Evaluation of Connection Number-Based Indices of Networks Derived from Triglycerides

By

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MS

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In partial fulfillment

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A Post Graduate Thesis is submitted to the Department of Mathematics as partial fulfillment of the requirement for the award of Degree of MS Mathematics.

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Declaration

I Hudaa Zaidi with registration number CIIT/FA21-RMT-036/LHR hereby declare that I have produced the work presented in this report, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.
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Certificate

It is certified that **Hudaa Zaidi with registration number CIIT/FA21-RMT-036/LHR** has carried out all the work related to this report under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore Campus, and the work fulfills the requirement for award of MS degree.

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Department of Mathematics

DEDICATION

To

To My Parents, My Family
And My All Teachers

ACKNOWLEDGEMENT

Praise to be ALLAH, the Cherisher and Lord of the World, Most gracious and Most Merciful

First and foremost, I would like to thank **ALLAH Almighty** (the most beneficent and most merciful) for giving me the strength, knowledge, ability, and opportunity to undertake this research study and to preserve and complete it satisfactorily. Without countless blessings of **ALLAH Almighty**, this achievement would not have been possible. May His peace and blessings be upon His messenger **Hazrat Muhammad (PBUH),** upon his family, companions and whoever follows him. My insightful gratitude to **Hazrat Muhammad (PBUH)** Who is forever a track of guidance and knowledge for humanity as a whole.

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ABSTRACT

Evaluation of Connection Number-Based Indices of Networks Derived from Triglycerides

Graph theory serves as the fundamental framework for chemical informatics, utilizing topological indices to generate chemical structures and establish connections between real numbers and molecular graphs. In 1974, Gutman and Trinajstic introduced connection-based Zagreb indices as molecular descriptors to analyze the topological properties of chemical compounds. These indices were subsequently named Zagreb indices. Building on this research, Ali et al. further explored the applicability and properties of Zagreb indices in theoretical chemistry and molecular modeling. This thesis specifically focuses on the computation of connection-based number indices for triglycerides, as well as their line and para-line graphs. By studying these indices, the aim is to gain insights into the topological characteristics of triglyceride structures and their corresponding graph representations. This research contributes to the broader field of chemical informatics, providing valuable information for the analysis and modeling of complex molecules.

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Chapter 1

Introduction and Historical Background

In this chapter of the thesis, we will discuss the history of graph theory, chemical graph theory, and theoretical concepts of topological indices. Additionally, we will explain essential definitions that will help readers understand the basic concepts related to the topic of graph theory.

1.1 History of Graph Theory

In 18th century *Swiss mathematician Leonhard Euler* was introduced graph theory. He used graph theoretic parameters to study a well-known K$\ddot{o}$nigsberg bridge. The city of K$\ddot{o}$nigsberg, today known as Kaliningrad in Russia, is located on the Pergola river and is made up of four main areas that are connected by seven bridges. Euler was given the challenge to assess if it was possible to pass each bridge only once in a single walk. Euler developed the first known visual representation of today's graph after recognizing all four earth bodies with seven bridges as necessary constraints. In modern-day graph, vertices or nodes are represented by points, while edges or connecting lines connect these points.

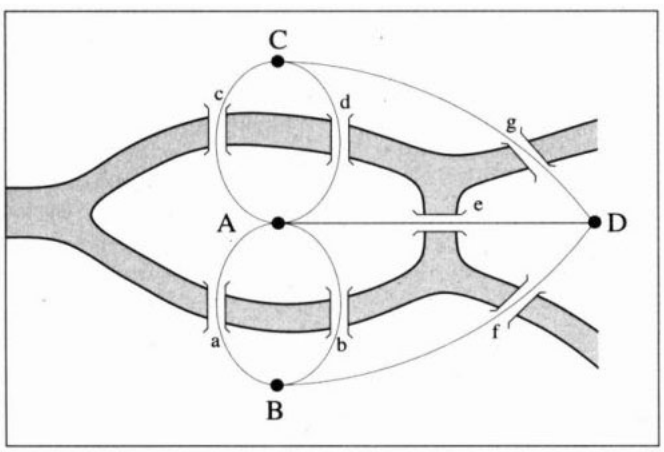

Figure 1.1: K$\ddot{o}$nigsberg Bridge

1.2 Chemical Graph Theory

A graph that re presents the bonds and atoms in a chemical structure is known as chemical graph. In this type of graph, vertices shows the atoms, while edges represent the bonds that connect them. The number of edges that incident on a vertex is defined as its degree.

Degree of any vertex in chemical graph is at most 5, as atoms can have at most 5 bonds with other atoms. The field of chemical graph theory emerged from the desire to combine natural sciences and mathematics. Chemical graphs have been used since the late 18th century, In which the fundamental concepts of matter and particles were discovered. In his lecture notes, a Scottish chemist [19] named these graphs "Affinity diagrams", they were used to explain the forces that occur between paired molecules during chemical processes. Arthur Cayley and James Sylvester were pioneers in developing structural formulae for [24] chemical graphs. Later, Harry Wiener established a connection between the molecular structure of alkanes and their boiling points using a distance-based index, which he initially called "path number". The sum of the distance between any two carbon atoms in the molecule measured in term of carbon-carbon bonds was used to establish this index. Eventually, it became known as the Wiener index, and it is commonly used to describe tree [23] structures.

1.3 Topological Indices and its Applications

The topological index (TI) is a numerical value that quantifies the overall structure of a molecular graph. Topological index are useful in analyzing quantitative structures attributes [9, 8, 20] and qualitative structures with active relationships QSPR between chemical compounds. In 1947, Wiener investigated the first topological index based on distance to determine the boiling point [22] of paraffin. As we mentioned before, chemical graph theory, QSPR, and QSAR are all fields where topological indices are often used. Researchers such as Ali et al. have shown that connection-based TIs are more effective than the commonly used degree-based topological indices (TIs). Many studies have focused on connection-based topological indices (TIs) for different types of chemical structures and families of graphs, including the work of Reti et al. on derived graphs based on connection numbers in 2012. In 2017, Kavithaa et al. introduced three different forms of pseudo-regular graphs, while Naji and Soner computed leap Zagreb indexes for certain graphs in the same year. More recently, Tang et al. utilized T-sum graphs to investigate new sequences using modified 1^{st} and 2^{nd} Zagreb connection indices. Usman et al [3] have developed new connection number-based TIs and compared them to previous TIs using numerical methods, 3D plots, and line graphs.

1.4 Elements of Graph Theory

[21] A *graph* is a mathematical concept that is beneficial in solving various problems. There is a relationship between lines and nodes in a graph.G is graph made up of a collection of objects $V(G)$ called vertices, together with a collection of unordered pairs of distinct vertices $E(G)$ known as edges. A graph is typically represented using symbolic notation "$G = (V(G), E(G))$." For example:- The representation of graph doesn't indicate meaning of vertices and edges, such as the countries and cities connected by roads.

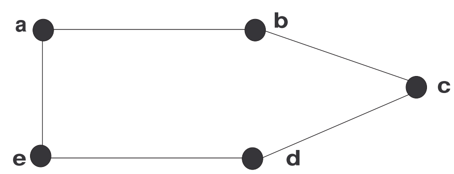

Figure 1.2: Graph

1.4.1 Degree of Graph

[21] The number of edges connected to a single node represent **vertex's degree**. The minimum count of edges joining a single node is called **graph's minimum degree**. It is denoted by $\delta(G)$. The maximum count of edges joining a single node is called **graph's maximum degree**. "$\Delta(G)$ is a symbol that represents degree of graph."

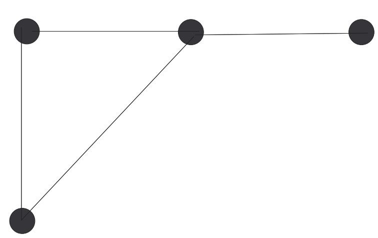

Figure 1.3: Degree of Graph

1.4.2 Order of Graph

[21] The total "***number of vertices***" in a graph is called the graph's order represents as $|V|$. As shown in Figure 1.3. The graph is of order 4.

1.4.3 Size of Graph

[21] Total "***number of Edges***" in a graph is called the graph's size represents as $|E|$. As shown in Figure 1.3.The graph is of size 6.

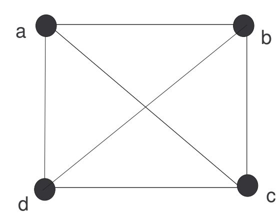

Figure 1.4: Size and Order of Graph

1.4.4 Adjacent Vertex

[21] We say two vertices adjacent if and only if they are connected by an edge.

1.4.5 Bipartite Graph

[21] Bipartite graph is basically the graph in which vertices can be divided into two distinct sets, with no edge between any two vertices of the same set. In other words, a bipartite graph has only even cycle.

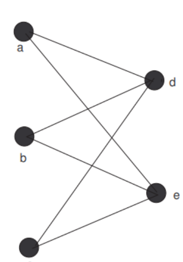

Figure 1.5: Bipartite Graph

1.4.6 Complete Graph

[21] A *graph's order* $n \geq 1$ with exactly one edge connecting each pair of vertices is known as Complete graph.It is represented using symbolic notation K_n. Every vertex is adjacent to other vertices.

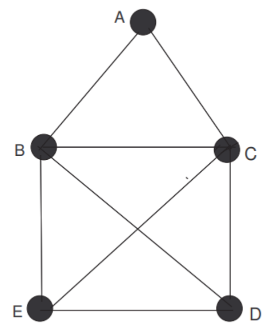

Figure 1.6: Complete Graph

1.4.7 Connected Graph

[21] Graph is said to be **connected graph** iff it contain at least one path to every other vertex.
Here,

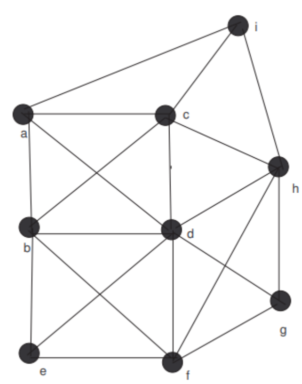

Figure 1.7: Connected Graph

- This above figure is connected as it fulfil the condition of the definition.

1.4.8 Disconnected Graph

[21] G is a graph that we said to be connected if every pair of its vertex set is part of a path; otherwise it is said to be disconnected.

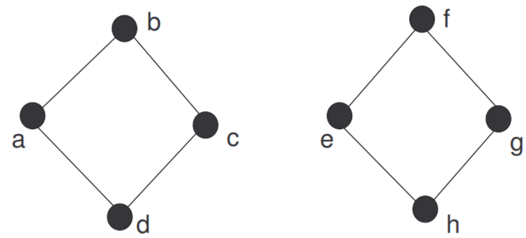

Figure 1.8: Disconnected Graph

1.4.9 Cycle Graph

[21] A cycle graph is a cycle in which a path begins and ends at the same vertex but contains no additional vertices that are repeated.It is denoted by C_n. Take a look at the following graph.

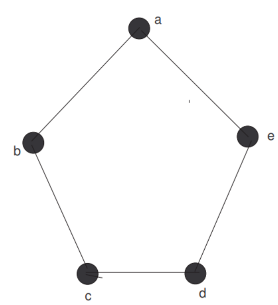

Figure 1.9: Cycle Graph

1.4.10 Cyclic Graph

[21] A **cyclic** graph is one that contains at least one cycle.

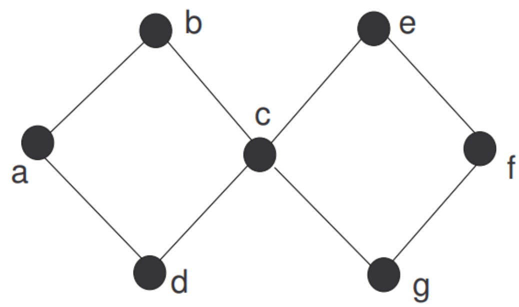

Figure 1.10: Cyclic Graph

The above figureshows two cycles: $a-b-c-d-a$ *and* $c-e-f-g-c$. As a result, it is known as a cyclic graph.

