{\beta r_2, \beta r_3\})$

$= (min \alpha r_1, (min\{\alpha r_2, \alpha r_1\}, max \beta r_1 max\{\beta r_2, \beta r_3\})$

$= (min\{min\{\alpha r_1, \alpha r_2\}, \alpha r_1\}, max \beta r_1 max\{\beta r_2, \beta r_3\})$

$(min\{\alpha r_1, \alpha r_2\}, max\{\beta r_1, \beta r_2\}) \cap (\alpha r_3, \beta r_3)$

$(F_1 \cap F_2) \cap F_3$

$(4) F_1 \cup (F_2 \cup F_3)$

$(\alpha r_1, \beta r_1) = (max\{\alpha r_2, \alpha r_3\}, min\{\beta r_2, \beta r_3\})$

$= (max \alpha r_1, (max\{\alpha r_2, \alpha r_1\}, min \beta r_1 min\{\beta r_2, \beta r_3\})$

$= (max\{max\{\alpha r_1, \alpha r_2\}, \alpha r_1\}, min \beta r_1 min\{\beta r_2, \beta r_3\})$

$(max\{\alpha r_1, \alpha r_2\}, min\{\beta r_1, \beta r_2\}) \cap (\alpha r_3, \beta r_3)$

$(F_1 \cap F_2) \cap F_3$

□

3.2 Fermatean Fuzzy Topological Space

Let be a family of Fermatean fuzzy subsets of X and X be a non-empty set [15]. If

$c[1]$: $\phi, U \in \tau$.

$c[2]$: if $R_1, R_2 \in \tau$ then $R_1 \cap R_2 \in \tau$.

$c[3]$: if $R_\iota \in \tau$ for all $\iota \in I$ then $\bigcup_{\iota \in I} R_\iota \in \tau$.

When ι is an arbitrary index set, is known as a **FFT** in U.

pair (U, τ) is **FFTS**. every member of τ are called open **FF** subset. The complement in open **FF** subset is called closed **FF** subset.

Let X=v_1, v_2. A family of **FF** subsets to consider $\tau = \{\phi, U, R_1, R_2, R_3, R_4, R_5\}$where

$F_1 = \{< v_1, \alpha_{F_1}(v_1) = 0.4, \beta_{F_1}(v_1) = 0.6 >, < v_2, \alpha_{F_1}(v_2) = 0.1, \beta_{F_1}(v_2) = 0.3 >\}$

$F_2 = \{< v_1, \alpha_{F_2}(v_1) = 0.5, \beta_{F_2}(v_1) = 0.4 >, < v_2, \alpha_{F_2}(v_2) = 0.2, \beta_{F_2}(v_2) = 0.8 >\}$

$F_3 = \{< v_1, \alpha_{F_3}(v_1) = 0.3, \beta_{F_3}(v_1) = 0.7 >, < v_2, \alpha_{F_3}(v_2) = 0, \beta_{F_3}(v_2) = 0.7 >\}$

$F_4 = \{< v_1, \alpha_{F_4}(v_1) = 0.5, \beta_{F_4}(v_1) = 0.4 >, < v_2, \alpha_{F_4}(v_2) = 0.2, \beta_{F_4}(v_2) = 0.3 >\}$

and

$F_5 = \{< v_1, \alpha_{F_5}(v_1) = 0.4, \beta_{F_5}(v_1) = 0.6 >, < v_2, \alpha_{F_5}(v_2) = 0.1, \beta_{F_5}(v_2) = 0.8 >\}$

Note that (X, τ) is a fermatean fuzzy topological space.

Example :-

Let $X = v_1, v_2$, the **FFS** to consider $\tau = \{\phi, U, R_1, R_2, R_3, R_4, R_5\}$ where

$F_1 = \{< v_1, \alpha_{F_1}(v_1) = 0.4, \beta_{F_1}(v_1) = 0.6 >, < v_2, \alpha_{F_1}(v_2) = 0.1, \beta_{F_1}(v_2) = 0.3 >\}$

and

$F_2 = \{< v_1, \alpha_{F_2}(v_1) = 0.5, \beta_{F_2}(v_1) = 0.4 >, < v_2, \alpha_{F_2}(v_2) = 0.2, \beta_{F_2}(v_2) = 0.8 >\}$

(X, τ) is **FFTS**, but that (X, τ) is neither an **IFTS** or **PFTS**.

3.3 Fermatean Fuzzy Soft Set

If V is denoted the universe and G represent the parameter of universe. If $\mathscr{F}(V)$ denoted the subset of **FF** and $\ell \subseteq G$ on V, pair $(\digamma, \ell)$ is then referred to as a **FFSS**. If R is function given by $\digamma : \ell \to \mathscr{F}(V)$. Let $\nu \in V$ and $\ell \in L$ then $\digamma(\ell)$ is a **FF**. It is defined by

$$R(L) = \{< \nu, (\eta_R^+(l)(\nu), \eta_R^-(l)(\nu)) > | \nu \in V\}$$

Where the function $(\eta_R^+(l)(\nu) : L \to [0,1]$ and $(\eta_R^-(l)(\nu) : L \to [0,1]$

Example

$$\nu_R^+(l)(v) = (0.4)^3 \text{ and } \nu_R^-(l)(v) = (0.8)^3$$

$$0 \leq (0.4)^3 + (0.8)^3 \leq 1$$

$$0 \leq 0.064 + 0.512 \leq 1$$

$$0 \leq 0.576 \leq 1$$

3.3.1 Fermatean Bipolar Soft Set

If G is parameter of whole set and ν be whole set. For every $B \subseteq G$, triplet $(\digamma, \daleth, B)$ are said to **BSS** in ν. then $\digamma$ and $\daleth$ are function define

$$\digamma : B \to P(V)$$
$$\daleth : \neg B \to P(V)$$

$\digamma(\nu) \cap \daleth(\neg\nu) = \phi, \forall \nu \in B, \neg\nu \in \neg B$

3.3.2 Fermatean Fuzzy Bipolar Soft set

Let G collection of parameters and ν be a universe. The triplet $p \in \digamma, q \in \daleth$ now $(\digamma, \daleth, \ell)$ is represent to **FFBSS** if and only if $L \subseteq G$. they are mapped as $\digamma : \ell \to F(V)$ and $\daleth : (\neg\ell) :\to F(V)$ respectively, then criteria must be met:

$$0 \leq (\eta_P^+(\ell)(\nu))^3 + (\zeta_Q^+(\neg\ell)(\nu))^3 \leq 1$$

$$0 \leq (\eta_P^-(\ell)(\nu))^3 + (\zeta_Q^-(\neg\ell)(\nu))^3 \leq 1$$

for all $\ell \in \ell, \neg\ell \in \neg\ell$, and $\nu \in V$

$$(\eta_P^+(l)(\vartheta)), (\eta_P^-(l)(\vartheta)), (\zeta_Q^+(\neg l)(\vartheta)), (\zeta_Q^-(\neg l)(\vartheta)) \in [0,1]$$

$(\eta_P^+(l)(\nu)), (\zeta_Q^+(\neg l)(\nu))$ are Membership value to $\digamma$ and $\daleth$. now, $(\eta_P^-(l)(\nu)), (\zeta_Q^-(\neg l)(\vartheta))$ are the non membership values.

Example

let $\nu = \{w_1, w_2, w_3, w_4\}$ set of four staff having different skills. Let $G = \{l_1 = \text{passionate},\ l_2 = \text{work on time},\ l_3 = \text{hard work},\ l_4 = \text{good communicate}\}$ be the collection of parameters.

let the not set be $\neg G = \{\neg l_1 = \text{negligent performance},\ \neg l_2 = \text{servitude},\ \neg l_3 = \text{hush communicate},\ \neg l_4 = \text{inexpert}\}$ for $L = \{L_1, L_2, L_3, L_4\} \subseteq G$. We describe skills and incomplete of the staff.

$\digamma(l_1) = \{(w_1, 0.3, 0.8), (w_2, 0.6, 0.7), (w_3, 0.6, 0.8), (w4, 0.4, 0.5)\}$

$\digamma(l_2) = \{(w_1, 0.2, 0.7), (w_2, 0.5, 0.6), (w_3, 0.8, 0.4), (w4, 0.4, 0.5)\}$

$\digamma(l_4) = \{(w_1, 0.6, 0.3), (w_2, 0.5, 0.4), (w_3, 0.6, 0.8), (w4, 0.8, 0.5)\}$

And

$\daleth(\neg l_1) = \{(w_1, 0.4, 0.1), (w_2, 0.1, 0.3), (w_3, 0.1, 0.2), (w4, 0.2, 0.4)\}$

$\daleth(\neg l_2) = \{(w_1, 0.5, 0.3), (w_2, 0.1, 0.5), (w_3, 0.1, 0.9), (w4, 0.4, 0.9)\}$

$\daleth(\neg l_4) = \{(w_1, 0.2, 0.3), (w_2, 0.4, 0.3), (w_3, 0.8, 0.0), (w4, 0.0, 0.4)\}$

These are all values is **FFBSS** now we check these value is it hold the condition of fermatean fuzzy bipolar soft set or not.

$$0 \leq (\eta_P^+(l)(\nu))^3 + (\zeta_Q^+(\neg l)(\nu))^3 \leq 1$$

$$0 \leq (0.1)^3 + (0.8)^3 \leq 1$$

$$0 \leq 0.001 + 0.512 \leq 1$$

$$0 \leq 0.513 \leq 1$$

This value is satisfied the condition so,this this **FFBSS**. Now we check the value which is not **FFBSS**

$$0 \leq (\eta_P^+(l)(\nu))^3 + (\zeta_Q^+(\neg l)(\nu))^3 \leq 1$$

$$0 \leq (0.8)^3 + (0.9)^3 \leq 1$$

$$0 \leq 0.512 + 0.729 \leq 1$$

$$0 \nleq 1.241 \nleq 1$$

It is not hold the condition of **FFBSS**.

