Oxygen Cyclotron Harmonic Waves in the Inner Magnetosphere

By

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CIIT/FA17-RPH-037/LHR

MS Thesis

In

Physics

COMSATS University Islamabad, Lahore Campus – Pakistan

Spring, 2019

COMSATS University Islamabad, Lahore

Oxygen Cyclotron Harmonic Waves in the Inner Magnetosphere

A Thesis Presented to

COMSATS University Islamabad, Lahore Campus

In partial fulfillment
of the requirement for the degree of

MS (Physics)

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A Post Graduate Thesis submitted to the Department of Physics as partial fulfillment of the requirement for the award of Degree of M.S(Physics).

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I Abdul Waheed (CIIT/FA17-RPH-037/LHR) hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within the acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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It is certified that Abdul Waheed (CIIT/FA17-RPH-037/LHR) has carried out all the work related to this thesis under my supervision at the Department of Physics, COMSATS University Islamabad, Lahore, and the work fulfills the requirement for the award of MS degree.

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Oh, Allah Almighty open our eyes,

To see what is beautiful,

Our minds to know what is true,

Our heart to love what is Allah

Dedicated To

MY FAMILY & TEACHERS

Whose encouragement, spiritual inspiration, well wishes, sincere prayers and an atmosphere that initiate me to achieve high academic goals

ACKNOWLEDGEMENTS

First of all, I would like to bow my head before **“ALMIGHTY ALLAH”**, The Merciful, the Beneficent who presented me in a Muslim community and also blessed me with such a lucid intelligence as I could endeavor my services toward this manuscript. Countless salutations are upon the ***HOLY PROPHET MUHAMMAD (May Peace Be upon Him),*** the fountains of knowledge, who guided his “Ummah” to seek knowledge from cradle to grave.

The work presented in this manuscript was accomplished under the sympathetic attitude, animate direction, observant pursuit, scholarly criticism, cheering perspective and enlightened supervision of **Dr. Muhammad Jamil** and **Dr. Muhammad Fraz Bashir**, Department of Physics, COMSATS University Islamabad, Lahore. Their thorough analysis and rigorous critique improved not only the quality of this dissertation but also my overall understanding of the subject. I am grateful to their inspiring guidance, keen interest, scholarly comments and constructive suggestion throughout the course of my studies.

Words are lacking to express my humble obligation to my affectionate, *Father* ***(Shabbir Ahmad),*** Mother (***Naseem Akhtar***), and Brother (***Abdul Hayee***) for their love, good wishes, inspirations and unceasing prayers for me, without which the present destination would have been merely a dream.

Abdul Waheed (CIIT/FA17-RPH-037/LHR)

ABSTRACT

Oxygen Cyclotron Harmonic Waves in the Inner Magnetosphere

The Banded emission of Oxygen cyclotron harmonic waves has been observed by Van Allen probes in the Earth inner magnetosphere. The general dispersion relation for electrostatic Oxygen cyclotron harmonic waves is derived by using Lerche-Newberger sum rule for Maxwellian and Kappa distributions. Oxygen ion kappa spectral index (κ_{iO}) has a significant impact on the Oxygen cyclotron harmonic waves. The quasi Maxwellian behavior is being observed as $\kappa_{iO} \geq 5$. The curves shift toward the higher value of wavenumber, as the value of κ_{iO} reduce. There is an increase in super-thermal particles as we reduce the value of κ_{iO}. By decreasing the value of κ_{iO}, the super-thermal particles reduce the frequency , ω_{peak}, where the group velocity vanish and the associated k_{peak} get increase. These cyclotron harmonic waves may be useful for the diagnostic for the velocity distribution characteristics. Oxygen cyclotron harmonic waves can be helpful for the diagnostic of Van Allen Radiation Belts.

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Chapter 1

Introduction

1 Introduction

1.1 Plasma

1.1.1 A Brief History of Plasma

The Plasma word came from the Greek word Plasma meaning ”Moldable substance” or ”jelly”. In 1879, Sir Willian Crooks discovered Plasma as radiant material. The J.J Thomson experiments on cathode rays urged him to tell about the new state of matter which was composed of positively and negatively subatomic charged particles. Johannes Purkinje names the clear blood (from its various corpuscles) liquid as Plasma. But the word Plasma for ionized gas was first used by an American Nobel prize chemist Irving Langmuir in 1927 [24].