1.4.11 Acyclic Graph

[21] An **acyclic** graph is a graph having no cycle.

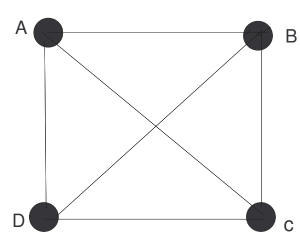

Figure 1.11: Acyclic Graph

1.4.12 Degree of vertex

[21] The total number of edges incident to a vertex is known as degree of vetex.

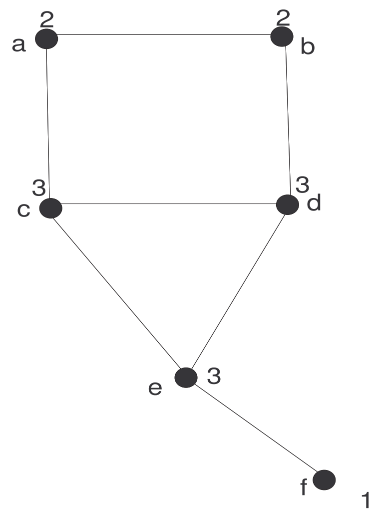

Figure 1.12: Degree of Vertex

1.4.13 Pendant Vertex

[21] All those vertices in graph *G* that have degree 1 (one) are said to be pendant vertices. **Pendant vertex** is also known as terminal node.

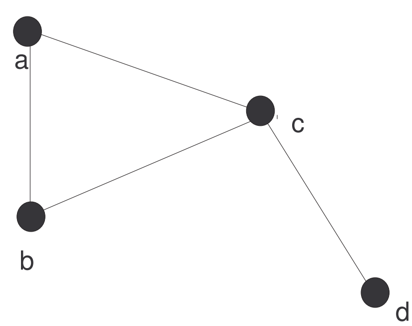

Figure 1.13: Pendant Vertex

1.4.14 Planar Graph

[21] A **planar** graph is one that has no two edges that intersect. In other words, we can draw it as a plane with no two edges intersecting.

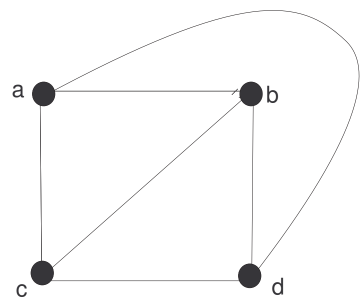

Figure 1.14: Planar Graph

1.4.15 Wheel Graph

[21] In a concept of graph theory,a wheel graph is formed when a single universal vertex is connected with all vertices of a cycle in a graph then it is called wheel graph. It is represented by W_n. For order, a wheel graph W_n is defined as $W_n = K_1 + C_n$.

Trail

[21] A walk where all of the edges are distinct are called trail.

1.4.16 Path

[21] A *Path* is a walk where vertices and edges are distinct.

Figure 1.15: Path Graph

1.4.17 Degree Sequence

[21] A graph's degree sequence is a list that contains all of the vertices in graph.Degree of a vertex is a number of edges that connect with that vertex. In degree sequence, we arrange these degrees in non-increasing order meaning from the largest degree to smallest degree.

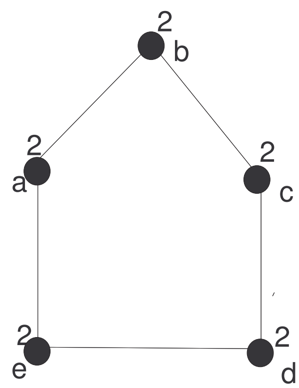

Figure 1.16: Degree Sequence (2,2,2,2,2)

1.4.18 Line Graph

[21] A **line graph L(G)**of a graph *G* is a graph where the vertices represent the *G's* edges. In the line graph, two vertices are connected if the related edges in *G* are incident to the same *G's* vertex.

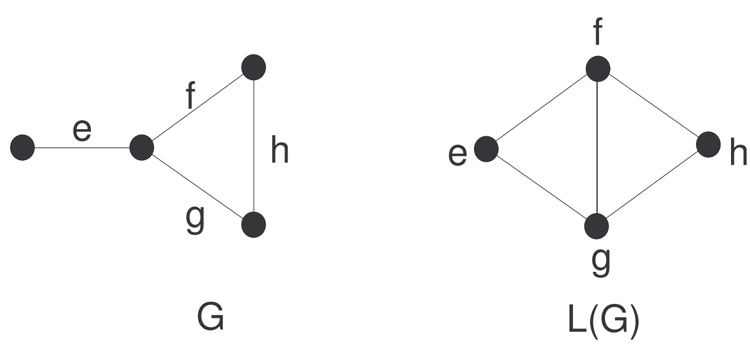

Figure 1.17: Line Graph

1.4.19 Para-line Graph

[21] A *line graph of G subdivision*, commonly referred to as a **para-line graph** of G. We can write it as $L(S(G))$.

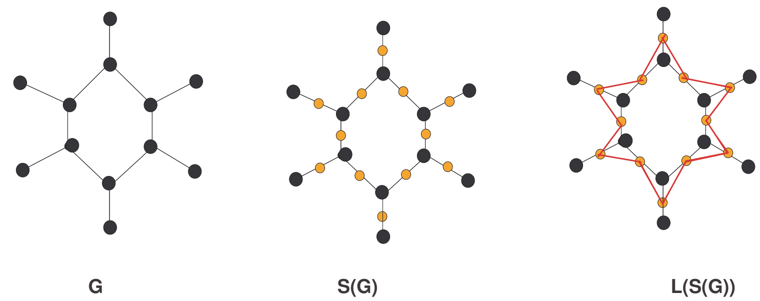

Figure 1.18: Para-Line Graph

1.5 Construction of Triglyceride

Three fatty acids and glycerol combine to generate the esters known as triglycerides. They are present in body fat and act as a source of energy for cells. Unused bits of a meal containing fat are kept in fat cells as triglycerides.

There are two types of fatty acid triglyceride residues: saturated and unsaturated. Carbon atoms in unsaturated fatty acids are connected by double bonds, whereas carbon atoms in saturated fatty acids are joined by single bonds [13]. Here, our focus is on saturated fatty acid carbon atoms.

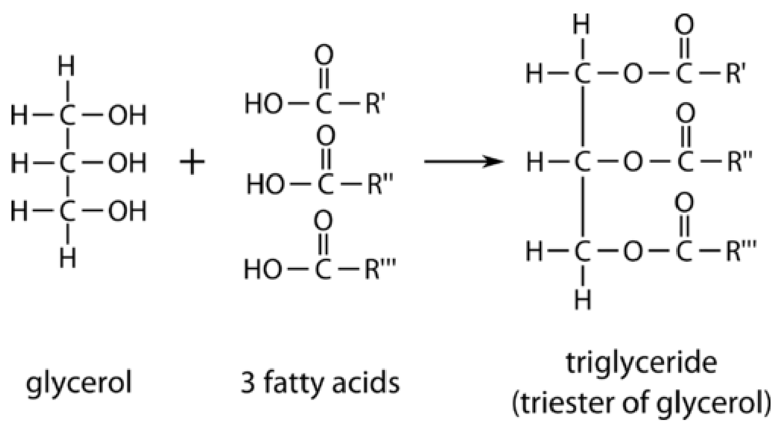

Figure 1.19: Chemical Structure of Triglycerides

Henceforward, in the chapter-3, these chains were converted into simple graphs.

Chapter 2

Literature Review

2.1 Introduction

In mathematical chemistry and chemical graph theory, the structural formula of a compound represents its molecular graph using graph theory concepts. Topological indices are numerical measures that provide information about the properties of a molecule. These indices, known as molecular descriptors, are widely used in theoretical chemistry, especially in QSAR/QSPR research [10]. Gutman and Trinajstipresented connection-based Zagreb indices as molecular descriptors for understanding chemical compound topological features in 1974. Ali et al. later named these as Zagreb indices. Afterwards, they continued their investigation on connection-based Zagreb indices, exploring further into their applications and qualities in theoretical chemistry and molecular modelling [6].

Recently, Naji et al. introduced three new topological indices:-

(1) 1^{st} Connection Zagreb Index

(2) 2^{nd} Second Connection Zagreb Index

(3) 3^{rd} Connection Zagreb Index

These indices aim to capture specific aspects of molecular structure [15] and have potential applications in various field of chemical Research [18, 17].

2.2 Connection Number of a Vertex

The connection number of a vertex,represents number of edges incident on a vertex v exactly at a distance of two. In similar words, the connectivity of a vertex refers to the number of edges that are connected to other vertices exactly two edges from the vertex v . It is denoted by $\bar{w}(v)$.

In figure below the *connection number* of $v_1, v_2, v_4, v_5, v_7, v_8, v_9$ is 2 , *connection number* of v_6 and v_{10} is 1, while *connection number* of v_3 is 4.

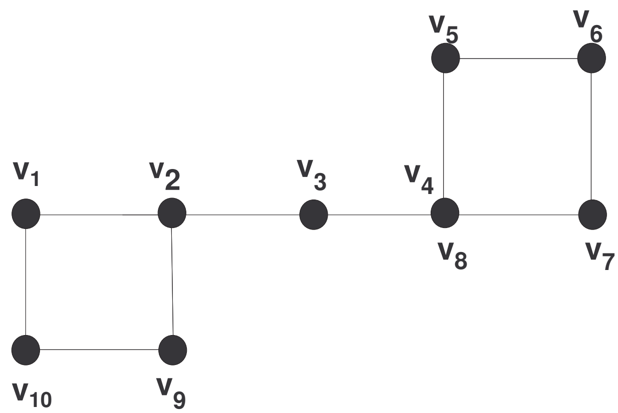

Figure 2.1: Connection Number

2.3 Connection Based Zagreb Indices

Following are the *Connection-based Zagreb indices*:

2.4 Zagreb Connection Index

The 1^{st} and 2^{nd} Zagreb connection indices were proposed by Ali and Trinajstic [1] in 2018:

(a) The 1^{st} *Zagreb connection index* of graph G_n is:

$$\hat{Z}C_1(G_n) = \sum_{t \in V(G_n)} [\bar{w}_{G_n}(t)]^2.$$

(b) The 2^{nd} *zagreb connection index* of graph G_n is:

$$\hat{Z}C_2(G_n) = \sum_{st \in E(G_n)} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)].$$

Here, the term $\bar{\omega}(t)$ represents the vertex'connection number t, while the term $\bar{\omega}(s)$ represents the vertex' connection number s for the edge st.

2.5 Modified Zagreb Connection Index mZCI

These modified Zagreb indices were proposed by Ali and Ali et al [3] in (2020):

(a) The 1^{st} *modified Zagreb connection index* of graph G_n is:

$$\hat{Z}C_1^*(G_n) = \sum_{st \in E^c(G_n)} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)].$$

(b) The 2^{nd} *modified Zagreb connection index* of graph G_n is:

$$\hat{Z}C_2^*(G_n) = \sum_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)].$$

(c) The 3^{rd} *modified Zagreb connection index* of graph G_n is:

$$\hat{Z}C_3^*(G_n) = \sum_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)].$$

(d) The 4^{th} *modified Zagreb connection index* of graph G_n is:

$$\hat{Z}C_4^*(G_n) = \sum_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].$$

here, $\bar{\omega}(t)$ represents the vertex's connection number t and $\bar{\omega}(s)$ represents the vertex's connection number s for the edge st. Similarly, $d_{G_n}(s)$ represents the degree number of s and $d_{G_n}(t)$ represents the degree number of t.