Properties

Let $v_1 = (P_1, Q_1, L_1)$ and $v_2 = (P_2, Q_2, L_2)$ be two **FFBSS** in the whole set v_1 then v is said to be **FFBS** Subset of v_2, respectively by $v_1 \subseteq v_2$

- $L_1 \subseteq L_2$
- $p_1(l) \subseteq p_2(l)$, for Example $\{\eta^+_{p_1}(l)(v) \leq \eta^+_{p_2}(l)(v), \eta^-_{p_2}(l)(v) \geq \eta^-_{p_1}(l)(v)\}$

If v_2 is **FFBSS** of v_1 then v_1 is referred as **FFBS** subset of v_2

Complement

Let $v = (\digamma, \daleth, \beth)$ be **FFBSS** over the whole set of v. Its complement is defined as $v^c = (\digamma^c, \daleth^c, \beth^c)$ in V. Where $\digamma^c : \beth \to [0,1]$ and $\daleth^c : \neg\beth \to [0,1]$ mapping are respectively.
$\digamma^c(\beth)(v) = \eta^-_{\digamma}(\beth)(v), (\eta^+_{\digamma}(\beth)(v)$
$\daleth^c(\neg\beth)(v) = \zeta^-_{\daleth}(\neg\beth)(v), (\zeta^+_{\daleth}(\neg\beth)(v)$
$\forall \ell \in \beth, \neg\ell \in \neg\beth$ and $v \in V$
$\digamma^c(l_1) = \{(w_1, 0.08, 0.03), (w_2, 0.07, 0.06), (w_3, 0.08, 0.06), (w4, 0.05, 0.04)\}$
$\digamma^c(l_2) = \{(w_1, 0.07, 0.02), (w_2, 0.06, 0.05), (w_3, 0.04, 0.08), (w4, 0.05, 0.04)\}$
$\digamma^c(l_4) = \{(w_1, 0.03, 0.06), (w_2, 0.04, 0.05), (w_3, 0.08, 0.06), (w4, 0.05, 0.08)\}$
And
$Q^(\neg l_1) = \{(w_1, 0.01, 0.04), (w_2, 0.03, 0.01), (w_3, 0.02, 0.01), (w4, 0.04, 0.02)\}$
$Q^c(\neg l_2) = \{(w_1, 0.03, 0.05), (w_2, 0.05, 0.01), (w_3, 0.09, 0.01), (w4, 0.09, 0.04)\}$
$Q^c(\neg l_4) = \{(w_1, 0.30, 0.02), (w_2, 0.03, 0.40), (w_3, 0.00, 0.08), (w4, 0.04, 0.00)\}$
These are all values are complement of **FFBSS** fulfil the conditions.

Relative Null

A fermatean over ν are called relative null **FFBSS**, is shown by $(\phi, \nu, \daleth)$

If $\{\phi(\ell)(\nu) = \eta_{\phi}^{+}(l\ell)(\nu) = 0, \eta_{\phi}^{-}(\ell)(\nu) = 1\}$; $V(\neg\ell)(\nu) = [1,0]$

Relative Absolute

A fermatean over ν are called relative absolute fermatean fuzzy bipolar soft set, is shown by (ν, ϕ, L)

let $\{V(\ell)(\nu) = \eta_{\phi}^{+}(\ell)(\nu) = 1, \eta_{\phi}^{-}(\ell)(\nu) = 0\}$; $\Phi(\neg\ell)(\nu) = [0,1]$

Chapter 4

Fermatean Fuzzy Bipolar Soft on Topological Space

4.1 Fermatean Fuzzy Bipolar Soft Topological Spaces

A triplet (R,T,S) is define as (**FFBSS**)if $A \subseteq S$ and S are universes of parameters.If $\digamma$ and $\daleth$ are mapped as $\digamma : W \to F(Z)$ and $\daleth : (\neg W) \to F(Z)$ respectively, then:

$$\digamma = \{< z, \gamma^{+}_{\digamma}(w)(z), \delta^{-}_{\digamma}(w)(z) >: z \in Z\}$$

$$\daleth = \{< z, \gamma^{+}_{\daleth}(\neg w)(z), \delta^{-}_{\daleth}(\neg w)(z) >: z \in Z\}$$

with the following conditions:

$$0 \leq (\gamma^{+}_{\digamma}(w)(z))^3 + (\delta^{+}_{\daleth}(\neg w)(z))^3 \leq 1$$

$$0 \leq (\gamma^{-}_{\digamma}(w)(z))^3 + (\delta^{-}_{\daleth}(\neg w)(z))^3 \leq 1$$

Null Fermatean

A (**FFBSS**)over Z is $F_B = (R,T,S)$ for which $(\gamma^{+}_{r}(w)(z) = 0, \gamma^{+}_{r}(w)(z)) = 1)$ and $(\delta^{+}_{T}(\neg w)(z) = 1, \delta^{+}_{T}(\neg w)(z)) = 0)$ for all $w \in W$ $(\neg w) \in \neg W z \in Z$ is called null (**FFBSS**) , is denote by (ϕ, Z, S).

Absolute Fermatean

A (**FFBSS**)over Z is $F_B = (R,T,S)$ for which $(\gamma^{+}_{r}(w)(z) = 1, \gamma^{+}_{r}(w)(z)) = 0)$ and $(\delta^{+}_{T}(\neg w)(z) = 0, \delta^{+}_{T}(\neg w)(z)) = 1)$ for all $w \in W$ $\neg w \in \neg$W z $\in$Z is called absolute (**FFBSS**) , is denote by (Z, ϕ, S).

4.1.1 Fermatean Fuzzy Bipolar Soft Topology

A (**FFBSTS**) is a triplet (Z, τ, W). Where τ is the family of fermatean fuzzy bipolar soft set over (Z,W) fulfilling the following conditions.

$c[1] : (\phi, Z, W), (Z, \phi, W) \in \tau$.

$c[2]$: If $(\digamma_1, \daleth_1, W_1), (\digamma_2, \daleth_2, W_2) \in \tau$ then $(\digamma_1, \daleth_1, W_1) \cap (\digamma_2, \daleth_2, W_2) \in \tau$.

$c[3]$: If $(\digamma_i, \daleth_i, W_i) \in \tau$ for all $\iota \in$ I then $\bigcup_{\iota \in I} (\digamma_i, \daleth_i, W_i) \in \tau$.

τ are called **FFBSTS** in (Z,W).

4.1.2 Fermatean Fuzzy Bipolar soft Closed set

(Z,τ,W) be (**FFBSTS**). The member of τ called open of (**FFBSTS**). A (**FFBSS**) (R,T,S) over (U,A) is Called (**FFBSCS**) if complement of the set is a (**FFBSS**).

Theorem 4.1.1. *Let* (Z,τ,W) *is a (**FFBSTS**).*
(1) (ϕ,Z,W), (Z,ϕ,W) *are (**FFBSCS**).*
(2) *The arbitrary union of (**FFBSCSs**) is (**FFBSS**).*
(3) *The finite intersection of two (**FFBSCSs**) is (**FFBSS**).*

Proof. (1) By using the complement operation, it is clear. $(U,\phi,A)^c,(\phi,U,A)^c$ are (**FFBSCSs**) since their complements $(\phi,U,S),(U,\phi,S)\in\tau$ respectively.
(2) Let $(R_i,T_i,S_i):(R_i,T_i,S_i)^c\in\tau$. then

$$\left(\bigcup_{i=1}^{n}(RT,S)\right)^c=\bigcap_{\imath=I}^{n}(RT,S)^c\in\tau.$$

Therefore, $(\bigcup_{i=1}^{n}(R,T,S))$ is a (**FFBSCSs**).

(3) Let $(R_i,T_i,S_i):(R_i,T_i,S_i)^c\in\tau$. then

$$\left(\bigcap_{i=1}^{n}(RT,S)\right)^c=\bigcup_{\imath=I}^{n}(R.T,S)^c\in\tau$$

.

Therefore,$(\bigcap_{\imath=I}^{n}(R,T,S))$ is a (**FFBSCSs**).