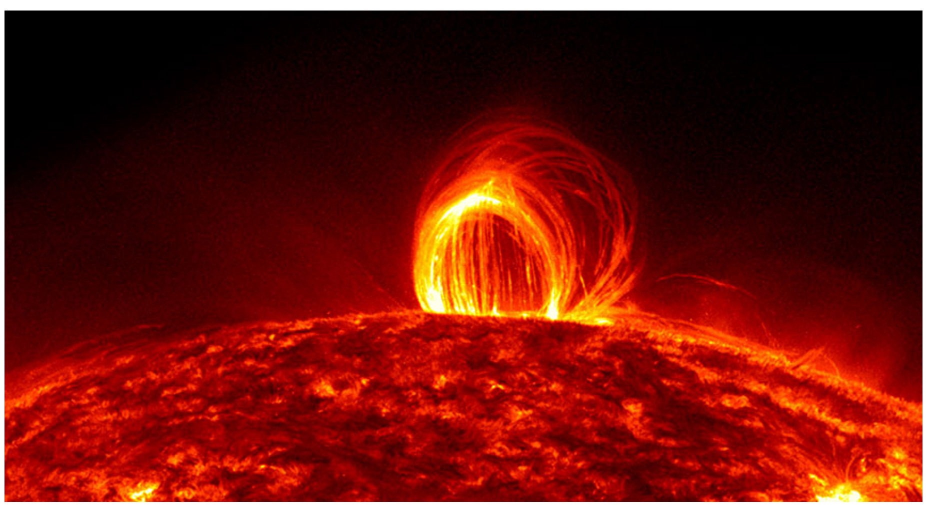

Fig. 1.1: Naturally Existed Plasma

Fig. 1.2: Northern Lights

1.1.2 Definition of Plasma

Plasma is called the fourth state of Matter but some scientist calls it the first state. The most common definition for plasma used is the "Ionized Gas". And from our thinking ionized gas means atoms are converted into ions and electrons. But basically, this definition does not explain the term of plasma correctly.

The Exact Definition of Plasma is as follows:

" The quasineutral gas of charge and neutral particles which exhibit collective behavior is called Plasma ".

The terms "quasineutrality" and "collective behavior" are the main part of the definition. Both terms perfectly define Plasma behavior and nature [4].

Quasineutrality term was introduced in Plasma in 1929 by Irving Langmuir and Tewi Tonks. The Latin word "Quasi" means " as if " or " resembling ". That means the overall Plasma is neutral. In overall Plasma, the number density of positively charged particles is equal to the negatively charged particles. But in Plasma the positive and negative charged particles make small charged and electric filed region.

Collective Behavior term define as the Plasma is just act like a mind. In Plasma the charged particles interact with each other by Coulomb electric field. When we apply a small disturbance in the system the whole system will respond to it. That means the electromagnetic forces in Plasma are much larger than the local collision. That's why we called Plasma collisionless. The motion in Plasma is not just depended on the local region but also on the remote region as well.

1.2 Quantities of Plamsa

1.2.1 Plasma Frequency

The linear oscillation of charged particles about there means the position is called Plasma frequency.

$$\omega_p = \sqrt{\frac{n_0 e^2}{m\epsilon_0}}, [rad/s](SI - unit) \tag{1.1}$$

$$\omega_p = \sqrt{\frac{4\pi n_0 e^2}{m}}, (cgs - unit) \tag{1.2}$$

where m is the mass of species, ϵ_0 is permittivity of free space, e is an electric charge of species and n_0 is number density of Plasma.
From above both relation, it is clear that Plasma frequency is inversely proportioned to the mass of species. In the case of electrons and ions, the electrons have high plasma frequency as compared to ions due to less mass.

1.2.2 Cyclotron Frequency

The charge particles motion in a cyclic path under the presence of a magnetic field. The path of motion is perpendicular to the magnetic field. This type of motion is called the cyclotron frequency or gyrofrequency. The cyclic path of charged particles is created due to the presence of Lorentz force. This force keeps the charged particles in the cyclic path.

$$\omega_c = \frac{|q|B}{m}, [rad/s](SI - units) \tag{1.3}$$

$$\omega_c = \frac{|q|B}{mc}, (cgs - units) \tag{1.4}$$

From above both relation, it is clear that cyclotron frequency is inversely proportioned to the mass of species. In the case of electrons and ions, the electrons have high cyclotron frequency as compared to ions due to less mass.

1.2.3 Larmor Radius

The radius formed due to the cyclic motion of charged particles in the presence of a uniform magnetic field is called the Larmor radius or Gyroradius.

$$r_L = \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|B}, [m](SI - units) \tag{1.5}$$

$$r_L = \frac{mcv_\perp}{|q|B}, (cgs - units) \tag{1.6}$$

$v_\perp$ is the perpendicular component of velocity. As we take $v_\perp = v_{th}$ the analogy to the product $\lambda_D \omega_p$ that $r_L \omega_c = v_{th}$

1.3 Waves

Any kind of disturbance in the equilibrium medium that propagates with time from one region to another is called a wave. This disturbance propagates in the medium due to the transfer of disturbance from one particle to another and so on. The wave has crest and trough.