2.6 Multiplicative Zagreb Connection Index MZCI

These multiplicative Zagreb connection indices were proposed by Javaid et al. [14] in 2021:

(a) The 1^{st} *multiplicative Zagreb connection index* of graph G_n is:

$$M\hat{Z}C_1(G_n) = \prod_{t \in V(n)} [\bar{w}_n(t)]^2.$$

(b) The 2^{nd} *multiplicative Zagreb connection index* of graph G_n is:

$$M\hat{Z}C_2(G_n) = \prod_{st \in E(n)} [\bar{w}_n(s) \times \bar{w}_n(t)].$$

(c) The 3^{rd} *multiplicative Zagreb connection index* of graph G_n is:

$$M\hat{Z}C_3(G_n) = \prod_{t \in V(n)} [d_n(t) \times (\bar{w}_n(t))].$$

(d) The 4^{th} *Multiplicative Zagreb Connection Index* of graph G_n is:

$$M\hat{Z}C_4(G_n) = \prod_{st \in E^c(n)} [\bar{w}_n(s) + \bar{w}_n(t)].$$

2.7 Modified Multiplicative Zagreb Connection Index mMZCI

These modified multiplicative Zagreb connection indices were proposed by Javaid et al. [14] in 2021:

(a) The 1^{st} *modified multiplicative Zagreb connection index* of graph G_n is:

$$M\hat{Z}C_1^*(G_n) = \prod_{st \in E(n)} [d_n(s)\bar{w}_n(t) + d_n(t)\bar{w}_n(s)].$$

(b) The 2^{nd} *modified multiplicative Zagreb connection index* of graph G_n is:

$$M\hat{Z}C_2^*(G_n) = \prod_{st \in E(n)} [d_n(s)\bar{w}_n(s) + d_n(t)\bar{w}_n(t)].$$

(c) The 3^{rd} *modified multiplicative Zagreb connection index* of graph G_n is:

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Chapter 3

Evaluation of Connection Number-Based Indices of Networks Derived from Triglycerides

3.1 Introduction

In this chapter, we will study different kind of graphs such as triglycerides, its line and para-line graphs. For these graphs, we will find connection number based indices.

3.2 Triglycerides

In this section, we will compute connection number-based indices for the triglyceride.

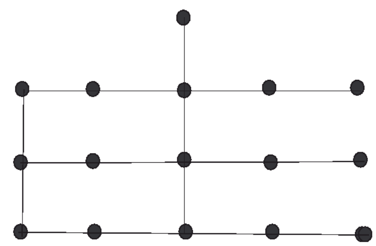

Figure 3.1: Triglycerides Graph of Unit 1

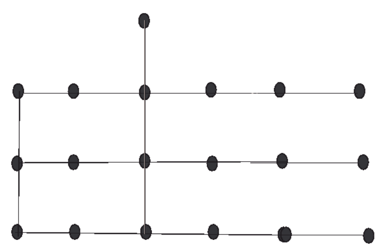

Figure 3.2: Triglycerides Graph of Unit 2

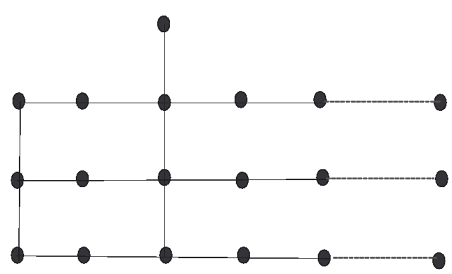

Figure 3.3: Triglycerides Graph of Unit n

3.3 Order and Size of Triglycerides

The results for triglycerides below are determined by applying edge partitioning based on the degree-connection number, vertex distribution based on the degree-connection of their vertices, and edge partitioning based on their connection number.

$E^{n}_{\bar{w}(n_1),\bar{w}(n_2)}$	No of Edges	$E^{n}_{\bar{w}(n_1),\bar{w}(n_2)}$	No of Edges
(2,1)	3	(3,4)	1
(2,2)	$3(n-2)$	(4,2)	1
(1,1)	3	(3,3)	4
(3,2)	8	0	0

Table 3.1: Edges are divided based on their connection number

$E^{\bar{w}(s),\bar{w}(t)}_{d(s),d(t)}$	No of Edges	$E^{\bar{w}(s),\bar{w}(t)}_{d(s),d(t)}$	No of Edges
$E^{3,3}_{2,2}$	2	$E^{4,2}_{2,3}$	1
$E^{3,2}_{2,3}$	5	$E^{3,4}_{3,2}$	1
$E^{3,3}_{2,3}$	2	$E^{2,2}_{3,1}$	3
$E^{1,1}_{2,1}$	3	$E^{2,1}_{2,2}$	3
$E^{2,2}_{2,2}$	$3(n-3)$	$E^{3,2}_{2,2}$	3

Table 3.2: Edge partitioning depending on their degree-connection number

$V^{\bar{w}(s)}_{d(s)}$	No of Vertices	$V^{\bar{w}(s)}_{d(s)}$	No of Vertices
V^{1}_{1}	3	V^{1}_{2}	3
V^{2}_{1}	3	V^{2}_{2}	$3(n-2)$
V^{2}_{3}	3	V^{3}_{2}	7
V^{4}_{2}	1	V^{3}_{3}	1

Table 3.3: Vertex division based on degree-connection of their vertices

Theorem 3.3.1. *For graph G_n. The first Zagreb connection index of triglyceride is*

$$\hat{Z}C_1(G_n) = 12n + 94$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 1^{st} ZCI we have,

$$\hat{Z}C_1(G_n) = \sum_{t \in V(G_n)} [\bar{w}_{G_n}(t)]^2.$$

$$\hat{Z}C_1(G_n) = \sum_{t \in V_1(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_2(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_3(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_4(G_n)} [\bar{w}_{G_n}(t)]^2$$

from Table 3.9, we have,

$$\hat{Z}C_1(G_n) = (6)(1)^2 + (3n)(2)^2 + (8)(3)^2 + (1)(4)^2$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_1(G_n) = 12n + 94.$ □

Theorem 3.3.2. *For graph G_n. The second Zagreb connection index of triglyceride is*

$$\hat{Z}C_2(G_n) = 12n + 65$$

.

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd} ZCI we have,

$$\hat{Z}C_2(G_n) = \sum_{st \in E(G_n)} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)].$$

$$
\begin{aligned}
\hat{Z}C_2(G_n) &= \sum_{st\in E^c_{2,1}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{2,2}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{1,1}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\\
&\quad+\sum_{st\in E^c_{3,2}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,3}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{4,2}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\\
&\quad+\sum_{st\in E^c_{3,4}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]
\end{aligned}
$$

from Table 3.7, we have,

$$
\begin{aligned}
\hat{Z}C_2(G_n) &= (3)(2\times 1)+(3n-6)(2\times 2)+3(1\times 1)\\
&\quad+(8)(3\times 2)+(4)(3\times 3)+1(4\times 2)+1(3\times 4)
\end{aligned}
$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_2(G_n)=12n+65.$ □

Theorem 3.3.3. *For graph G_n. The first modified Zagreb connection index of triglyceride is*

$$\hat{Z}C_1^*(G_n)=12n+68$$

Proof. Let G_n be a triglyceride graph with $n\geq 3$, where n is an integer. Then from the definition of 1^{st} mZCI we have,

$$\hat{Z}C_1^*(G_n)=\sum_{st\in E^c(G_n)}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)].$$

$$
\begin{aligned}
\hat{Z}C_1^*(G_n) &= \sum_{st\in E^c_{2,1}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{2,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{1,1}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\\
&\quad+\sum_{st\in E^c_{3,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,3}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{4,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\\
&\quad+\sum_{st\in E^c_{3,4}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]
\end{aligned}
$$

from Table 3.7, we have,

$$\begin{aligned}\hat{Z}C_1^*(G_n) &= (2+1)(3)+(2+2)(3(n-2))+(1+1)(3)+(3+2)(8)\\&\quad+(3+3)(4)+(4+2)(1)+(3+4)(1)\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_1^*(G_n) = 68+12n. \qquad \square$$

Theorem 3.3.4. *For graph G_n. The second modified Zagreb connection index of triglyceride is*

$$\hat{Z}C_2^*(G_n) = 24n+162$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd} mZCI we have,

$$\hat{Z}C_2^*(G_n) = \sum_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)].$$

$$\begin{aligned}\hat{Z}C_2^*(G_n) &= \sum_{st\in E_{2,2}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st\in E_{2,3}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&\quad+\sum_{st\in E_{2,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st\in E_{2,3}^{4,2}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&\quad+\sum_{st\in E_{3,2}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st\in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&\quad+\sum_{st\in E_{3,1}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st\in E_{2,1}^{1,1}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&\quad+\sum_{st\in E_{2,2}^{2,1}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st\in E_{2,2}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\end{aligned}$$

from Table 3.8, we have

$$\begin{aligned}\hat{Z}C_2^*(G_n) &= 2(12)+5(13)+2(15)+1(16)+1(18)\\ &\quad +3(10)+3(8)+3(3)+3(6)+(8)(3(n-9))\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_2^*(G_n) = 162+24n. \qquad \square$$

Theorem 3.3.5. *For graph G_n. The third modified Zagreb connection index of triglyceride is*

$$\hat{Z}C_3^*(G_n) = 24n+154$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 3^{rd} mZCI we have,

$$\hat{Z}C_3^*(G_n) = \sum_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}\hat{Z}C_3^*(G_n) &= \sum_{st\in E_{2,2}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,3}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\ &\quad + \sum_{st\in E_{2,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,3}^{4,2}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\ &\quad + \sum_{st\in E_{3,2}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\ &\quad + \sum_{st\in E_{3,1}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,1}^{1,1}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\ &\quad + \sum_{st\in E_{2,2}^{2,1}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,2}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\end{aligned}$$

from Table 3.8, we have,

$$\begin{aligned}\hat{Z}C_3^*(G_n) &= 2(12)+5(12)+2(15)+1(14)+1(17) \\ &\quad +3(10)+3(8)+(3)(3)+(3)(6)+8(3(n-9))\end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_3^*(G_n) = 154+24n.$ □

Theorem 3.3.6. *For graph G_n. The fourth modified Zagreb connection index of triglyceride is*

$$\hat{Z}C_4^*(G_n) = 48n+530$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 4^{th} mZCI we have,

$$\hat{Z}C_4^*(G_n) = \sum_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}\hat{Z}C_4^*(G_n) &= \sum_{st\in E_{2,2}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,3}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\ &\quad + \sum_{st\in E_{2,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,3}^{4,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\ &\quad + \sum_{st\in E_{3,2}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\ &\quad + \sum_{st\in E_{3,1}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,1}^{1,1}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\ &\quad + \sum_{st\in E_{2,2}^{2,1}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,2}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)]\end{aligned}$$

from Table 3.8, we have,

$$\begin{aligned}\hat{Z}C_4^*(G_n) &= 2(36)+3(36)+2(54)+1(32)+1(72)\\ &\quad +3(24)+3(12)+3(2)+3(6)+(3(n-3))(16)\end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_4^*(G_n) = 48n+530.$ □

Theorem 3.3.7. *For graph G_n. The first multiplicative Zagreb connection index of triglyceride is*

$$M\hat{Z}C_1(G_n) = (2)^{6n} \times (688747536)$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 1^{st} MZCI we have,

$$M\hat{Z}C_1(G_n) = \prod_{t \in V(G_n)} [\bar{w}_{G_n}(t)]^2.$$

$$M\hat{Z}C_1(G_n) = \prod_{t\in V_1(G_n)} [\bar{w}_{G_n}(t)]^2 \times \prod_{t\in V_2(G_n)} [\bar{w}_{G_n}(t)]^2 \times \prod_{t\in V_3(G_n)} [\bar{w}_{G_n}(t)]^2 \times \prod_{t\in V_4(G_n)} [\bar{w}_{G_n}(t)]^2$$

from Table 3.9, we have,

$$M\hat{Z}C_1(G_n) = 1 \times (2)^{6n} \times (3)^{16} \times (4)^2$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_1(G_n) = (2n)^{6n} \times (688747536).$ □

Theorem 3.3.8. *For graph G_n. The second multiplicative Zagreb connection index of triglyceride is*

$$M\hat{Z}C_2(G_n) = (4)^{3n} \times (2066242608)$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd} MZCI we have,

$$M\hat{Z}C_2(G_n) = \prod_{st \in E(G_n)} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)].$$

$$\begin{aligned} M\hat{Z}C_2(G_n) &= \prod_{st \in E^c_{2,1}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{2,2}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{1,1}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\ &\times \prod_{st \in E^c_{3,2}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{3,3}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{4,2}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\ &\times \prod_{st \in E^c_{3,4}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \end{aligned}$$

from Table 3.7, we have,

$$M\hat{Z}C_2(G_n) = (2^3) \times (4)^{3n-6} \times (1) \times (6^8) \times (9^4) \times (8) \times (12)$$