□

Example :-
Let $U=\{x,y\}$, $S=\{s_1,s_2,s_3\}$. then
$\tau=\{(\phi,U,A),(U,\phi,A),(R_1,T_1,W_1),(R_2,T_2,W_2),(R_3,T_3,W_3)\}$ is a (**FFBSTS**) where:

$$R_1(w_1) = \{< x, \gamma_{R_1}(w_1)(x) = 0.3, \delta_{R_1}(w_1)(x) = 0.7 >\}$$
$$T_1(\neg w_1) = \{< x, \gamma_{T_1}(\neg w_1)(x) = 0.4, \delta_{T_1}(\neg w_1)(x) = 0.3 >\}$$

$$R_1(w_1) = \{< y, \gamma_{R_1}(w_1)(y) = 0.2, \delta_{R_1}(w_1)(y) = 0.6 >\}$$
$$T_1(\neg w_1) = \{< y, \gamma_{T_1}(\neg w_1)(y) = 0.3, \delta_{T_1}(\neg w_1)(y) = 0.7 >\}$$

$$R_1(w_2) = \{< x, \gamma_{R_1}(w_2)(x) = 0.6, \delta_{R_1}(w_2)(x) = 0.7 >\}$$
$$T_1(\neg w_2) = \{< x, \gamma_{T_1}(\neg w_2)(x) = 0.1, \delta_{T_1}(\neg w_2)(x) = 0.3 >\}$$

$$R_1(w_2) = \{< y, \gamma_{R_1}(w_2)(y) = 0.8, \delta_{R_1}(w_2)(y) = 0.1 >\}$$
$$T_1(\neg w_2) = \{< y, \gamma_{T_1}(\neg w_2)(y) = 0.2, \delta_{T_1}(\neg w_2)(y) = 0.5 >\}$$

$$R_2(w_1) = \{< x, \gamma_{R_2}(w_1)(x) = 0.2, \delta_{R_2}(w_1)(x) = 0.7 >\}$$
$$T_2(\neg w_1) = \{< x, \gamma_{T_2}(\neg w_1)(x) = 0.5, \delta_{T_2}(\neg w_1)(x) = 0.3 >\}$$

$$R_2(w_1) = \{< y, \gamma_{R_2}(w_1)(y) = 0.1, \delta_{R_2}(w_1)(y) = 0.8 >\}$$
$$T_2(\neg w_1) = \{< y, \gamma_{T_2}(\neg w_1)(y) = 0.4, \delta_{T_2}(\neg w_1)(y) = 0.6 >\}$$

$$R_2(w_3) = \{< x, \gamma_{R_2}(w_3)(x) = 0.6, \delta_{R_2}(w_3)(x) = 0.7 >\}$$
$$T_2(\neg w_3) = \{< x, \gamma_{T_2}(\neg w_3)(x) = 0.1, \delta_{T_2}(\neg w_3)(x) = 0.9 >\}$$

$$R_2(w_3) = \{< y, \gamma_{R_2}(w_3)(y) = 0.8, \delta_{R_2}(w_3)(y) = 0.2 >\}$$
$$T_2(\neg w_3) = \{< y, \gamma_{T_2}(\neg w_3)(y) = 0.5, \delta_{T_2}(\neg w_3)(y) = 0.4 >\}$$

$$R_3(w_1) = \{< x, \gamma_{R_3}(w_1)(x) = 0.6, \delta_{R_3}(w_1)(x) = 0.3 >\}$$
$$T_3(\neg w_1) = \{< x, \gamma_{T_1}(\neg w_3)(x) = 0.5, \delta_{T_1}(\neg w_3)(x) = 0.3 >\}$$

$$R_3(w_1) = \{< y, \gamma_{R_3}(w_1)(y) = 0.9, \delta_{R_3}(w_1)(y) = 0.1 >\}$$
$$T_3(\neg w_1) = \{< y, \gamma_{T_1}(\neg w_3)(y) = 0.2, \delta_{T_1}(\neg w_3)(y) = 0.7 >\}$$

$R_3(w_2) = \{< x, \gamma_{R_3}(w_2)(x) = 0.5, \delta_{R_3}(w_2)(x) = 0.7 >\}$
$T_3(\neg w_2) = \{< x, \gamma_{T_3}(\neg w_2)(x) = 0.4, \delta_{T_3}(\neg w_2)(x) = 0.3 >\}$

$R_3(w_2) = \{< y, \gamma_{R_3}(w_2)(y) = 0.4, \delta_{R_3}(w_2)(y) = 0.1 >\}$
$T_3(\neg w_2) = \{< y, \gamma_{T_3}(\neg w_2)(y) = 0.2, \delta_{T_3}(\neg w_2)(y) = 0.5 >\}$

$R_3(w_3) = \{< x, \gamma_{R_3}(w_3)(x) = 0.5, \delta_{R_3}(w_3)(x) = 0.7 >\}$
$T_3(\neg w_3) = \{< x, \gamma_{T_3}(\neg w_3)(x) = 0.8, \delta_{T_3}(\neg w_3)(x) = 0.0 >\}$

$R_3(w_3) = \{< y, \gamma_{R_3}(w_3)(y) = 0.8, \delta_{R_3}(w_3)(y) = 0.3 >\}$
$T_3(\neg w_3) = \{< y, \gamma_{T_3}(\neg w_3)(y) = 0.7, \delta_{T_3}(\neg w_3)(y) = 0.1 >\}$
As we know $(\phi, U, A), (U, \phi, A) \in \tau$ intersection of two (**FFBSS**) open set $(R_1, T_1, W_1) \cap (R_2, T_2, W_2)$ belongs to τ and union of $(R_1, T_1, W_1), (R_2, T_2, W_2), (R_3, T_3, W_3)$ these (**FFBSS**) are belongs to τ.
$(\phi, U, A), (U, \phi, A), (R_1, T_1, W_1), (R_2, T_2, W_2), (R_3, T_3, W_3)$ all the members of τ. So these are called open sets of (**FFBSTS**). These are all value hold the topological space.
Properties of (**Union**):-

$$R_1(s) \cup R_2(s) = \{< u, \gamma^+_{R_1}(a_1)(u) \vee \gamma^+_{R_2}(a_2)(u), \gamma^-_{R_1}(a_1)(u) \wedge \gamma^-_{R_2}(a_2)(u) >: u \in U\}$$

$$T_1(s) \cap T_2(s) = \{< u, \delta^+_{T_1}(\neg a_1) \wedge \delta^+_{T_2}(\neg a_2)(u), \delta^-_{T_1}(\neg a_1)(u) \vee \delta^-_{T_2}(\neg a_2)(u) >: u \in U\}$$

.

Properties of (**Intersection**):-

$$R_1(s) \cap R_2(s) = \{< u, \gamma^+_{R_1}(a_1)(u) \wedge \gamma^+_{R_2}(a_2)(u), \gamma^-_{R_1}(a_1)(u) \vee \gamma^-_{R_2}(a_2)(u) >: u \in U\}$$

,

$$T_1(s) \cup T_2(s) = \{< u, \delta^+_{T_1}(\neg a_1) \vee \delta^+_{T_2}(\neg a_2)(u), \delta^-_{T_1}(\neg a_1)(u) \wedge \delta^-_{T_2}(\neg a_2)(u) >: u \in U\}$$

.

now check $(R_1, T_1, S)(R_3, T_3, S_3)$ is it topological space or not.

Union:

$$R_1 \cup R_3(S_1) = \{x, 0.3, 0.7\} \cup \{x, 0.6, 0.3\}$$

$$R_1 \cup R_3(S_1) = \{x, (0.3 \vee 0.6\}, \{0.7 \wedge 0.3\}$$

$$R_1 \cup R_3(S_1) = \{x, 0.6, 0.3\}$$

$$T_1 \cap T_3(\neg S_1) = \{x, 0.4, 0.3\} \cup \{x, 0.5, 0.3\}$$

$$T_1 \cap T_3(\neg S_1) = \{x, 0.4 \wedge 0.5\} \cup \{x, 0.3 \vee 0.3\}$$

$$T_1 \cap T_3(\neg S_1) = \{x, 0.4, 0.3\}$$

Intersection:

$$R_1 \cap R_3(S_1) = \{x, 0.3, 0.7\} \cap \{x, 0.6, 0.3\}$$

$$R_1 \cap R_3(S_1) = \{x, 0.3 \wedge 0.6\}\{0.7 \vee 0.3\}$$

$$R_1 \cap R_3(S_1) = \{x, 0.3, 0.7\}$$

$$T_1 \cup T_3(\neg S_1) = \{x, 0.4, 0.3\} \cup \{x, 0.5, 0.3\}$$

$$T_1 \cup T_3(\neg S_1) = \{x, 0.4 \vee 0.5\} \cup \{x, 0.3 \wedge 0.3\}$$

$$T_1 \cup T_3(\neg S_1) = \{x, 0.5, 0.3\}$$

now check $(R_1, T_1, S_1)(R_2, T_2, S_1)$ is it topological space or not.

Union:

$$R_1 \cup R_2(S_1) = \{y, 0.2, 0.6\} \cup \{y, 0.1, 0.8\}$$

$$R_1 \cup R_2(S_1) = \{y, (0.2 \vee 0.1\}, \{0.6 \wedge 0.8\}$$

$$R_1 \cup R_2(S_1) = \{y, 0.2, 0.6\}$$

$$T_1 \cap T_2(\neg S_1) = \{y, 0.3, 0.7\} \cup \{y, 0.4, 0.6\}$$

$$T_1 \cap T_2(\neg S_1) = \{y, 0.3 \wedge 0.4\} \cup \{y, 0.7 \vee 0.6\}$$

$$T_1 \cap T_2(\neg S_1) = \{y, 0.3, 0.7\}$$

Intersection:

$$R_1 \cap R_2(S_1) = \{y, 0.2, 0.6\} \cap \{y, 0.1, 0.8\}$$

$$R_1 \cap R_2(S_1) = \{y, 0.2 \wedge 0.1\}\{0.6 \vee 0.8\}$$

$$R_1 \cap R_2(S_1) = \{y, 0.1, 0.8\}$$

$$T_1 \cup T_2(\neg S_1) = \{y, 0.3, 0.7\} \cup \{y, 0.4, 0.6\}$$

$$T_1 \cup T_2(\neg S_1) = \{y, 0.3 \vee 0.4\} \cup \{y, 0.7 \wedge 0.6\}$$

$$T_1 \cup T_2(\neg S_1) = \{y, 0.4, 0.6\}$$

So, this values is the hold the condition of topological space. Now satisfy the condition of fermatean fuzzy bipolar soft set.

$$R_1(w_1) = \{< x, \gamma_{R_1}(x)(w_1) = 0.3, \delta_{R_1}(x)(w_1) = 0.7 >\}$$

$$T_1(\neg w_1) = \{< x, \gamma_{T_1}(x)(\neg w_1) = 0.4, \delta_{T_1}(x)(\neg w_1) = 0.3 >\}$$

$$0 \leq (0.3)^3 + (0.7)^3 \leq 1 \qquad 0 \leq (0.4)^3 + (0.3)^3 \leq 1$$

$$0 \leq 0.027 + 0.343 \leq 1 \qquad 0 \leq 0.064 + 0.027 \leq 1$$

$$0 \leq 0.37 \leq 1 \qquad 0 \leq 0.091 \leq 1$$

Now check for y

$$R_1(w_1) = \{< y, \gamma_{R_1}(y)(w_1) = 0.2, \delta_{R_1}(y)(w_1) = 0.6 >\}$$

$$T_1(\neg w_1) = \{< y, \gamma_{T_1}(y)(\neg w_1) = 0.3, \delta_{T_1}(y)(\neg w_1) = 0.7 >\}$$

$$0 \leq (0.2)^3 + (0.6)^3 \leq 1 \qquad 0 \leq (0.3)^3 + (0.7)^3 \leq 1$$

$$0 \leq 0.008 + 0.216 \leq 1 \qquad 0 \leq 0.027 + 0.343 \leq 1$$

$$0 \leq 0.224 \leq 1 \qquad 0 \leq 0.37 \leq 1$$

So, these values are satisfied.