After simplifying the above equation we get,

$$\Rightarrow M\hat{Z}C_2(G_n) = (4)^{3n} \times (2066242608).$$ □

Theorem 3.3.9. *For graph G_n. The third multiplicative Zagreb connection index of triglyceride is*

$$M\hat{Z}C_3(G_n) = (4)^{3n} \times (68024448)$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 3^{rd} MZCI we have,

$$M\hat{Z}C_3(G_n) = \prod_{t \in V(G_n)} [d_{G_n}(t) \times (\bar{w}_{G_n}(t))].$$

$$
\begin{aligned}
M\hat{Z}C_3(G_n) &= \prod_{t\in V_1^1}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]\times\prod_{t\in V_2^1}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]\times\prod_{t\in V_1^2}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]\\
&\times\prod_{t\in V_2^2}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]\times\prod_{t\in V_3^2}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]\times\prod_{t\in V_2^3}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]\\
&\times\prod_{t\in V_2^4}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]\times\prod_{t\in V_3^3}[d_{G_n}(t)\times(\bar{w}_{G_n}(t))]
\end{aligned}
$$

from Table 3.9, we have,

$$M\hat{Z}C_3(G_n) = 1^3\times2^3\times2^3\times4^{3n-6}\times6^3\times6^7\times8\times9$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_3(G_n) = (4)^{3n}\times(68024448).$ □

Theorem 3.3.10. *For graph G_n. The fourth multiplicative Zagreb connection index of triglyceride is*

$$M\hat{Z}C_4(G_n) = 4^{3n}\times(1121264648)$$

Proof. Let G_n be a triglyceride graph with $n\geq3$, where n is an integer. Then from the definition of 4^{th} MZCI we have,

$$M\hat{Z}C_4(G_n) = \prod_{st\in E^c(G_n)}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)].$$

$$
\begin{aligned}
M\hat{Z}C_4(G_n) &= \prod_{st\in E^c_{2,1}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\prod_{st\in E^c_{2,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\prod_{st\in E^c_{1,1}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\\
&\prod_{st\in E^c_{3,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\prod_{st\in E^c_{3,3}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\prod_{st\in E^c_{4,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\\
&\prod_{st\in E^c_{3,4}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)].
\end{aligned}
$$

from Table 3.7, we have,

$$M\hat{Z}C_4(G_n) = 3^3\times4^{3n-6}\times2^3\times5^8\times6^4\times6^1\times7^1$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_4(G_n) = 4^{3n} \times 1121264648.$ □

Theorem 3.3.11. *For graph G_n. The first modified multiplicative Zagreb connection index of triglyceride is*

$$M\hat{Z}C_1^*(G_n) = 8^{3n} \times (7.7077825 \times 10^{13})$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 1^{st} mMZCI we have,

$$M\hat{Z}C_1^*(G_n) = \prod_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)].$$

$$\begin{aligned}
\hat{Z}C_1^*(G_n) \quad = \quad & \prod_{st \in E_{2,2}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{2,3}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{2,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{2,3}^{4,2}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{3,2}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{3,1}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{2,1}^{1,1}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod st \in E_{2,2}^{2,1} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{2,2}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)].
\end{aligned}$$

from Table 3.8, we have,

$$\begin{aligned}
\hat{Z}C_1^*(G_n) \quad = \quad & (12)^2 \times (13)^5 \times (15)^2 \times (16)^1 \times (18)^1 \times (8)^3 \\
& \times (3)^3 + (6)^3 + 8^{3(n-9)} \times (10)^3
\end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_1^*(G_n) = 8^{3n} \times (7.7077825 \times 10^{13}).$ □

Theorem 3.3.12. *For graph G_n.The second modified multiplicative Zagreb connection in-*

dex of triglyceride is

$$M\hat{Z}C_2^*(G_n) = 8^{3n} \times (4.268799 \times 10^{13})$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd} mMZCI we have,

$$M\hat{Z}C_2^*(G_n) = \prod_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
\hat{Z}C_2^*(G_n) \quad = \quad & \prod_{st \in E_{2,2}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{2,3}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{2,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{2,3}^{4,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{3,2}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{3,1}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{2,1}^{1,1}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{2,2}^{2,1}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{2,2}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) + d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}$$

from Table 3.8, we have,

$$\begin{aligned}
\hat{Z}C_2^*(G_n) \quad = \quad & (12)^2 \times (12)^5 \times (15)^2 \times (14)^1 \times (17)^1 \times (10)^3 \times (8)^3 \\
& \times (3)^3 \times (6)^3 \times 8^{(3(n-9))}
\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_2^*(G_n) = 8^{3n} \times (4.268799 \times 10^{13}). \qquad \square$$

Theorem 3.3.13. *For graph G_n. The third modified multiplicative Zagreb connection index of triglyceride is*

$$M\hat{Z}C_3^*(G_n) = 16^{3n} \times (7.4962673 \times 10^{11})$$

Proof. Let G_n be a triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 3^{rd} mMZCI we have,

$$M\hat{Z}C_3^*(G_n) = \prod_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)]$$

$$\begin{aligned}
\hat{Z}C_3^*(G_n) \quad = \quad & \prod_{st\in E_{2,2}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{2,3}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{2,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{2,3}^{4,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{3,2}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{3,1}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{2,1}^{1,1}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{2,2}^{2,1}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{2,2}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}$$

from Table 3.8, we have,

$$\begin{aligned}
\hat{Z}C_3^*(G_n) \quad = \quad & (36)^2 \times (36)^5 \times (54)^2 \times (32)^1 \times (72)^1 \times (12)^3 \times (2)^3 \\
& \times (8)^3 \times (24)^3 \times 16^{(3(n-9))}
\end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_3^*(G_n) = 16^{3n} \times (7.4962673 \times 10^{11}).$ $\square$

3.4 Line Graph of Triglycerides

In this section, we will compute connection number-based indices for line-graph of triglyceride.

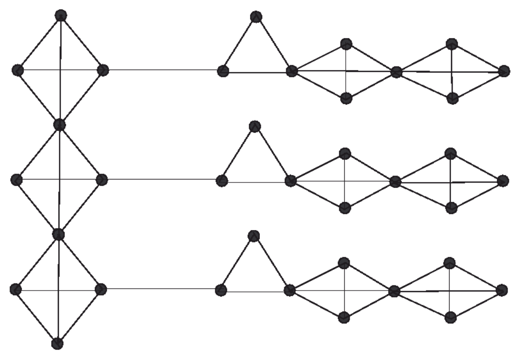

Figure 3.4: Line Graph of Triglycerides Unit 1

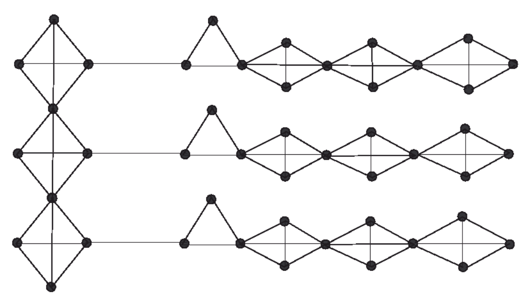

Figure 3.5: Line Graph of Triglycerides Unit 2

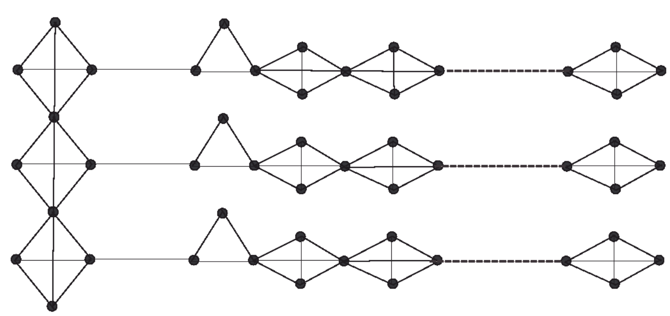

Figure 3.6: Line Graph of Triglycerides Unit n

3.5 Order and Size of Line Graph of Triglycerides

The results for line graph of triglycerides below are determined by applying edge partitioning based on the degree-connection number, vertex distribution based on the degree-connection of their vertices, and edge partitioning based on their connection number.

$E^{n}_{\bar{w}(n_1),\bar{w}(n_2)}$	No of Edges	$E^{n}_{\bar{w}(n_1),\bar{w}(n_2)}$	No of Edges
(3,3)	18	(4,4)	5
(4,5)	17	(4,6)	6
(5,5)	12	(5,6)	11
(5,7)	2	(6,3)	9
(6,6)	$18(n-2)$	(6,8)	1
(7,8)	1	(8,5)	2

Table 3.4: Edges are divided based on their connection number

$E^{\bar{w}(s),\bar{w}(t)}_{d(s),d(t)}$	No of Edges	$E^{\bar{w}(s),\bar{w}(t)}_{d(s),d(t)}$	No of Edges
$E^{4,5}_{3,4}$	4	$E^{4,4}_{3,3}$	2
$E^{5,5}_{6,4}$	2	$E^{4,5}_{3,6}$	4
$E^{5,5}_{6,6}$	1	$E^{5,5}_{3,6}$	6
$E^{4,5}_{5,3}$	3	$E^{5,6}_{4,3}$	2
$E^{7,8}_{3,4}$	1	$E^{8,6}_{4,3}$	1
$E^{7,5}_{3,6}$	2	$E^{5,8}_{6,4}$	2
$E^{4,4}_{2,5}$	3	$E^{6,4}_{3,2}$	3
$E^{3,6}_{6,3}$	6	$E^{6,4}_{3,5}$	3
$E^{5,5}_{3,3}$	3	$E^{5,6}_{6,3}$	6
$E^{5,6}_{6,6}$	3	$E^{3,3}_{6,3}$	9
$E^{3,3}_{3,3}$	9	$E^{6,6}_{6,6}$	$3(n-3)$
$E^{6,3}_{6,6}$	3	$E^{6,6}_{3,3}$	$3(n-1)$
$E^{6,6}_{6,3}$	$12(n-2)$	0	0

Table 3.5: Edge partitioning depending on their degree-connection number

$V_{d(s)}^{\bar{w}(s)}$	No of Vertices	$V_{d(s)}^{\bar{w}(s)}$	No of Vertices
V_3^4	4	V_4^5	2
V_4^8	1	V_3^7	1
V_6^5	5	V_2^4	3
V_5^4	3	V_3^5	6
V_3^6	$6n-3$	V_3^3	9
V_6^6	$3n-6$	V_6^3	3

Table 3.6: Vertex division based on degree-connection of their vertices

Theorem 3.5.1. *For graph G_n. The first Zagreb connection index of line graph of triglyceride is*

$$\hat{Z}C_1(G_n) = 324n + 382$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 1^{st}ZCI we have,

$$\hat{Z}C_1(G_n) = \sum_{t \in V(G_n)} [\bar{w}_n(t)]^2.$$

$$\begin{aligned} \hat{Z}C_1(G_n) &= \sum_{t \in V_3(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_4(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_5(G_n)} [\bar{w}_{G_n}(t)]^2 \\ &\quad + \sum_{t \in V_6(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_7(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_8(G_n)} [\bar{w}_{G_n}(t)]^2 \end{aligned}$$

from Table 3.9, we have,

$$\hat{Z}C_1(G_n) = (12)(3)^2 + (10)(4)^2 + (13)(5)^2 + (9n-9)(6)^2 + 1(7)^2 + 1(8)^2$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_1(G_n) = 324n + 382. \qquad \square$$

Theorem 3.5.2. *For graph G_n. The second Zagreb connection index of line graph of triglyceride is*

$$\hat{Z}C_2(G_n) = 648n + 476$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd}ZCI we have,

$$\hat{Z}C_2(G_n) = \sum_{st \in E(G_n)} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)].$$

$$\begin{aligned}
\hat{Z}C_2(G_n) \quad = \quad & \sum_{st \in E^c_{3,3}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{4,4}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\
& + \sum_{st \in E^c_{4,5}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{4,6}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\
& + \sum_{st \in E^c_{5,7}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{5,5}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\
& + \sum_{st \in E^c_{5,6}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{6,8}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\
& + \sum_{st \in E^c_{7,8}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{8,5}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\
& + \sum_{st \in E^c_{6,6}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{6,3}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)]
\end{aligned}$$

from Table 3.7, we have,

$$\begin{aligned}
\hat{Z}C_2(G_n) \quad = \quad & 18(3 \times 3) + 5(4 \times 4) + 17(4 \times 5) + 6(4 \times 6) \\
& + 2(5 \times 7) + 12(5 \times 5) + 11(5 \times 6) + 1(6 \times 8) + 1(7 \times 8) \\
& + 2(8 \times 5) + (18n - 36)(6 \times 6) + 9(6 \times 3)
\end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_2(G_n) = 648n + 476.$ □

Theorem 3.5.3. *For graph G_n. The first modified Zagreb connection index of line Graph of triglyceride is*

$$\hat{Z}C_1^*(G_n) = 330 + 216n$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 1^{st} mZCI we have,

$$\hat{Z}C_1^*(G_n) = \sum_{st \in E^c(G_n)} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)].$$

$$\begin{aligned} \hat{Z}C_1^*(G_n) &= \sum_{st \in E^c_{3,3}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{4,4}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\ &+ \sum_{st \in E^c_{4,5}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{4,6}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\ &+ \sum_{st \in E^c_{5,7}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{5,5}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\ &+ \sum_{st \in E^c_{5,6}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{6,8}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\ &+ \sum_{st \in E^c_{7,8}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{8,5}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\ &+ \sum_{uv \in E^c_{6,6}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] + \sum_{st \in E^c_{6,3}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \end{aligned}$$

from Table 3.7, we have,

$$\begin{aligned} \hat{Z}C_1^*(G_n) &= 18(3+3) + 5(4+4) + 17(4+5) + 6(4+6) + 2(5+7) + 12(5+5) \\ &+ 11(5+6) + 1(6+8) + 1(7+8) + 2(8+5) + (18n-36)(6+6) + 9(6+3) \end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_1^*(G_n) = 330 + 216n. \qquad \square$$

Theorem 3.5.4. *For graph G_n. The second modified Zagreb connection index of line graph of triglyceride is*

$$\hat{Z}C_2^*(G_n) = 972n + 1216$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd} mZCI we have,

$$\hat{Z}C_2^*(G_n) = \sum_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_(t) + d_{G_n}(t)\bar{w}_{G_n}(s)].$$

$$\begin{aligned}
\hat{Z}C_2^*(G_n) = & \sum_{st \in E_{4,4}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{3,4}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{4,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{6,4}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{6,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{6,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{3,4}^{5,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{6,4}^{5,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{3,5}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{4,7}^{3,8}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{3,8}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{6,7}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{4,5}^{6,8}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{5,4}^{2,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{2,6}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{3,3}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{5,6}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{3,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{3,5}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{6,5}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& + \sum_{st \in E_{3,3}^{6,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] + \sum_{st \in E_{3,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)]
\end{aligned}$$

$$+\sum_{st\in E_{6,6}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E_{6,6}^{6,3}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$
$$+\sum_{st\in E_{3,6}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E_{3,6}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$

from Table 3.8, we have,

$$\begin{aligned}\hat{Z}C_2^*(G_n) &= 4(15+16)+2(12+12)+2(30+20)+4(15+24)+1(30+30)+6(15+30)\\ &+6(25+12)+3(25+24)+2(24+15)+1(24+28)+1(24+24)+2(15+42)\\ &+2(48+20)+3(8+20)+3(12+12)+6(36+9)+3(12+30)+3(15+15)\\ &+6(36+15)+3(36+30)+9(18+9)+9(9+9)+(3n-9)(36+36)\\ &+3(18+36)+(3n-3)(18+18)+(12n-24)(36+18)\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_2^*(G_n)=972n+1216. \qquad \square$$

Theorem 3.5.5. *For graph G_n. The third modified Zagreb connection index of line graph of triglyceride is*

$$\hat{Z}C_3^*(G_n)=972n+1122$$

Proof. Let G_n be a line graph of triglyceride with $n\geq 3$, where n is an integer. Then from the definition of 3^{rd}mZCI we have,

$$\hat{Z}C_3^*(G_n)=\sum_{st\in E(G_n)}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}\hat{Z}C_3^*(G_n) &= \sum_{st\in E_{4,5}^{3,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,4}^{3,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\ &+\sum_{st\in E_{4,5}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{6,5}^{3,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\ &+\sum_{st\in E_{6,5}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{6,5}^{3,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\end{aligned}$$

$$
\begin{aligned}
&+\sum_{st\in E_{3,5}^{5,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{6,5}^{5,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{3,6}^{4,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{4,8}^{3,7}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{3,6}^{4,8}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{6,5}^{3,7}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{4,8}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{5,4}^{2,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{2,4}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,6}^{6,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{5,4}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,5}^{3,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{3,6}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{6,6}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{3,3}^{6,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,3}^{3,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{6,6}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{6,3}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{3,6}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,6}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}
$$

from Table 3.8, we have,

$$
\begin{aligned}
\hat{Z}C_3^*(G_n) \quad = \quad & 4(12+20)+2(12+12)+2(30+20)+4(12+30)\\
&+1(30+30)+6(15+30)+6(20+15)+3(20+30)+2(20+18)+1(21+32)\\
&+1(32+18)+2(21+30)+2(30+32)+3(8+20)+3(18+8)+6(18+18)\\
&+3(18+20)+3(15+15)+6(30+18)+3(30+36)+9(18+9)+9(9+9)\\
&+(3n-9)(36+36)+3(36+18)+(3n-3)(18+18)+(12n-24)(36+18)
\end{aligned}
$$

After simplifying the above equation we get with,

$$\Rightarrow \hat{Z}C_3^*(G_n)=972n+1122.$$

$\square$

Theorem 3.5.6. *For graph G_n. Thefourth modified Zagreb connection index of line graph of triglyceride is*

$$\hat{Z}C_4^*(G_n) = 12636n + 3320$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 4^{th}mZCI we have,

$$\hat{Z}C_4^*(G_n) = \sum_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
\hat{Z}C_4^*(G_n) \quad = \quad & \sum_{st \in E_{4,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{3,4}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{4,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{6,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{6,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{6,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{3,5}^{5,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{6,5}^{5,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{3,6}^{4,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{4,8}^{3,7}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{3,6}^{4,8}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{6,5}^{3,7}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{4,8}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{5,4}^{2,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{2,4}^{3,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{3,6}^{6,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{5,4}^{3,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{3,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{3,6}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{6,6}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{3,3}^{6,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{3,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st \in E_{6,6}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st \in E_{6,3}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}$$

$$+\sum_{st\in E_{3,6}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)\times d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,6}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)\times d_{G_n}(t)\bar{w}_{G_n}(t)]$$

from Table 3.8, we have,

$$\begin{aligned}
\hat{Z}C_4^*(G_n) &= 4(12\times 20)+2(12\times 12)+2(30\times 20)+4(12\times 30)+1(30\times 30)\\
&\quad +6(15\times 30)+6(20\times 15)+3(20\times 30)+2(20\times 18)\\
&\quad +1(21\times 32)+1(32\times 18)+2(21\times 30)+2(30\times 32)\\
&\quad +3(8\times 20)+3(18\times 8)+6(18\times 18)+3(18\times 20)\\
&\quad +3(15\times 15)+6(30\times 18)+3(30\times 36)+9(18\times 9)\\
&\quad +9(9\times 9)+(3n-9)(36\times 36)+3(36\times 18)+(3n-3)(18\times 18)\\
&\quad +(12n-24)(36\times 18)
\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_4^*(G_n) = 12636n+3320. \qquad \square$$

Theorem 3.5.7. *For graph G_n. The first multiplicative Zagreb connection index of line graph of triglyceride is*

$$M\hat{Z}C_1(G_n) = (6)^{18n}\times(1.42884\times 10^{31})$$

Proof. Let G_n be a line graph of triglyceride with $n\geq 3$, where n is an integer. Then from the definition of 1^{st} MZCI we have,

$$M\hat{Z}C_1(G_n) = \prod_{t\in V(G_n)}[\bar{w}_{G_n}(t)]^2.$$

$$\begin{aligned}
M\hat{Z}C_1(G_n) &= \prod_{t\in V_3(G_n)}[\bar{w}_{G_n}(t)]^2\times\prod_{t\in V_4(G_n)}[\bar{w}_{G_n}(t)]^2\times\prod_{t\in v_5(G_n)}[\bar{w}_{G_n}(t)]^2\\
&\quad\times\prod_{t\in V_6(G_n)}[\bar{w}_{G_n}(t)]^2\times\prod_{t\in v_7(G_n)}[\bar{w}_{G_n}(t)]^2\times\prod_{t\in V_8(G_n)}[\bar{w}_{G_n}(t)]^2
\end{aligned}$$

from Table 3.9, we have,

$$M\hat{Z}C_1(G_n) \quad = \quad [(3)^2]^{12} \times [(4)^2]^{10} \times [(5)^2]^{13} \times [(6)^2]^{9n-9} \times [(7)^2]^1 \times [(8)^2]^1$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_1(G_n) = (6)^{18n} \times (1.42884 \times 10^{31}).$ □

Theorem 3.5.8. *For graph G_n. The second multiplicative Zagreb connection index of line graph of triglyceride is*

$$M\hat{Z}C_2(G_n) = (36)^{18n} \times (1.635551453 \times 10^{50})$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd} MZCI we have,

$$M\hat{Z}C_2(G_n) = \prod_{st \in E(G_n)} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)].$$

$$\begin{aligned} M\hat{Z}C_2(G_n) \quad = \quad & \prod_{st \in E^c_{3,3}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{4,4}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{4,5}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\ & \times \prod_{st \in E^c_{4,6}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{5,7}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{5,5}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\ & \times \prod_{st \in E^c_{5,6}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{6,8}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{7,8}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \\ & \times \prod_{st \in E^c_{8,5}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{6,6}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{6,3}} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)] \end{aligned}$$

from Table 3.7, we have,

$$M\hat{Z}C_2(G_n) \quad = \quad (3 \times 3)^{18} \times (4 \times 4)^5 \times (4 \times 5)^{17} \times (4 \times 6)^6 \times (5 \times 7)^2 \times (5 \times 5)^{12}$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_2(G_n) = (36)^{18n} \times (1.635551453 \times 10^{50})$

. ∎

Theorem 3.5.9. *For graph G_n. The third multiplicative Zagreb connection index of line graph of triglyceride is*

$$M\hat{Z}C_3(G_n) = (2^{12n} \times 3^{18n}) \times (1.124697 \times 10^{30})$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 3^{rd} MZCI we have,

$$M\hat{Z}C_3(G_n) = \prod_{t \in V(G_n)} [d_{G_n}(t) \times \bar{w}_{G_n}(t)].$$

$$\begin{aligned} M\hat{Z}C_3(G_n) &= \prod_{t \in V_3^4} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{t \in V_4^5} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{t \in V_4^8} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \\ &\times \prod_{t \in V_3^7} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{t \in V_6^5} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{t \in V_2^4} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \\ &\times \prod_{t \in V_5^4} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{V_3^5} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{t \in V_3^6} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \\ &\times \prod_{t \in V_3^3} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{t \in V_6^6} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \times \prod_{t \in V_6^3} [d_{G_n}(t) \times \bar{w}_{G_n}(t)] \end{aligned}$$

from Table 3.9, we have,

$$\begin{aligned} M\hat{Z}C_3(G_n) &= (4 \times 3)^4 \times (5 \times 4)^2 \times (8 \times 4)^1 \times (7 \times 3)^1 \times (5 \times 6)^5 \times (4 \times 2)^3 \\ &\times (4 \times 5)^3 \times (5 \times 3)^6 \times (6 \times 3)^{6n-3} \times (3 \times 3)^9 \times (6 \times 6)^{3n-6} \times (3 \times 6)^3 \end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_3(G_n) = (2^{12n} \times 3^{18n}) \times (1.124697 \times 10^{30}).$