4.1.3 Fermatean Fuzzy Bipolar Soft Point

(R, T, s) is called (**FFBSP**) in (Z, W). If there exist $m \in Z$ then $R(s)(m) = P, (0 \leq p \leq 1), T(\neg s)(m) = Q, (0 \leq p \leq 1)$ and $0 \leq p^3 + q^3 \leq 1$. The (**FFBSP**)is denoted by $(e_m^p, \neg e_m^q)$. Let$(e_x^p, \neg e_x^q)$ be a (**FFBSP**) and (R, T, S) be **BSS** in (Z, W). $(e_x^p, \neg e_x^q)$ are called an element of (R, T, S) and it is denoted by $(e_x^p, \neg e_x^q) \in (R, T, S)$ iff $p \leq R(s)(m), G(\neg s)(m) \leq q$.

Example

Let $Z = \{x, y\}$, $S = \{s_1, s_2, s_3\}$ and (R, T, S) be a (**FFBSS**) in (z, A) where

$R(s_1) = \{x, 0.2, 0.3\}, T(\neg s_1) = \{x, 0.4, 0.5\}$

$R(s_2) = \{y, 0.5, 0.8\}, T(\neg s_2) = \{y, 0.3, 0.5\}$

Then $(e_{1_x}^{0.2^3}, \neg e_{1_x}^{0.4^3}) \in (R, T, S)$. The value is $(e_{1_x}^{o.oo8}, \neg e_{1_x}^{0.064})$

Lemma 4.1.2. *Let (J, K, Y) be a (**FFBSS**) and $(R, T, S) = (e_x^p, \neg e_x^q)$ be a (**FFBSP**) in (U, A), then the following holds.*

$(1)(R, T, s) \in (J, K, Y)$ iff $(R, T, s) \subseteq (J, K, Y)$.

(2) If $(R, T, s) \cap (J, K, Y) = (\phi, U, A)$ then $(R, T, s) \notin (J, K, Y)$.

Proof. (1) Let (R, T, s) be (**FFBSP**)of $(e_x^p, \neg e_x^q)$ then there exist fermatean fuzzy bipolar soft open set(**FFBoS**) (J, K, Y) such that $(e_x^p, \neg e_x^q) \in (J, K, Y) \subseteq (R, T, w)$. Hence $(e_x^p, \neg e_x^q) \in (J, K, Y) \subseteq (R, T, w)$ this Shows that (R, T, s) is a fermatean fuzzy bipolar soft of (e_x^p, e_x^q).

(2) Let (R, T, s) are (**FFBSS**) of $(e_x^p, \neg e_x^q)$ then there exist (**FFBoS**) (J, K, Y) such that $(e_x^p, \neg e_x^q) \in (J, K, Y) \subseteq (R, T, s)$. Hence, $(R, T, s) \cap (J, K, Y) = (\phi, U, A)$ and $(R, T, s) \notin$

$(J,K,y) = (U,\phi,A)$. □

Theorem 4.1.3. *Let (U,ϕ,A) be a (**FFBSTS**). Then the collection $\tau_s = \{R_s : (R,T,S) \in \tau\}$ defines a **FFSTS** on (U,A).*

Proof. (1) Since ϕ(s) = 0 and $U(s)$ = 1, for all $s \in S$.
(2) Since $(R_1,T_1,W_1),(R_2,T_2,W_2) \in \tau$ then $(R_1,T_1,W_1) \cap (R_2,T_2,W_2) \in \tau$ and so $R_{1S} \wedge R_{2S} \in \tau_s$.
(3) Since $(R_i,T_i,W_i) \in \tau$ then $\bigcup_{i\in I} \in \tau$.

□

4.1.4 Discrete and Indiscrete Fermatean

Let $\tau = \{(\phi,z,A)(Z,\phi,A)\}\tau$ are called indiscrete(**FFBSTS**).
The discrete is set of all fermatean.(**FFBSTS**). A Fermatean Fuzzy topology τ_1 is said to be coarser than a Fermatean Fuzzy Topology τ_2 . $\tau_1 \subseteq \tau_2$.

Theorem 4.1.4. *IF (U,τ_1,A) and (U,τ_2,A) be Two (**FFBSTS**), Then $(U,\tau_1 \cap \tau_2,A)$ is a (**FFBSTS**).*

Proof. (1) Since $(\phi,Z,W)(Z,\phi,W) \in \tau_1,\tau_2$ then $(\phi,Z,W)(Z,\phi,W) \in \tau_1 \cap \tau_2$.
(2) Since $(R_1,T_1,W_1),(R_2,T_2,W_2) \in \tau_1,\tau_2$ then $(R_1,T_1,W_1) \cap (R_2,T_2,W_2) \in \tau_1 \cap \tau_2$.
(3) Since $(R_i,T_i,W_i) \in \tau_1,\tau_2$ for all i ∈ I then $\bigcup_{\iota\in I}(R_i,T_i,W_i) \in \tau_1 \cup \tau_2$. □

Example:-

Let $U = \{x,y\}$, $W = \{w_1,w_2,w_3\}$. then
$\tau = \{(\phi,Z,W),(Z,\phi,W),(R_1,T_1,W_1),(R_2,T_2,W_2),(R_3,T_3,W_3)\}$.

$$R_1(w_1) = \{< x, \gamma_{R_1}(x)(w_1) = 0.1, \delta_{R_1}(x)(w_1) = 0.3 >\}$$

$$T_1(\neg w_1) = \{< x, \gamma_{T_1}(x)(\neg w_1) = 0.7, \delta_{T_1}(x)(\neg w_1) = 0.3 >\}$$

$$R_1(w_1) = \{< y, \gamma_{R_1}(y)(w_1) = 0.6, \delta_{R_1}(y)(w_1) = 0.5 >\}$$

$$T_1(\neg w_1) = \{< y, \gamma_{T_1}(y)(\neg w_1) = 0.8, \delta_{T_1}(y)(\neg w_1) = 0.2 >\}$$

$$R_1(w_2) = \{< x, \gamma_{R_1}(x)(w_2) = 0.9, \delta_{R_1}(x)(w_2) = 0.1 >\}$$

$$T_1(\neg w_2) = \{< x, \gamma_{T_1}(x)(\neg w_2) = 0.2, \delta_{T_1}(x)(\neg w_2) = 0.4 >\}$$

$$R_1(w_2) = \{< y, \gamma_{R_1}(y)(w_2) = 0.7, \delta_{R_1}(y)(w_2) = 0.2 >\}$$

$$T_1(\neg w_2) = \{< y, \gamma_{T_1}(y)(\neg w_2) = 0.3, \delta_{T_1}(y)(\neg w_2) = 0.4 >\}$$

$$R_2(w_1) = \{< x, \gamma_{R_2}(x)(w_1) = 0.1, \delta_{R_2}(x)(w_1) = 0.6 >\}$$

$$T_2(\neg w_1) = \{< x, \gamma_{T_2}(x)(\neg w_1) = 0.4, \delta_{T_2}(x)(\neg w_1) = 0.2 >\}$$

$$R_2(w_1) = \{< y, \gamma_{R_2}(y)(w_1) = 0.1, \delta_{R_2}(y)(w_1) = 0.7 >\}$$

$$T_1(\neg w_1) = \{< y, \gamma_{T_2}(y)(\neg w_1) = 0.3, \delta_{T_2}(y)(\neg w_1) = 0.1 >\}$$

now check $(R_1, T_1, W)(R_1, T_1, W_1)$ is it topological space or not.

$$(R_1 \cup R_1)S_1 = \{x, 0.3, 0.7\} \cup \{x, 0.1, 0.3\}$$

$$(R_1 \cup R_1)S_1 = \{x, (0.3 \vee 0.1\}, \{0.7 \wedge 0.3\}$$

$$(R_1 \cup R_1)S_1 = \{x, 0.3, 0.3\}$$

Intersection:

$$(R_1 \cap R_1)S_1 = \{x, 0.3, 0.7\} \cap \{x, 0.1, 0.3\}$$

$$(R_1 \cap R_1)S_1 = \{x, 0.3 \wedge 0.1\}\{0.7 \vee 0.3\}$$

$$(R_1 \cap R_1)S_1 = \{x, 0.1, 0.7\}$$

for $(T_1, T_3)\neg s_1$

$$(T_1 \cup T_1)\neg S_1 = \{x, 0.4, 0.3\} \cup \{x, 0.7, 0.3\}$$

$$(T_1 \cup T_1)\neg S_1 = \{x, 0.4 \vee 0.7\} \cup \{x, 0.3 \wedge 0.3\}$$

$$(T_1 \cup T_1)\neg S_1 = \{x, 0.7, 0.3\}$$

Intersection:

$$(T_1 \cap T_1)\neg S_1 = \{x, 0.4, 0.3\} \cup \{x, 0.7, 0.3\}$$

$$(T_1 \cap T_1)\neg S_1 = \{x, 0.4 \wedge 0.7\} \cup \{x, 0.3 \vee 0.3\}$$

$$(T_1 \cap T_1)\neg S_1 = \{x, 0.4, 0.3\}$$

$(R_1, T_1, W_1)(R_2, T_2, W_1)$

$$(R_1 \cup R_2)S_1 = \{x, 0.3, 0.7\} \cup \{x, 0.1, 0.6\}$$

$$(R_1 \cup R_2)S_1 = \{x, (0.3 \vee 0.1\}, \{0.7 \wedge 0.6\}$$

$$(R_1 \cup R_2)S_1 = \{x, 0.3, 0.6\}$$

$R_1 \cup R_2$ is not topological space.

Intersection:

$$(R_1 \cap R_2)S_1 = \{x, 0.3, 0.7\} \cap \{x, 0.1, 0.6\}$$

$$(R_1 \cap R_2)S_1 = \{x, 0.3 \wedge 0.1\}\{0.7 \vee 0.6\}$$

$$(R_1 \cap R_2)S_1 = \{x, 0.1, 0.7\}$$

So, this values is the hold the condition of topological space. Now satisfy the condition of fermatean fuzzy bipolar soft set.

$$R_1(w_1) = \{< x, \gamma_{R_1}(x)(w_1) = 0.1, \delta_{R_1}(x)(w_1) = 0.3 >\}$$

$$T_1(\neg w_1) = \{< x, \gamma_{T_1}(x)(\neg w_1) = 0.7, \delta_{T_1}(x)(\neg w_1) = 0.3 >\}$$

$$0 \leq (0.1)^3 + (0.3)^3 \leq 1 \qquad 0 \leq (0.7)^3 + (0.3)^3 \leq 1$$

$$0 \leq 0.001 + 0.027 \leq 1 \qquad 0 \leq 0.343 + 0.027 \leq 1$$

$$0 \leq 0.028 \leq 1 \qquad 0 \leq 0.37 \leq 1$$

Now check for y

$$R_1(w_1) = \{< y, \gamma_{R_1}(y)(w_1) = 0.6, \delta_{R_1}(y)(w_1) = 0.5 >\}$$

$$T_1(\neg w_1) = \{< y, \gamma_{T_1}(y)(\neg w_1) = 0.8, \delta_{T_1}(y)(\neg w_1) = 0.2 >\}$$

$$0 \leq (0.6)^3 + (0.5)^3 \leq 1 \qquad 0 \leq (0.8)^3 + (0.2)^3 \leq 1$$

$$0 \leq 0.216 + 0.125 \leq 1 \qquad 0 \leq 0.512 + 0.008 \leq 1$$

$$0 \leq 0.341 \leq 1 \qquad 0 \leq 0.52 \leq 1$$

So, these values are satisfied.

Although they are Fermatean, as may be seen but $\tau_1 \cup \tau_2$ is not fermatean fuzzy bipolar soft topology space.

4.1.5 Fermatean Fuzzy Bipolar Soft Neighborhood

Let (U,τ,A) be Fermatean Fuzzy Bipolar Soft Topological Space (R,T,S) be **FBSS** and $(e_x^p,\neg e_x^q)$ be a **FFBSP** in (U,A). (R,T,S) is called **FFBS**neighborhood of $(e_x^p,\neg e_x^q)$ Bipolar fuzzy Open Set (R_1,T_1,S_1) s.t $(e_x^p,\neg e_x^q) \in (R_1,T_1,S_1) \subseteq (R,T,S)$.

Let (U,τ,A) be Fermatean Fuzzy Bipolar Soft Topological Space. (R,T,S), (R_1,T_1,S_1) be two sets in (U,A).(R_1,T_1,S_1) are called a Fermatean Fuzzy Bipolar Soft Neighborhood of (R,T,W) If exist a fermatean bipolar fuzzy soft open set (R_2,T_2,W_2) such that $(R,T,W) \subseteq (R_2,T_2,W_2) \subseteq (R_1,T_1,W_1)$.

Example :-

Let $U=\{x,y\}$, $W=\{w_1,w_2,w_3\}$. Then

$\tau = \{(\phi,Z,W),(Z,\phi,W),(R_1,T_1,W_1),(R_2,T_2,W_2),(R_3,T_3,W_3)\}$

$Z=\{(R_1,T_1,W_1)\}$ is a set.

$(e_x^p,\neg e_x^q)$ be a point.(R_1,T_1,W_1) be a neighborhood of this point $(e_x^p,\neg e_x^q)$.(R_1,T_1,W_1) is a open set such that $(e_x^p,\neg e_x^q) \in (R_1,T_1,W_1) \subseteq (R,T,W)$.

$$R_1(w_1) = \{< x, \gamma_{R_1}(x)(w_1) = 0.3, \delta_{R_1}(x)(w_1) = 0.7 >\}$$

$$T_1(\neg w_1) = \{< x, \gamma_{T_1}(x)(\neg w_1) = 0.4, \delta_{T_1}(x)(\neg w_1) = 0.3 >\}$$

$$R_1(w_1) = \{< y, \gamma_{R_1}(y)(w_1) = 0.2, \delta_{R_1}(y)(w_1) = 0.6 >\}$$

$$T_1(\neg w_1) = \{< y, \gamma_{T_1}(y)(\neg w_1) = 0.3, \delta_{T_1}(y)(\neg w_1) = 0.7 >\}$$

And

$$R(w_1) = \{< x, \gamma_R(x)(w_1) = 0.2, \delta_R(x)(w_1) = 0.3 >\}$$

$$T(\neg w_1) = \{< x, \gamma_T(x)(\neg w_1) = 0.4, \delta_T(x)(\neg w_1) = 0.5 >\}$$

$$R(w_1) = \{< y, \gamma_R(y)(w_1) = 0.5, \delta_R(y)(w_1) = 0.8 >\}$$

$$T(\neg w_1) = \{< y, \gamma_T(y)(\neg w_1) = 0.3, \delta_T(y)(\neg w_1) = 0.5 >\}$$

$(R_1, T_1, W_1) \subseteq (R, T, W)$ now point $\in$ to $(e_{1_y}^{0.2}, \neg e_{1_y}^{0.3})$.
so $(e_x^p, \neg e_x^q) \in (R_1, T_1, W_1) \subseteq (R, T, W)$.

Theorem 4.1.5. *Let (U, ϕ, A) be a **(FFBSTS)**, then the following holds:*
(1) *There exist a **(FFBSoS)** neighborhood (R, T, S) for each **(FFBSP)** $(e_x^p, \neg e_x^q)$.*
(2) *If $(R_1, T_1, W_1), (R_2, T_2, W_2)$ are fermatean bipolar fuzzy soft neighborhood of $(e_x^p, \neg e_x^q)$ then $(R_1, T_1, W_1) \cap (R_2, T_2, W_2)$ is a **(FFBS)** neighborhood of $(e_x^p, \neg e_x^q)$.*
(3) *If (R_1, T_1, W_1) is a fermatean bipolar fuzzy soft neighborhood **(FFBS)**of $(e_x^p, \neg e_x^q)$ and $(R_1, T_1, W_1) \subseteq (R_2, T_2, W_2)$ Then (R_2, T_2, W_2) is a **(FFBS)**neighborhood of $(e_x^p, \neg e_x^q)$.*

Proof. (1) Let (U, τ, A) be a **(FFBSTS)**, (R, T, W) be a **(FBSS)** and $(e_x^p, \neg e_x^q)$ be a **(FFBSP)**in (U, A). (R, T, W) is called a **(FFBS)** neighborhood of $(e_x^p, \neg e_x^q)$ IF there exist Bipolar Fuzzy open Set (R_1, T_1, W_1) such that $(e_x^p, \neg e_x^q) \in (R_1, T_1, W_1) \subseteq (R, T, W)$.
(2) Let $(R_1, T_1, W_1), (R_2, T_2, W_2)$ are **(FFBSS)** neighborhood of $(e_x^p, \neg e_x^q)$ then there exists fermatean fuzzy bipolar soft open set $(J_1, K_1, Y_1), (J_2, K_2, Y_2)$ such that $(e_x^p, \neg e_x^q) \in (J_1, K_1, Y_1) \subseteq (R_1, T_1, W_1)$ and $(e_x^p, e_x^q) \in (J_2, K_2, Y_2) \subseteq (R_2, T_2, W_2)$. Hence $(e_x^p, \neg e_x^q) \in (J_1, K_1, Y_1) \cap (J_2, K_2, Y_2) \subseteq (R_1, T_1, W_1) \cap (R_2, T_2, W_2)$. Thus $(R_1, T_1, W_1) \cap (R_2, T_2, W_2)$ is a **(FFBS)** neighborhood of $(e_x^p, \neg e_x^q)$.
(3) Let (R_1, T_1, W_1) is a **(FFBS)** neighborhood of $(e_x^p, \neg e_x^q)$ and $(R_1, T_1, W_1) \subseteq (R_2, T_2, W_2)$ Then there exists a **(FFBSoS)**(J_1, K_1, Y_1) such that $(e_x^p, \neg e_x^q) \in (J_1, K_1, Y_1) \subseteq (R_1, T_1, W_1)$. Hence $(e_x^p, \neg e_x^q) \in (J_1, K_1, Y_1) \subseteq (R_2, T_2, W_2)$. This shows that (R_2, T_2, W_2) is a neighborhood of $(e_x^p, \neg e_x^q)$. □

4.1.6 Fermatean Fuzzy Bipolar Soft Interior Point

Let (Z, ϕ, W) be **(FFBSTS)**. $(e_x^p, \neg e_x^q)$ be a **(FFBSP)** and (R, T, W) be **(FFBSS)** in (U, S). $(e_x^p, \neg e_x^q)$ is called **(FFBSIP)** of (R, T, S) IF there exist fermatean bipolar Fuzzy Soft open Set $((J, K, Y)$ Such That $(e_x^p, \neg e_x^q) \in (J, K, Y) \subseteq (R, T, W)$. The all fermatean fuzzy

Bipolar Soft interior Points of (R,T,W) are called (**FFBSIP**) of (R,T,W) and is shown by FFBSINT(R,T,W).