∎

Theorem 3.5.10. *For graph G_n. The fourth multiplicative Zagreb connection index of line graph of triglyceride is*

$$M\hat{Z}C_4(G_n) = 12^{18n} \times (4.4232436 \times 10^{4n})$$

Proof. Let G_n be a line graph of triglyceride graph with $n \geq 3$, where n is an integer. Then from the definition of 4^{th} MZCI we have,

$$M\hat{Z}C_4(G_n) = \prod_{st \in E^c(G_n)} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)].$$

$$\begin{aligned}
M\hat{Z}C_4(G_n) &= \prod_{st \in E^c_{3,3}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{4,4}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{4,5}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\
&\times \prod_{st \in E^c_{4,6}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{5,7}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{5,5}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\
&\times \prod_{st \in E^c_{5,6}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{6,8}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{7,8}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \\
&\times \prod_{st \in E^c_{8,5}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{6,6}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)] \times \prod_{st \in E^c_{6,3}} [\bar{w}_{G_n}(s) + \bar{w}_{G_n}(t)].
\end{aligned}$$

from Table 3.7, we have,

$$\begin{aligned}
M\hat{Z}C_4(G_n) &= (3+3)^{18} \times (4+4)^5 \times (4+5)^{17} \times (4+6)^{10} \\
&\times (5+7)^2 \times (5+5)^{12} \times (5+6)^{11} \times (6+8)^1 \\
&\times (7+8)^1 \times (8+5)^2 \times (6+6)^{18(n-2)} \times (6+3)^9
\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow M\hat{Z}C_3(G_n) = 12^{18n} \times (4.4232436 \times 10^{4n}).$$

□

Theorem 3.5.11. *For graph G_n. The first modified multiplicative Zagreb connection index* *of line graph of triglyceride is*

$$M\hat{Z}C_1^*(G_n) = 2^{27n} \times 3^{48n} \times (1.216226 \times 10^{36})$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 1^{st} mMZCI we have,

$$M\hat{Z}C_1^*(G_n) = \prod_{st \in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)].$$

$$\begin{aligned}
M\hat{Z}C_1^*(G_n) \quad = \quad & \prod_{st \in E_{4,4}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{3,4}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{4,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{6,4}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{6,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{6,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{3,4}^{5,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{6,4}^{5,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{3,5}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{4,7}^{3,8}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{3,8}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{6,7}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{4,5}^{6,8}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{5,4}^{2,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{2,6}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{3,3}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{5,6}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{3,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{3,5}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{6,5}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{3,3}^{6,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{3,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st \in E_{6,6}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st \in E_{6,6}^{6,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)]
\end{aligned}$$

$$\times \prod_{st\in E_{3,6}^{3,6}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E_{3,6}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)].$$

from Table 3.8, we have,

$$\begin{aligned}
M\hat{Z}C_1^*(G_n) &= (15+16)^{4)} \times (12+12)^2 \times (30+20)^2 \times (15+24)^4 \times (30+30)^1 \\
&\times (15+30)^6 \times (25+12)^6 \times (25+24)^3 \times (24+15)^2 \times (24+28)^1 \\
&\times (24+24)^1 \times (15+42)^2 \times (48+20)^2 \times (8+20)^3 \times (12+12)^3 \\
&\times (36+9)^6 \times (12+30)^3 \times (15+15)^3 \times (36+15)^6 \times (36+30)^3 \\
&\times (18+9)^9 \times (9+9)^9 \times (36+36)^{(3n-9)} \times (18+36)^3 \\
&\times (18+18)^{(3n-3)} \times (36+18)^{(12n-24)}
\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow M\hat{Z}C_1^*(G_n) = 2^{27n} \times 3^{48n} \times (1.216226 \times 10^{36}). \qquad \square$$

Theorem 3.5.12. *For graph G_n. The second modified multiplicative Zagreb connection index of line graph of triglyceride is*

$$M\hat{Z}C_2^*(G_n) = 2^{27n} \times 3^{48n} \times (4.303226 \times 10^{34}) \times (1.595598 \times 10^{74}) \times (3.7234444 \times 10^{16})$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 2^{nd}mMZCI we have,

$$M\hat{Z}C_2^*(G_n) = \prod_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
M\hat{Z}C_2^*(G_n) &= \prod_{st\in E_{4,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,4}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \\
&\times \prod_{st\in E_{4,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{6,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \\
&\times \prod_{st\in E_{6,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{6,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}$$

$$\times \prod_{st\in E_{3,5}^{5,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{6,5}^{5,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{3,6}^{4,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{4,8}^{3,7}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{3,6}^{4,8}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{6,5}^{3,7}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{4,8}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{5,4}^{2,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{2,4}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,6}^{6,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{5,4}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,5}^{3,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{3,6}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{6,6}^{6,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{3,3}^{6,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,3}^{3,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{6,6}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{6,3}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{3,6}^{3,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,6}^{6,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]$$

from Table 3.8, we have,

$$\begin{aligned}
M\hat{Z}C_2^*(G_n) &= (12+20)^4 \times (12+12)^2 \times (30+20)^2 \times (12+30)^4 \times (30+30)^1 \\
&\times (15+30)^6 \times (20+15)^6 \times (20+30)^3 \times (20+18)^2 \times (21+32)^1 \\
&\times (32+18)^1 \times (21+30)^2 \times (30+32)^2 \times (8+20)^3 \times (18+8)^3 \\
&\times (18+18)^6 \times (18+20)^3 \times (15+15)^3 \times (30+18)^6 \times (30+36)^3 \\
&\times (18+9)^9 \times (9+9)^9 \times (36+36)^{(3n-9)} \\
&\times (36+18)^3 \times (18+18)^{3n-3} \times (36+18)^{12n-24}
\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow M\hat{Z}C_2^*(G_n) = 2^{27n} \times 3^{48n} \times (4.303226 \times 10^{34}) \times (1.595598 \times 10^{74}) \times (3.7234444 \times 10^{16}).$$

□

Theorem 3.5.13. *For graph G_n. The third modified multiplicative Zagreb connection index of Line Graph of Triglyceride is*

$$M\hat{Z}C_3^*(G_n) = 2^{54n} \times 3^{72n} \times (8.98259149 \times 10^{93}) \times (2.3485854 \times 10^{68}) \times (3.042848323 \times 10^{17})$$

Proof. Let G_n be a line graph of triglyceride with $n \geq 3$, where n is an integer. Then from the definition of 3^{rd} mMZCI we have,

$$M\hat{Z}C_3^*(G_n) = \prod_{st \in (G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
M\hat{Z}C_2^*(G_n) \quad = \quad & \prod_{st \in E_{4,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{3,4}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{4,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{6,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{6,5}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{6,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{3,5}^{5,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{6,5}^{5,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{3,6}^{4,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{4,8}^{3,7}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{3,6}^{4,8}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{6,5}^{3,7}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{4,8}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{5,4}^{2,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{2,4}^{3,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{3,6}^{6,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{5,4}^{3,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{3,5}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{3,6}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{6,6}^{6,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st \in E_{3,3}^{6,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st \in E_{3,3}^{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}$$

$$\times \prod_{st\in E_{6,6}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{6,3}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)]$$
$$\times \prod_{st\in E_{3,6}^{3,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,6}^{6,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].$$

from Table 3.8, we have,

$$\begin{aligned} M\hat{Z}C_2^*(G_n) &= (12\times 20)^4 \times (12\times 12)^2 \times (30\times 20)^2 \times (12\times 30)^4 \times (30\times 30)^1 \times (15\times 30)^6 \\ &\times (20\times 15)^6 \times (20\times 30)^3 \times (20\times 18)^2 \times (21\times 32)^1 \times (32\times 18)^1 \times (21\times 30)^2 \\ &\times (30\times 32) \times^2 \times (8\times 20)^3 \times (18\times 8)^3 \times (18\times 18)^6 \times (18\times 20)^3 \times (15\times 15)^3 \\ &\times (30\times 18)^6 \times (30\times 36)^3 \times (18\times 9)^9 \times (9\times 9)^9 \times (36\times 36)^{(3n-9)} \times (36\times 18)^3 \\ &\times (18\times 18)^{(3n-3)} \times (36\times 18)^{(12n-24)} \end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_2^*(G_n) = 2^{54n} \times 3^{72n} \times (8.98259149\times 10^{93}) \times (2.3485854\times 10^{68}) \times (3.042848323\times 10^{17}).$ □

3.6 Para-Line Graph of Triglycerides

In this section, we will compute number-based connection indices for para-line graph of triglyceride.

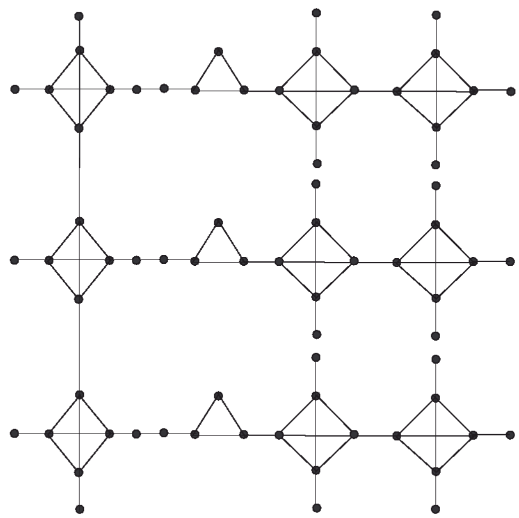

Figure 3.7: Para-Line Graph of Triglycerides Unit 1

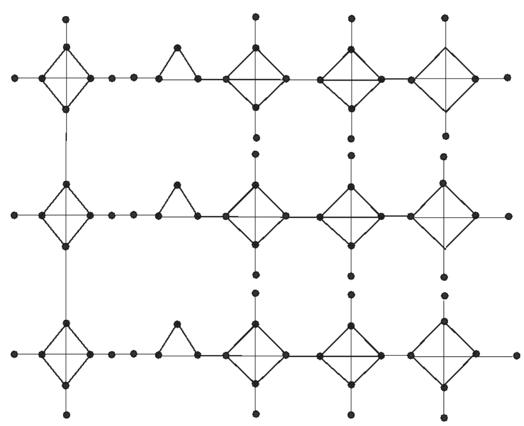

Figure 3.8: Para-Line Graph of Triglycerides Unit 2

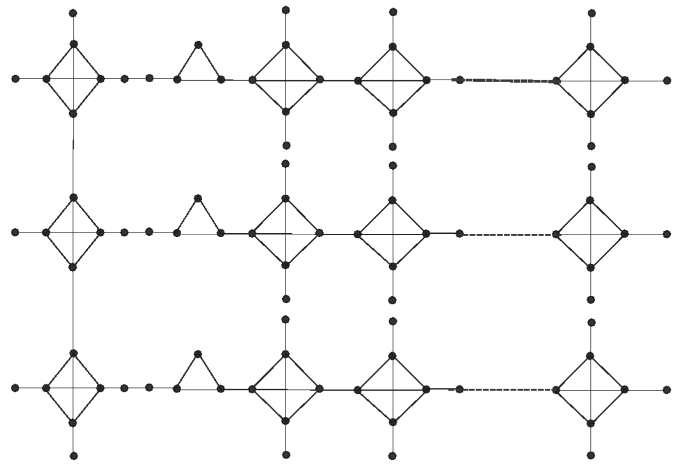

Figure 3.9: Para-Line Graph of Triglycerides Unit n

3.7 Order and Size of Para-Line Graph of Triglycerides

The results for para-line graph of triglycerides below are determined by applying edge partitioning based on the degree-connection number, vertex distribution based on the degree-connection of their vertices, and edge partitioning based on their connection number.