Theorem 4.1.6. *Let (U,ϕ,A) be a (**FFBSTS**) and (R,T,W) be **FFBSS** in (Z,W), Then the followings hold:*

(1) *FFBSINT (R,T,W) contains all the fermatean fuzzy bipolar soft open sets contained in (R,T,W).*

(2) *FFBSINT $(R,T,W) \subseteq (R,T,W)$.*

(3) *FFBSINT (R,T,W) is a fermatean fuzzy bipolar soft open set.*

(4) *FFBSINT (R,T,W) is the biggest (**FFBSoS**) contained in (R,T,W).*

Proof. (1) Let $(e_x^p, \neg e_x^q)$ be a(**FFBSIP**) of (R, T, S). Then $(e_x^p, \neg e_x^q) \in^{\sim} (J_i,K_i,Y_i) \subseteq (R,T,S)$ Where $(R_i,T_i,S_i) \in^{\sim} \tau$. Thus $(e_x^p, \neg e_x^q) \in^{\sim} \bigcup^{\sim}(J_i,K_i,Y_i)$. Conversely Let $(e_x^p, \neg e_x^q) \in^{\sim} (J_i,K_i,Y_i) \subseteq (R,T,S)$ where $(J_i,K_i,Y_i)\tau$. Hence $(e_x^p, \neg e_x^q) \in^{\sim}$ **FFBSINT** (R,T,S).

(2) Let (U,ϕ,A) $(e_x^p, \neg e_x^q)$ be a (**FFBSP**) and (R, T, S) be a (**FFBSS**) in (U,A). $(e_x^p, \neg e_x^q)$ is called a (**FFBSIP**) of (R, T, S) IF there exist (**FFBSoS**) (J,K,Y) Such That $(e_x^p, \neg e_x^q) \in (J,K,Y) \subseteq$ (R, T, S). The set of all **FFBSIP** of (R,T,S) is called the **FFBS** interior of (R,T,S) and is shown by **FFBSINT** (R,T,S).

(3) Since we know that **FFBSINT** (R,T,S) is the Union of fermatean Fuzzy Soft open set.so **FFBSINT** (R,T,S) is fermatean Fuzzy bipolar Soft open set.

(4) we already discuss in 1. So **FFBSINT** (R,T,S) is the biggest(**FFBSoS**) contained in (R,T,S)

(5) Let (R,T,W) be a (**FFBSoS**). in 1 the fermatean fuzzy bipolar soft interior of (R,T,W) is the biggest fermatean fuzzy bipolar soft open set contained in (R,T,W). Hence **FFBSINT**t (R,T,W) = (R,T,W). Conversely let **FFBSINT** (R,T,W) = (R,T,W). Since the fermatean fuzzy bipolar soft interior of (R,T,W) is a fermatean fuzzy bipolar soft open set (R,T,W) is a FFBS open. □

Example :-

Let U = $\{x,y\}, S = \{s_1,s_2,s_3\}$, Then

$(R,T,S) = \{(s_2,\{0.1,0.3,0.4\},\{0.2\}),(s_3,\{0.2,0.3,0.4\},\phi)\}$

$\tau = \{(Z,\phi,A),(\phi,Z,A),(R_1,T_1,W),(R_2,T_2,W),(R_3,T_3,W)\}$ be a fermatean fuzzy bipolar soft topology on (Z,ϕ,A) where,

$$R_1(w_2)=\{<x,\gamma_{R_1}(x)(w_2)=(0.1,0.4),\delta_{R_1}(x)(w_2)=0.2>\}$$

$$T_1(\neg w_2)=\{<x,\gamma_{T_1}(x)(\neg w_2)=0.4,\delta_{T_1}(x)(\neg w_2)=0.1,0.3>\}$$

$$R_2(w_2)=\{<y,\gamma_{R_2}(y)(w_2)=0.3,\delta_{R_2}(y)(w_2)=(0.1,0.2)>\}$$

$$T_2(\neg w_2)=\{<y,\gamma_{T_2}(y)(\neg w_2)=(0.2,0.3,0.4)\delta_{T_2}(y)(\neg w_2)=0.1>\}$$

$$R_3(w_3)=\{<x,\gamma_{R_3}(x)(w_3)=(0.1,0.3,0.4)\delta_{R_2}(x)(w_1)=0.2>\}$$

$$T_3(\neg w_3)=\{<x,\gamma_{T_3}(x)(\neg w_3)=(0.2,0.3,0.4)\delta_{T_3}(x)(\neg w_3)=0.1>\}$$

$$R_3(w_3)=\{<y,\gamma_{R_2}(y)(w_1)=\phi,\delta_{R_2}(y)(w_1)=0.1,0.2>\}$$

$$T_3(\neg w_3)=\{<y,\gamma_{T_3}(y)(\neg w_3)=0.4,\delta_{T_3}(y)(\neg w_3)=(0.1,0.3)>\}$$

Let

$$G_1(w_1)=\{<x,\gamma_{G_1}(x)(w_1)=(0.1,0.4)\delta_{G_1}(x)(w_1)=0.2>\}$$

$$H_1(\neg w_1)=\{<x,\gamma_{H_1}(x)(\neg w_1)=(0.2,0.4),\delta_{H_1}(x)(\neg w_1)=0.1>\}$$

$$G_2(w_2)=\{<y,\gamma_{G_2}(y)(w_2)=0.3,\delta_{G_2}(y)(w_2)=(0.1,0.2)>\}$$

$$H_2(\neg w_2)=\{<y,\gamma_{H_2}(y)(\neg w_2)=(0.2,0.3,0.4)\delta_{H_2}(y)(\neg w_2)=\phi>\}$$

Then

$(G_1,H_1,S)^{int} = (R,T,S)$ and $(G_2,H_2,S)^{int} = (R_2,T_2,S)$

Thus

$(G_1,H_1,S)^{int} \cup (G_2,H_2,S)^{int} = (R_3,T_3,S)$

Now,

$$(G_1,H_1,S),(G_2,H_2,S) = \{< x, \gamma_{R_3}(x)(w_3) = (0.1,0.3,0.4)\delta_{R_2}(x)(w_1) = 0.2 >\}$$

$$T_3(\neg w_3) = \{< x, \gamma_{T_3}(x)(\neg w_3) = (0.2,0.3,0.4), \delta_{T_3}(x)(\neg w_3) = \phi >\}$$

$(G_1 \cup G_2) = \{x,(00.1,00.4),(00.2)\} \cup \{x,(00.3),(00.1,00.2)\}$

$(G_1 \cup G_2) = \{x,(00.1,00.4) \cup (00.3),(00.2) \cap (00.1,00.2)\}$

$(G_1 \cup G_2) = \{x,(00.1,00.3,00.4),(00.2)\}$

$(H_1 \cup H_2) = \{x,(0.2,0.4),(0.1)\} \cup \{x,(0.2,0.3,0.4),\phi\}$

$(H_1 \cup H_2) = \{x,(0.2,0.4) \cup (0.2,0.3,0.4),(0.1) \cap \phi\}$

$(H_1 \cup H_2) = \{x,(0.2,0.3,0.4),\phi\}$

Therefore,

$[(G_1,H_1,S) \cup (G_2,H_2,S)]^{int} = (U,\phi,A)$

$\neq (G_1,H_1,S)^{int} \cup (G_2,H_2,S)^{int}$

Theorem 4.1.7. *Let (Z,ϕ,W) be a FFBST space, $(R,T,W),(J, K, W)$ be* ***(FFBSS)*** *in (Z,W). Then the following holds:*

(1) *FFBSINT $(\phi,U,W) = (\phi,U,W)$ and* ***FFBSINT*** *$(U,\phi,W) = (U,\phi,W)$*

(2) *FFBSINT(FFBSINT(R,T,W)) = FFBSINT (R,T,W).*

(3) *If $(R,T,W) \subseteq^{\sim} (J, K, W)$ then ffbsint $(R,T,W) \subseteq^{\sim}$ FFBSINT$((J,K,W))$.*

(4) *FFBSINT $(R,T,W) \subseteq^{\sim}$* ***FFBSINT*** *$((J,K,W))$ =* ***FFBSINT*** *$[(R,T,W) \cap^{\sim} ((J,K,W))]$.*

(5) *FFBSINT$(R,T,W) \cup^{\sim}$* ***FFBSINT*** *$((J,K,W)) \subseteq^{\sim}$* ***FFBSINT*** *$[(R,T,W) \cup^{\sim} ((J,K,W))]$.*

Proof. (1) The proof is done.

(2) Let **FFBSINT** $(R,T,W) = (J,K,Y)$. Since (J,K,Y) is a (**FFBSoS**) FFBSINT$(J,K,Y) = (J,K,Y)$. Hence the proof is done.