$E^c_{\bar{w}(n_1),\bar{w}(n_2)}$	No of Edges	$E^c_{\bar{w}(n_1),\bar{w}(n_2)}$	No of Edges
(2,2)	3	(2,4)	6
(3,2)	6	(3,3)	$9n+25$
(3,4)	8	(3,6)	$12n+9$
(3,5)	6	(4,5)	3
(4,4)	3	(5,6)	3
(6,4)	4	(6,6)	$6n$

Table 3.7: Edges are divided based on their connection number

$E_{d(s),d(t)}^{\bar{w}(s),\bar{w}(t)}$	No of Edges	$E_{d(s),d(t)}^{\bar{w}(s),\bar{w}(t)}$	No of Edges
$E_{1,4}^{3,3}$	$6n+14$	$E_{3,2}^{2,2}$	3
$E_{2,3}^{2,4}$	3	$E_{2,3}^{3,2}$	3
$E_{4,4}^{3,3}$	$3n+11$	$E_{4,4}^{3,4}$	5
$E_{4,4}^{3,6}$	$12n+9$	$E_{4,2}^{4,4}$	3
$E_{2,4}^{3,3}$	3	$E_{4,4}^{3,5}$	6
$E_{4,4}^{5,6}$	3	$E_{4,4}^{6,6}$	$6n$
$E_{4,4}^{6,4}$	4	$E_{2,2}^{4,3}$	3
$E_{3,4}^{4,5}$	3	0	0

Table 3.8: Edge partitioning depending on their degree-connection number

$V_{d(s)}^{\bar{w}(s)}$	No of Vertices	$V_{d(s)}^{\bar{w}(s)}$	No of Vertices
V_1^3	$6n+14$	V_2^2	3
V_3^2	3	V_3^4	3
V_2^3	3	V_4^4	3
V_4^6	$6n+4$	V_4^5	3
V_4^3	$6n+14$	V_2^4	3

Table 3.9: Vertex division based on degree-connection of their vertices

Theorem 3.7.1. *For graph G_n. The first Zagreb connection index of para-line graph of triglyceride is*

$$\hat{Z}C_1(G_n) = 324n + 666$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 1^{st} ZCI we have,

$$\hat{Z}C_1(G_n) = \sum_{t \in V(G_n)} [\bar{w}_{G_n}(t)]^2.$$

$$\begin{aligned} \hat{Z}C_1(G_n) \quad &= \quad \sum_{t \in V_3(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_4(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_5(G_n)} [\bar{w}_{G_n}(t)]^2 + \\ & \quad \sum_{t \in V_2(G_n)} [\bar{w}_{G_n}(t)]^2 + \sum_{t \in V_6(G_n)} [\bar{w}_{G_n}(t)]^2 \end{aligned}$$

from Table 3.9, we have,

$$\hat{Z}C_1(G_n) \quad = \quad (12n+31)(3)^2 + (9)(4)^2 + (3)(5)^2 + (6)(2)^2 + (6n+14)(6)^2$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_1(G_n) = 324n + 666.$ □

Theorem 3.7.2. *For graph G_n. The second Zagreb connection index of para-line graph of triglyceride is*

$$\hat{Z}C_2(G_n) = 513n + 1719$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 2^{nd} ZCI we have,

$$\hat{Z}C_2(G_n) = \sum_{st \in E(G_n)} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)].$$

$$\begin{aligned}\hat{Z}C_2(G_n) &= \sum_{st\in E^c_{2,2}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{2,4}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,2}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\\ &+\sum_{st\in E^c_{3,3}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,4}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,6}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\\ &+\sum_{st\in E^c_{3,5}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{4,5}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{4,4}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\\ &+\sum_{st\in E^c_{5,6}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{6,4}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{6,6}}[\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)].\end{aligned}$$

from Table 3.7, we have,

$$\begin{aligned}\hat{Z}C_2(G_n) &= 3(2+2)+6(2+4)+3(3+2)+(9n+25)(3+3)\\ &+8(3+4)+(12n+9)(3+6)+6(3+5)+3(4+5)+3(4+4)+3(5+6)\\ &+4(6+4)+(6n)(6+6)\end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_2(G_n) = 513n+1719.$ □

Theorem 3.7.3. *For the graph G_n. The first modified Zagreb connection index of para-line graph of triglyceride is*

$$\hat{Z}C_1^*(G_n) = 234n+522$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where$n$is an integer.Then from the definition of 1^{st} mZCI we have,

$$\hat{Z}C_1^*(G_n) = \sum_{st\in E^c(G_n)}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)].$$

$$\begin{aligned}\hat{Z}C_1^*(G_n) &= \sum_{st\in E^c_{2,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{2,4}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\\ &+\sum_{st\in E^c_{3,3}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,4}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{3,6}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\end{aligned}$$

$$+\sum_{st\in E^c_{3,5}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{4,5}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{4,4}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]$$
$$+\sum_{st\in E^c_{5,6}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{6,4}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]+\sum_{st\in E^c_{6,6}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)].$$

from Table 3.7, we have,

$$\begin{aligned}\hat{Z}C^*_1(G_n) &= 3(2+2)+6(2+4)+3(3+2)+(9n+25)(3+3)\\ &+8(3+4)+(12n+9)(3+6)+6(3+5)+3(4+5)+3(4+4)\\ &+3(5+6)+4(6+4)+(6n)(6+6)\end{aligned}$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C^*_1(G_n)=234n+522.$ □

Theorem 3.7.4. *For the graph G_n. The second modified Zagreb connection index of para-line graph of triglyceride is*

$$\hat{Z}C^*_2(G_n)=882n+1794$$

Proof. Let G_n be a para-Line graph of triglyceride with $n\geq 1$, where n is an integer. Then from the definition of2^{nd} mZCI we have,

$$\hat{Z}C^*_2(G_n)=\sum_{st\in E(G_n)}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)].$$

$$\begin{aligned}\hat{Z}C^*_2(G_n) &= \sum_{st\in E^{1,3}_{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E^{3,2}_{2,2}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\ &+\sum_{st\in E^{2,4}_{2,3}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E^{2,2}_{3,3}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\ &+\sum_{st\in E^{4,3}_{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E^{4,3}_{4,4}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\end{aligned}$$

$$
\begin{aligned}
&+\sum_{st\in E_{4,3}^{4,6}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E_{2,4}^{4,4}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&+\sum_{st\in E_{4,3}^{2,3}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E_{4,3}^{4,5}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&+\sum_{st\in E_{4,5}^{4,6}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E_{4,6}^{4,6}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&+\sum_{st\in E_{4,6}^{4,4}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]+\sum_{st\in E_{2,4}^{2,3}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]\\
&+\sum_{st\in E_{4,4}^{3,5}}[d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)].
\end{aligned}
$$

from Table 3.8, we have,

$$
\begin{aligned}
\hat{Z}C_2^*(G_n) &= (6n+14)(3+12)+3(6+4)+3(8+6)+3(4+9)+(3n+11)(12+12)\\
&\quad +5(16+12)+(12n+9)(24+12)+3(16+8)+3(6+12)+6(20+12)\\
&\quad +3(24+20)+(6n)(24+24)+4(16+24)+3(6+8)+3(15+16)
\end{aligned}
$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_2^*(G_n) = 882n+1794.$ □

Theorem 3.7.5. *For the graph G_n. The third modified Zagreb connection index of para-line graph of triglyceride is*

$$\hat{Z}C_3^*(G_n) = 882n+1668$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 3^{rd} mZCI we have,

$$\hat{Z}C_3^*(G_n) = \sum_{st\in E(G_n)}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$
\begin{aligned}
\hat{Z}C_3^*(G_n) \;=\; & \sum_{st\in E_{4,3}^{1,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{2,2}^{3,2}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{3,4}^{2,2}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,2}^{2,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{4,3}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{4,4}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{4,6}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{2,4}^{4,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{4,3}^{2,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{4,5}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{4,6}^{4,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{4,6}^{4,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{4,4}^{4,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]+\sum_{st\in E_{3,2}^{2,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&+\sum_{st\in E_{4,5}^{3,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)].
\end{aligned}
$$

from Table 3.8, we have,

$$
\begin{aligned}
\hat{Z}C_3^*(G_n) \;=\; & (6n+14)(3+12)+3(6+4)+3(4+12)+3(6+6)+(3n+11)(12+12)\\
&+5(12+16)+(12n+9)(12+24)+3(16+8)+3(6+12)+6(12+20)\\
&+3(20+24)+(6n)(24+24)+4(24+16)+3(8+6)+3(12+20)
\end{aligned}
$$

After simplifying the above equation we get,

$\Rightarrow \hat{Z}C_3^*(G_n) = 882n+1668.$ □

Theorem 3.7.6. *For the graph G_n. The fourth modified Zagreb connection index of para-line graph of triglyceride is*

$$\hat{Z}C_4^*(G_n) = 7560n11844$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 4^{th} mMZCI we have,

$$\hat{Z}C_4^*(G_n) = \sum_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
\hat{Z}C_4^*(G_n) \quad = \quad & \sum_{st\in E_{4,3}^{1,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st\in E_{3,4}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{3,2}^{2,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st\in E_{4,3}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{4,4}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st\in E_{4,6}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{2,4}^{4,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st\in E_{4,3}^{2,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{4,5}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st\in E_{4,6}^{4,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{4,6}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st\in E_{4,4}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] + \sum_{st\in E_{3,2}^{2,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& + \sum_{st\in E_{4,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].
\end{aligned}$$

from Table 3.8, we have,

$$\begin{aligned}
\hat{Z}C_4^*(G_n) \quad = \quad & (6n+14)(3\times 12) + 3(6\times 4) + 3(4\times 12) + 3(6\times 6) + (3n+11)(12\times 12) \\
& +5(12\times 16) + (12n+9)(12\times 24) + 3(16\times 8) + 3(6\times 12) + 6(12\times 20) \\
& +3(20\times 24) + (6n)(24\times 24) + 4(24\times 16) + 3(8\times 6) + 3(12\times 20)
\end{aligned}$$

After simplifying the above equation we get,

$$\Rightarrow \hat{Z}C_4^*(G_n) = 7560n + 11844. \qquad \square$$

Theorem 3.7.7. *For the graph G_n. The first multiplicative Zagreb connection index of para-line graph of triglyceride is*

$$M\hat{Z}C_1(G_n) = (3^{36n} \times 2^{12n} \times (1.030416065 \times 10^{70})$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 1^{nd} MZCI we have,

$$M\hat{Z}C_1(G_n) = \prod_{t \in V(G_n)} [\bar{w}_{G_n}(t)]^2.$$

$$\begin{aligned} M\hat{Z}C_1(G_n) &= \prod_{t \in V_3(G_n)} [\bar{w}_{G_n}(t)]^2 \times \prod_{t \in V_4(G_n)} [\bar{w}_{G_n}(t)]^2 \times \prod_{t \in V_5(G_n)} [\bar{w}_{G_n}(t)]^2 \\ &\quad \times \prod_{t \in V_2(G_n)} [\bar{w}_{G_n}(t)]^2 \times \prod_{t \in V_6(G_n)} [\bar{w}_{G_n}(t)]^2 \end{aligned}$$

from Table 3.9, we have,

$$M\hat{Z}C_1(G_n) = [(3)^2]^{12n+31} \times [(4)^2]^9 \times [(5)^2]^3 \times [(2)^2]^6 \times [(6)^2]^{6n+14}.$$

After simplifying the above equation we get,

$$\Rightarrow M\hat{Z}C_1(G_n) = (3^{36n} \times 2^{12n} \times (1.030416065 \times 10^{70}). \qquad \square$$

Theorem 3.7.8. *For the graph G_n. The second multiplicative Zagreb connection index of para-line graph of triglyceride is*

$$M\hat{Z}C_2(G_n) = (3)^{54n} \times (2)^{24n} \times (1.635551453 \times 10^{50}).$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 2^{nd} MZCI we have,

$$M\hat{Z}C_2(G_n) = \prod_{st \in E(G_n)} [\bar{w}_{G_n}(s) \times \bar{w}_{G_n}(t)].$$

$$
\begin{aligned}
M\hat{Z}C_2(G_n) \quad = \quad & \prod_{st\in E^c_{2,2}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{2,4}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)] \\
& \times\prod_{st\in E^c_{3,2}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{3,3}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)] \\
& \times\prod_{st\in E^c_{3,4}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{3,6}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)] \\
& \times\prod_{st\in E^c_{3,5}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{4,5}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)] \\
& \times\prod_{st\in E^c_{4,4}} [\bar{w}_{n}(s)\times\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{5,6}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)] \\
& \times\prod_{st\in E^c_{6,4}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{6,6}} [\bar{w}_{G_n}(s)\times\bar{w}_{G_n}(t)].
\end{aligned}
$$

from Table 3.7, we have

$$
\begin{aligned}
M\hat{Z}C_2(G_n) \quad = \quad & (2\times 2)^3\times(2\times 4)^6\times(3\times 2)^3\times(3\times 3)^{(9n+25)} \\
& \times(3\times 4)^8\times(3\times 6)^{(12n+9)}\times(3\times 5)^6\times(4\times 5)^3 \\
& \times(4\times 4)^3\times(5\times 6)^3\times(6\times 4)^4\times(6\times 6)^{6n}
\end{aligned}
$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_2(G_n) = (3)^{54n}\times(2)^{24n}\times(1.635551453\times 10^{50}).$ □

Theorem 3.7.9. *For the graph G_n. The third multiplicative Zagreb connection index of para-line graph of triglyceride is*

$$M\hat{Z}C_3(G_n) = (3^{18n}\times 2^{30n})\times(1.763725882\times 10^{47})$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 3^{rd} MZCI we have,

$$M\hat{Z}C_3(G_n) = \prod_{t\in V(G_n)} [d_{G_n}(t)\times\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
M\hat{Z}C_3(G_n) &= \prod_{t\in V_1^3}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\times\prod_{t\in V_2^2}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\times\prod_{t\in V_3^2}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\\
&\times\prod_{t\in V_3^4}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\times\prod_{t\in V_2^3}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\times\prod_{t\in V_4^4}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\\
&\times\prod_{t\in V_4^6}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\times\prod_{V_4^5}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\times\prod_{t\in V_4^3}[d_{G_n}(t)\times\bar{w}_{G_n}(t)]\\
&\times\prod_{t\in V_2^4}[d_{G_n}(t)\times\bar{w}_{G_n}(t)].
\end{aligned}$$

from Table 3.9, we have,

$$\begin{aligned}
M\hat{Z}C_3(G_n) &= (3\times1)^{6n+14}\times(2\times2)^3\times(2\times3)^6\times(4\times3)^3\times(3\times2)^3\times\\
&\times(4\times4)^3\times(6\times4)^{6n+4}\times(5\times4)^3\\
&\times(3\times4)^{6n+14}\times(4\times2)^3
\end{aligned}$$

After simplifying the above term equation we get ,

$\Rightarrow M\hat{Z}C_3(G_n) = (3^{18n}\times2^{30n})\times(1.763725882\times10^{47}).$ □

Theorem 3.7.10. *For the graph G_n. The fourth multiplicative Zagreb connection index of para-line graph of triglyceride is*

$$M\hat{Z}C_4(G_n) = 2^{21n}\times3^{39n}\times(3.0864633\times10^{61})$$

Proof. Let G_n be a para-line graph of triglyceride graph with $n\geq1$, where n is an integer. Then from the definition of 4^{th} MZCI we have,

$$M\hat{Z}C_4(G_n) = \prod_{st\in E^c(G_n)}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
M\hat{Z}C_4(G_n) &= \prod_{st\in E^c_{2,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{2,4}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E^c_{3,2}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]\times\prod_{st\in E^c_{3,3}}[\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)]
\end{aligned}$$

$$
\begin{aligned}
&\times \prod_{st\in E^c_{3,4}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)] \times \prod_{st\in E^c_{3,6}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)] \\
&\times \prod_{st\in E^c_{3,5}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)] \times \prod_{st\in E^c_{4,5}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)] \\
&\times \prod_{st\in E^c_{4,4}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)] \times \prod_{st\in E^c_{5,6}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)] \\
&\times \prod_{st\in E^c_{6,4}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)] \times \prod_{st\in E^c_{6,6}} [\bar{w}_{G_n}(s)+\bar{w}_{G_n}(t)].
\end{aligned}
$$

from Table 3.7, we have,

$$
\begin{aligned}
M\hat{Z}C_4(G_n) \quad = \quad & (2+2)^3 \times (2+4)^6 \times (3+2)^3 \times (3+3)^{(9n+25)} \\
& \times (3+4)^8 \times (3+6)^{(12n+9)} \times (3+5)^6 \times (4+5)^3 \\
& \times (4+4)^3 \times (5+6)^3 \times (6+4)^4 \times (6+6)^{6n}
\end{aligned}
$$

After simplifying the above equation we get,

$\Rightarrow M\hat{Z}C_4(G_n) = 2^{21n} \times 3^{39n} \times (3.0864633 \times 10^{61}).$ □

Theorem 3.7.11. *For the graph G_n. The first modified multiplicative Zagreb connection index of para-line graph of triglyceride is*

$$M\hat{Z}C_1^*(G_n) = 2^{57n} \times 3^{39n} \times 5^{6n} \times (7.223478872 \times 10^{98})$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 1^{st} mMZCI we have,

$$M\hat{Z}C_1^*(G_n) = \prod_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)].$$

$$
\begin{aligned}
M\hat{Z}C_1^*(G_n) \quad = \quad & \prod_{st\in E^{1,3}_{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E^{3,2}_{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \\
& \times \prod_{st\in E^{2,4}_{2,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E^{2,2}_{3,3}} [d_{G_n}(s)\bar{w}_{G_n}(t) + d_{G_n}(t)\bar{w}_{G_n}(s)]
\end{aligned}
$$

$$\times \prod_{st\in E_{4,3}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E_{4,4}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$
$$\times \prod_{st\in E_{4,3}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E_{2,4}^{4,4}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$
$$\times \prod_{st\in E_{4,3}^{2,3}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E_{4,3}^{4,5}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$
$$\times \prod_{st\in E_{4,5}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E_{4,6}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$
$$\times \prod_{st\in E_{4,6}^{4,4}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)] \times \prod_{st\in E_{2,4}^{2,3}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$
$$\times \prod_{st\in E_{4,4}^{3,5}} [d_{G_n}(s)\bar{w}_{G_n}(t)+d_{G_n}(t)\bar{w}_{G_n}(s)]$$

from Table 3.8, we have,

$$\begin{aligned} M\hat{Z}C_1^*(G_n) &= (3+12)^{(6n+14)} \times (6+4)^3 \times (8+6)^3 \times (4+9)^3 \times (12+12)^{3n+11} \\ &\quad \times (16+12)^5 \times (24+12)^{12n+9} \times (16+8)^3 \times (6+12)^3 \times (20+12)^6 \\ &\quad \times (24+20)^3 \times (24+24)^{6n} \times (16+24)^4 \times (6+8)^3 \times (15+16)^3 \end{aligned}$$

After simplifying the above equation we get,
$\Rightarrow M\hat{Z}C_1^*(G_n) = 2^{57n} \times 3^{39n} \times 5^{6n} \times (7.223478872 \times 10^{98}).$ □

Theorem 3.7.12. *For the graph G_n. The second modified multiplicative Zagreb connection index of para-line graph of triglyceride is*

$$M\hat{Z}C_2^*(G_n) = 2^{57n} \times 3^{39n} \times 5^{6n} \times (2.76306885 \times 10^{84} \times 10^{33})$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 2^{nd}mMZCI we have,

$$M\hat{Z}C_2^*(G_n) = \prod_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$
\begin{aligned}
M\hat{Z}C_2^*(G_n) \;=\; & \prod_{st\in E_{4,3}^{1,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\times\prod_{st\in E_{2,2}^{3,2}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E_{3,4}^{2,2}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\times\prod_{st\in E_{3,2}^{2,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E_{4,3}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\times\prod_{st\in E_{4,4}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E_{4,6}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\times\prod_{st\in E_{2,4}^{4,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E_{4,3}^{2,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\times\prod_{st\in E_{4,5}^{4,3}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E_{4,6}^{4,5}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\times\prod_{st\in E_{4,6}^{4,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E_{4,4}^{4,6}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\times\prod_{st\in E_{3,2}^{2,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]\\
&\times\prod_{st\in E_{4,5}^{3,4}}[d_{G_n}(s)\bar{w}_{G_n}(s)+d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}
$$

from Table 3.8, we have,

$$
\begin{aligned}
M\hat{Z}C_2^*(G_n) \;=\; & (3+12)^{6n+14}\times 3(6+4)^3\times(4+12)^3\times(6+6)^3\times(12+12)^{3n+11}\\
&\times(12+16)^5\times(12n+9)(12+24)^{12n+9}\times(16+8)^3\times(6+12)^3\times(12+20)6\\
&\times(20+24)^3\times(24+24)^{6n}\times(24+16)4\times(8+6)^3\times(12+20)6^3
\end{aligned}
$$

After simplifying the above equation we get ,
$\Rightarrow M\hat{Z}C_2^*(G_n) = 2^{57n}\times 3^{39n}\times 5^{6n}\times(2.76306885\times 10^{84}\times 10^{33})$. □

Theorem 3.7.13. *For the graph G_n. The third modified multiplicative Zagreb connection index of para-line graph of triglyceride is .*

$$M\hat{Z}C_3^*(G_n) = 2^{120n}\times 3^{54n}\times(1.238478\times 10^{95})\times(5.499217\times 10^{56})$$

Proof. Let G_n be a para-line graph of triglyceride with $n \geq 1$, where n is an integer. Then from the definition of 3^{rd}mMZCI we have,

$$M\hat{Z}C_3^*(G_n) = \prod_{st\in E(G_n)} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)].$$

$$\begin{aligned}
M\hat{Z}C_3^*(G_n) \quad = \quad & \prod_{st\in E_{4,3}^{1,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{2,2}^{3,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{3,4}^{2,2}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,2}^{2,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{4,3}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{4,4}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{4,6}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{2,4}^{4,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{4,3}^{2,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{4,5}^{4,3}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{4,6}^{4,5}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{4,6}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{4,4}^{4,6}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \times \prod_{st\in E_{3,2}^{2,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)] \\
& \times \prod_{st\in E_{4,5}^{3,4}} [d_{G_n}(s)\bar{w}_{G_n}(s) \times d_{G_n}(t)\bar{w}_{G_n}(t)]
\end{aligned}$$

from Table 3.8, we have,

$$\begin{aligned}
M\hat{Z}C_3^*(G_n) \quad = \quad & (3\times 12)^{6n+14} \times (6\times 4)^3 \times (4\times 12)^3 \times (6\times 6)^3 \times (12\times 12)^{3n+11} \\
& \times (12\times 16)^5 \times (12n+9)(12\times 24)^{12n+9} \times (16\times 8)^3 \times (6\times 12)^3 \times (12\times 20)^6 \\
& \times (20\times 24)^3 \times (24\times 24)^{6n} \times (24\times 16)^4 \times (8\times 6)63 \times (12\times 20)^3
\end{aligned}$$

After simplifying the above equation we get ,

$\Rightarrow M\hat{Z}C_3^*(G_n) = 2^{120n} \times 3^{54n} \times (1.238478 \times 10^{95}) \times (5.499217 \times 10^{56}).$ □

Chapter 4

Conclusion and Future Work

4.1 Conclusion

The aim of this study was to propose universal equations for calculating the 1^{st} and 2^{nd} Zagreb connection indices for triglycerides, as well as the corresponding line and para-line graphs. Specifically, we derived expressions for the 1^{st} MZCI, 2^{nd} MZCI, 3^{rd} MZCI, and the fourth MZCI, as well as their modified counterparts. These calculations apply to the chemical structure of triglycerides, their line graph and para-line graph. The expressions for triglycerides and their line graph depend on the value of $n \geq 3$. On the other hand, the expressions for the para-line graph of triglycerides depend on the value of $n \geq 1$.

4.2 Future Work

The following open issues are the ones we propose in [7] connection-based Zagreb indices:

(1) For the Sunlet network,compute connection-based Zagreb indices.

(2) For nanotubes and oxide nanotubes,compute connection-based Zagreb indices.

(3) For calcium chloride compute connection-based Zagreb indices.

Chapter 5

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