(3) Let $(R,T,W) \subseteq^{\sim} (J,K,Y)$. It is known that ffbsint $(R,T,W) \subseteq^{\sim} (R,T,W)$ and so, **FFBSINT** $(R,T,W) \subseteq^{\sim} (J,K,Y)$. Also **FFBSINT** $((J,K,W)) \subseteq^{\sim} ((J,K,W))$ and **FFBSINT** (J, K, W) is the biggest (**FFBSoS**) contained in (J,K,Y). Thus, **FFBSINT** $((J,K,W)) \subseteq^{\sim}$ **FFBSINT** (J,K,Y).

(4) It can be easy that FFBSINT $(R,T,W) \subseteq^{\sim} (R,T,W)$ and FFBSINT$(J,K,Y) \subseteq^{\sim} (J,K,Y)$. Hence FFBSINT$(R,T,W)\cap^{\sim}$ FFBSINT(J,K,Y) is a (**FFBSoS**) contained in $(R,T,W)\cap^{\sim}$ (J,K,Y). It is also known that FFBSINT$[(R,T,W)\cap^{\sim}(J,K,Y)]$ is the (**FFBSoS**) contained in $(R,T,W)\cap^{\sim}(J,K,Y)$. Thus, (**FFBSINT**)$(R,T,W)\cap^{\sim}$ FFBSINT$(J,K,Y) \subseteq^{\sim}$ FFBSINT$[(R,T,W)\cap^{\sim}$ $(J,K,Y]$.

Conversely, it is clear that (**FFBSINT**)$[(R,T,W)\cap^{\sim}(J,K,Y)] \subseteq^{\sim}$ (**FFBSINT**)(R,T,W) and (**FFBSINT**)$[(R,T,W)\cap^{\sim}(J,K,Y)] \subseteq^{\sim}$ (**FFBSINT**)(J,K,Y). Thus, (**FFBSINT**)$[(R,T,W)\cap^{\sim}$ $(J,K,Y)] \subseteq^{\sim}$ (**FFBSINT**)$(R,T,W)\cap^{\sim}$ (**FFBSINT**)(J,K,Y). Hence (**FFBSINT**)$(R,T,W)\cap^{\sim}$ (**FFBSINT**)(J,K,Y) = (**FFBSINT**)$[(R,T,W)\cap^{\sim}(J,K,Y)]$.

(5) It is known that (**FFBSINT**)$(R,T,W) \subseteq^{\sim} (R,T,W)$ and (**FFBSINT**)$(J,K,Y) \subseteq^{\sim} (J,K,Y)$. Hence (**FFBSINT**)$(R,T,W)\cup^{\sim}$ (**FFBSINT**)$(J,K,Y) \subseteq^{\sim}$ (F, G, A)$\cup^{\sim}(J,K,Y)$. Also the biggest fermatean fuzzy bipolar soft open set contained in $(R,T,W)^{\sim}(J,K,Y)$ is (**FFBSINT**)$[(R,T,W)\cup^{\sim}$ $(J,K,Y)]$. Thus (**FFBSINT**)$(R,T,W)\cup^{\sim}$ (**FFBSINT**)$(J,K,Y) \subseteq^{\sim}$ (**FFBSINT**)$[(R,T,W)\cup^{\sim}$ $(J,K,Y)]$. □

4.1.7 Fermatean Fuzzy Bipolar Soft Closure

Let (Z,τ,W) be a FFBST space and (R,T,W) be a ffbs set over (U,A). The intersection of all (**FFBSCL**) containing in (R,T,W) is called the fermatean fuzzy bipolar soft closure of (R,T,W) and is denote by (**FFBSCL**)(R,T,W).

Example :-

Let U = $\{x,y\}, W = \{w_1,w_2,w_3\}$, Then

$(R,T,W) = \{(w_2,Z,\phi),(w_3,Z,\phi)\} = (Z,\phi,W)$

$\tau = \{(Z,\phi,W),(\phi,Z,W),(R_1,T_1,W),(R_2,T_2,W),(R_3,T_3,W)\}$ be a fermatean fuzzy bipolar soft topology on (Z,ϕ,W) where,

$$R_1(s_2) = \{< x, \gamma_{R_1}(x)(s_2) = (0.1,0.2), \delta_{R_1}(x)(s_2) = 0.3 >\}$$

$$T_1(\neg s_2) = \{< x, \gamma_{T_1}(x)(\neg s_2) = (0.1, 0.3)\delta_{T_1}(x)(\neg s_2) = 0.2 >\}$$

$$R_2(s_2) = \{< y, \gamma_{R_2}(y)(s_2) = (0.2, 0.3), \delta_{R_2}(y)(s_2) = \phi >\}$$

$$T_2(\neg s_2) = \{< y, \gamma_{T_2}(y)(\neg s_2) = (0.1), \delta_{T_2}(y)(\neg s_2) = (0.2, 0.3) >\}$$

$$R_3(s_3) = \{< x, \gamma_{R_3}(x)(s_3) = (0.2), \delta_{R_2}(x)(s_1) = 0.3 >\}$$

$$T_3(\neg s_3) = \{< x, \gamma_{T_3}(x)(\neg s_3) = (0.1), \delta_{T_3}(x)(\neg s_3) = (0.2, 0.3) >\}$$

$$R_3(s_3) = \{< y, \gamma_{R_2}(y)(s_1) = W, \delta_{R_2}(y)(s_1) = \phi >\}$$

$$T_3(\neg s_3) = \{< y, \gamma_{T_3}(y)(\neg s_3) = (0.1, 0.3), \delta_{T_3}(y)(\neg s_3) = (0.2) >\}$$

Let

$$G_1(s_1) = \{< x, \gamma_{G_1}(x)(s_1) = \phi, \delta_{G_1}(x)(s_1) = (0.1, 0.2) >\}$$

$$H_1(\neg s_1) = \{< x, \gamma_{H_1}(x)(\neg s_1) = (0.2), \delta_{H_1}(x)(\neg s_1) = (0.1, 0.3) >\}$$

$$G_2(s_2) = \{< y, \gamma_{G_2}(y)(s_2) = \phi, \delta_{G_2}(y)(s_2) = (0.2, 0.3) >\}$$

$$H_2(\neg s_2) = \{< y, \gamma_{H_2}(y)(\neg s_2) = (0.3)\delta_{H_2}(y)(\neg s_2) = (0.1, 0.2) >\}$$

Then

$(G_1, H_1, W)^{cl} = (R_1, T_1, W_1)^c$ and $(G_2, H_2, W)^{cl} = (R_2, T_2, W_2)^c$

Thus

$(G_1, H_1, W)^{cl} \cap (G_2, H_2, W)^{cl} = (R_3, T_3, W)^c$

Now,

$$(G_1,H_1,W)\cap(G_2,H_2,W)=\{<x,\gamma_{G_3}(x)(s_3)=\phi,\delta_{G_2}(x)(s_1)=W>\}$$

$$T_3(\neg s_3)=\{<x,\gamma_{H_3}(x)(\neg s_3)=\phi,\delta_{H_3}(x)(\neg s_3)=W>\}$$

$(G_1\cap G_2)=\{x,\phi,(0.1,0.2)\}\cap\{x,\phi,(0.2,0.3)\}$
$(G_1\cap G_2)=\{x,\phi\cap\phi,(0.1,0.2)\cup(0.2,0.3)\}$
$(G_1\cap G_2)=\{x,\phi,W\}$
$(H_1\cap H_2)=\{x,(0.2),(0.1,0.3)\}\cap\{x,(0.3),(0.1,0.2)\}$
$(H_1\cap H_2)=\{x,(0.2)\cap(0.3),(0.1,0.3)\cup(0.1,0.2)\}$
$(H_1\cap H_2)=\{x,\phi,W\}=(\phi,U,A)$
Therefore,
$[(G_1,H_1,W)\cap(G_2,H_2,W)]^{cl}=(\phi,U,A)$
$\neq(G_1,H_1,W)^{cl}\cap(G_2,H_2,W)^{cl}$

Theorem 4.1.8. *Let (Z,τ,W) be a (**FFBSTS**) space and (R,T,W) and (J,K,Y) be two (**FFBS**) sets over (Z,W). Then*
(1) $[(R,T,W)^{cl}]^c=[(R,T,W)^c]^{int}$
(2) $[(R,T,W)^c]^{cl}=[(R,T,W)^{int}]^c$

Proof. (1) Let $\{(R_1,T_1,W):(R,T,W)\subseteq^{\sim}(R_1,T_1,W),(R_1,T_1,W)^c\in\tau 1\in I\}$

$$[(R,T,W)^{cl}]^c=[\widetilde{\bigcap_{1\in I}}(R_1,T_1,W)]^c$$

$$=\widetilde{\bigcup_{i\in I}}(R_1,T_1,W)^c$$

$$=[(R,T,W)^c]^{int}$$

(2) similar to (1). Let $\{(R_1,T_1,W):(R,T,W)\subseteq^{\sim}(R_1,T_1,W),(R_1,T_1,W)^c\in\tau 1\in I\}$

$$[(R,T,W)^c]^{cl}=[\widetilde{\bigcap_{1\in I}}(R_1,T_1,W)]^c$$

$$= \widetilde{\bigcup_{i \in I}} (R_1, T_1, W)^c$$

$$= [(R,T,W)^{int}]^c$$

□

Theorem 4.1.9. *Let (Z,τ,W) be $(\textbf{\textit{FFBSTS}})$ Space and (R,T,W) and (J,K,Y) be Two $(\textbf{\textit{FFBS}})$ Sets in (Z,W). Then the followings hold:*
(1) $(\textbf{\textit{FFBSCL}})(R,T,W)$ *is a* $(\textbf{\textit{FFBS}})$ *closed set.*
(2) $(R,T,W) \subseteq^{\sim} (\textbf{\textit{FFBSCL}})(R,T,W)$.
(3) $(\textbf{\textit{FFBSCL}})(R,T,W)$ *is the smallest* $(\textbf{\textit{FFBS}})$ *closed set containing* (R,T,W).
(4) *If* $(R,T,W) \subseteq (J,K,Y)$ *then* $(\textbf{\textit{FFBSCL}})(R,T,W) \subseteq (\textbf{\textit{FFBSCL}})(J,K,Y)$.
(5) (R,T,W) *is (FFBS) closed set iff* $(\textbf{\textit{FFBSCL}})(R,T,W) = (R,T,W)$.
(6) $(\textbf{\textit{FFBSCL}})((\textbf{\textit{FFBSCL}})(R,T,W))$ = $(\textbf{\textit{FFBSCL}})(R,T,W)$.
(7) $(\textbf{\textit{FFBSCL}})[(R,T,W) \cup (J,K,Y)]$ = $(\textbf{\textit{FFBSCL}})(R,T,W) \cup$ $(\textbf{\textit{FFBSCL}})(J,K,Y)$.
(8) $(\textbf{\textit{FFBSCL}})[(R,T,W) \cap (J,K,Y)] \subseteq (\textbf{\textit{FFBSCL}})(R,T,W) \cap$ $(\textbf{\textit{FFBSCL}})(J,K,Y)$.

Proof. (1) The proof is clear from definition so the intersection of (**FFBSCL**) is (**FFBSS**).
(2) The proof is clear from Definition. (U,τ,A) be a (**FFBSTS**) and (R,T,W) be a (**FFBS**) set over (U,A). The intersection of all (**FFBSCL**) containing in (R,T,W) is called the (**FFBSCL**) of (R,T,W) and is denoted by (**FFBSCL**)(R,T,W).
(3) The proof is clear same as explained in 2.
(4) Let $(R,T,W) \subseteq (J,K,Y)$. It can be easy seen that $(R,T,W) \subseteq$ (**FFBSCL**)(R,T,W) and $(J,K,Y) \subseteq$ (**FFBSCL**)(J,K,Y) so,$(R,T,W) \subseteq$ (**FFBSCL**)(J,K,Y). Since the smallest (**FFBSCL**) containing (R,T,W) is (**FFBSCL**)(R,T,W) we obtain that (**FFBSCL**)$(R,T,W) \subseteq$ (**FFBSCL**)(J,K,Y).
(5) The proof is obvious.
(6) Suppose that (**FFBSCL**)$(R,T,W) = (J,K,Y)$. Since (J,K,Y) is a fermatean fuzzy bipolar soft closed set (**FFBSCL**)$(J,K,Y) = (J,K,Y)$. Hence the proof is completed.
(7) It is known that $(R,T,W) \subseteq$ (**FFBSCL**)(R,T,W) and $(J,K,Y) \subseteq$ (**FFBSCL**)(J,K,Y) and hence $(R,T,W) \cup (J,K,Y) \subseteq$ (**FFBSCL**)$(R,T,W) \cup$ (**FFBSCL**)l(H, I, B). Also it can be easily seen that $(R,T,W) \cup (J,K,Y) \subseteq$ (**FFBSCL**)$[(R,T,W) \cup$(H, I, B)]. Since

the smallest fermatean fuzzy bipolar soft closed set containing $(R,T,W)\cup(J,K,Y)$ is (**FFBSCL**)$[(R,T,W)\cap(J,K,Y)]$ we obtain that (**FFBSCL**)$[(R,T,W)\cup(J,K,Y)]\subseteq$ (**FFBSCL**)$(R,T,W)\cup$ (**FFBSCL**)(J,K,Y).
Conversely, it is known that (**FFBSCL**)$(R,T,W)\subseteq$ (**FFBSCL**)$[(R,T,W)\cup(J,K,Y)]$ and (**FFBSCL**)$(J,K,Y)\subseteq$ (**FFBSCL**)$[(R,T,W)\cup(J,K,Y)]$. Hence we get (**FFBSCL**)$(R,T,W)\cup$ (**FFBSCL**)(J, K, Y) $\subseteq$ (**FFBSCL**)$[(R,T,W)\cup(J,K,Y)]$.
(8) It can be easily seen that (**FFBSCL**)$[(R,T,W)\cap(J,K,Y)]\subseteq$ (**FFBSCL**)(R,T,W) and (**FFBSCL**)$[(R,T,W)\cap(\mathtt{J,\ K,\ y})]\subseteq$ (**FFBSCL**)(J,K,Y). Thus, (**FFBSCL**)$[(R,T,W)\cap(J,K,Y)]\subseteq$(**FFBSCL**)$(R,T,W)\cap$ (**FFBSCL**)(J,K,Y). □

4.1.8 Fermatean Fuzzy Bipolar Soft Exterior

Let (Z,τ,W) be a **FFBST** space and (R,T,W) be a **FFBS** set over (Z,W). The interior of all (**FFBS**) complement of (R,T,W) is called the fermatean fuzzy bipolar soft exterior of (R,T,W) and is denote By (**FFBSE**)(R,T,W).

Example :-

Let U = $\{x,y\},W=\{w_1,w_2,w_3\}$, Then
$(R,T,W)=\{(s_2,(0.2,0.3),\phi),(s_3,0.1,0.2)\}Then(R,T,W)^c=(\phi,Z,W)$
$\tau=\{(Z,\phi,W),(\phi,Z,W),(R_1,T_1,W),(R_2,T_2,W),(R_3,T_3,W)\}$ be a fermatean fuzzy bipolar soft topology on (Z,ϕ,W) where,

$$R_1(s_2)=\{<x,\gamma_{R_1}(x)(s_2)=(0.1,0.2),\delta_{R_1}(x)(s_2)=0.3>\}$$

$$T_1(\neg s_2)=\{<x,\gamma_{T_1}(x)(\neg s_2)=(0.1,0.3)\delta_{T_1}(x)(\neg s_2)=0.2>\}$$

$$R_2(w_2)=\{<y,\gamma_{R_2}(y)(w_2)=(0.2,0.3),\delta_{R_2}(y)(w_2)=\phi>\}$$

$$T_2(\neg w_2)=\{<y,\gamma_{T_2}(y)(\neg w_2)=(0.1),\delta_{T_2}(y)(\neg w_2)=(0.2,0.3)>\}$$

$$R_3(s_3) = \{< x, \gamma_{R_3}(x)(s_3) = (0.2), \delta_{R_2}(x)(s_1) = 0.3 >\}$$

$$T_3(\neg s_3) = \{< x, \gamma_{T_3}(x)(\neg s_3) = (0.1), \delta_{T_3}(x)(\neg s_3) = (0.2, 0.3) >\}$$

$$R_3(s_3) = \{< y, \gamma_{R_2}(y)(s_1) = W, \delta_{R_2}(y)(s_1) = \phi >\}$$

$$T_3(\neg s_3) = \{< y, \gamma_{T_3}(y)(\neg s_3) = (0.1, 0.3), \delta_{T_3}(y)(\neg s_3) = (0.2) >\}$$

Let

$$G_1(s_1) = \{< x, \gamma_{G_1}(x)(s_1) = \phi, \delta_{G_1}(x)(s_1) = (0.1, 0.2) >\}$$

$$H_1(\neg s_1) = \{< x, \gamma_{H_1}(x)(\neg s_1) = (0.2), \delta_{H_1}(x)(\neg s_1) = (0.1, 0.3) >\}$$

$$G_2(w_2) = \{< y, \gamma_{G_2}(y)(w_2) = \phi, \delta_{G_2}(y)(w_2) = (0.2, 0.3) >\}$$

$$H_2(\neg w_2) = \{< y, \gamma_{H_2}(y)(\neg w_2) = (0.3)\delta_{H_2}(y)(\neg w_2) = (0.1, 0.2) >\}$$

Then

$(G_1, H_1, W)^e = (R_1, T_1, W_1)^c$ and $(G_2, H_2, W)^e = (R_2, T_2, W_2)^c$

Thus

$(G_1, H_1, W)^e \cap (G_2, H_2, W)^e = (R_3, T_3, W)^c$

Now,

$$(G_1, H_1, W) \cap (G_2, H_2, W) = \{< x, \gamma_{G_3}(x)(s_3) = \phi, \delta_{G_2}(x)(s_1) = W >\}$$

$$T_3(\neg s_3) = \{< x, \gamma_{H_3}(x)(\neg s_3) = \phi, \delta_{H_3}(x)(\neg s_3) = W >\}$$

$(G_1 \cap G_2) = \{x, \phi, (0.1, 0.2)\} \cap \{x, \phi, (0.2, 0.3)\}$

$(G_1 \cap G_2) = \{x, \phi \cap \phi, (0.1, 0.2) \cup (0.2, 0.3)\}$

$(G_1 \cap G_2) = \{x, \phi, W\}$

$(H_1 \cap H_2) = \{x, (00.2), (00.1, 00.3)\} \cap \{x, (00.3), 00.1, 00.2)\}$

$(H_1 \cap H_2) = \{x, (00.2) \cap (00.3), (00.1, 00.3) \cup (0.1, 0.2)\}$

$(H_1 \cap H_2) = \{x, \phi, W\} = (\phi, Z, W)$

Therefore,

$[(G_1, H_1, W) \cap (G_2, H_2, W)]^e = (\phi, Z, W)$

$\neq (G_1, H_1, W)^e \cap (G_2, H_2, W)^e$

Chapter 5

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