1.3.1 Velocity of Wave

There is two kinds of velocities are associated with the motion of wave as fellow:

Phase Velocity

Phase velocity is also called the wave velocity. The disturbance or monochromatic wave in medium move forward from one particle to another so as there is a progressive change in the phase of one particle to another.

For example, the x-axis propagating electromagnetic wave equation is

$$E = E_0 \cos(\omega t - kx) \tag{1.7}$$

from the above equation, ω, k are angular frequency and wave number or propagating constant respectively. The $(\omega t - kx)$ term represents the phase of the wave. For constant phase

$$(\omega t - kx) = constant \tag{1.8}$$

Differentiating w.r.t "time"

$$\omega - k\frac{dx}{dt} = 0 \tag{1.9}$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \upsilon_\varphi \tag{1.10}$$

$$\upsilon_\varphi = \frac{2\pi f\lambda}{2\pi} = f\lambda \tag{1.11}$$

The phase velocity a wave is just the product of its frequency and wavelength.

Group Velocity

The most important work of wave is to transfer information from one place to another. And the transformation of date by the single wave is not suitable. Therefore a group of waves with different wavelength and frequencies is formed by there superposition. And the velocity of that group or pulse is called the group velocity. By adding the propagation equation of two waves of nearly equal frequencies, and that group or pulse of wave carry information with velocity as.

$$\upsilon_g = \lim_{\Delta\omega\to 0} \frac{\Delta\omega}{\Delta k} \tag{1.12}$$

$$\upsilon_g = \frac{d\omega}{dk} \tag{1.13}$$

1.3.2 Relation between Phase Velocity and Group Velocity

From above relations

$$\upsilon_g = \frac{d\omega}{dk} \tag{1.14}$$

and

$$
\upsilon_\varphi = \frac{\omega}{k} \tag{1.15}
$$

so

$$
\omega = k\upsilon_\varphi \tag{1.16}
$$

$$
\upsilon_g = \frac{d(k\upsilon_\varphi)}{dk} \tag{1.17}
$$

$$
\upsilon_g = \upsilon_\varphi + k\frac{d\upsilon_\varphi}{dk} \tag{1.18}
$$

we know that

$$
k = \frac{2\pi}{\lambda} \tag{1.19}
$$

so

$$
\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2} = -\frac{k}{\lambda} \tag{1.20}
$$

$$
dk = -k\frac{d\lambda}{\lambda} \tag{1.21}
$$

Final result come as

$$
\upsilon_g = \upsilon_\varphi - \lambda\frac{d\upsilon_\varphi}{d\lambda} \tag{1.22}
$$

The above relation shows that in the dispersive medium the phase velocity is always greater than the group velocity. But in non-dispersive medium, all waves have the same velocity as

$$\frac{d\upsilon_\varphi}{d\lambda} = 0 \tag{1.23}$$

and

$$\upsilon_g = \upsilon_\varphi \tag{1.24}$$

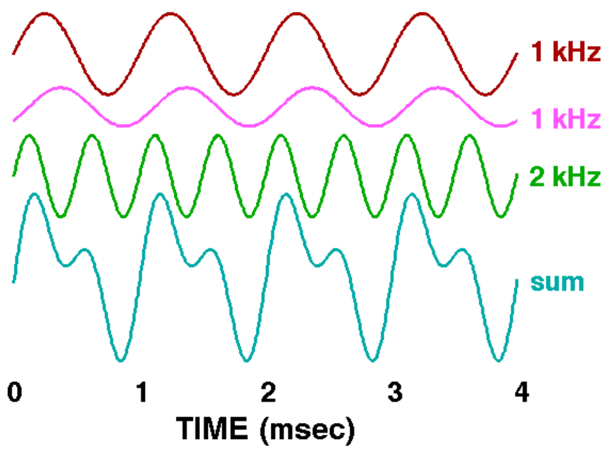

Fig. 1.3: Phase Velocity and Group velocity

1.4 Plasma Waves

Plasma is simply composed of electrons and ions, but it may contain, plenty of ions species including negative ions too. And in some cases, Plasma also contains neutrals particles. These particles generate plenty of waves in Plasma. There is a wide range of Plasma frequency in optical, radio and acoustic frequencies. The phase velocity of the wave in Plasma can be greater than the speed of light. But that does not violate the law of general theory of relativity. A single wave of constant amplitude with Phase velocity cannot carry information. For information transformation, we need to modulate the wave. Only modulated wave with group velocity can carry information. And group velocity is less than the speed of light.

1.4.1 Dispersion Relation

For any dispersive medium, the relation between frequency ω and k wave number of the wave is called Dispersion relation.

some waves have linear dispersion and some have nonlinear dispersion relation. Like light has linear dispersion relation $\omega = ck$.

1.4.2 Classification of Waves

The waves can be classified in different ways. But we are going to classify them from their oscillating magnetic field. And by that way there are two categories of waves, one is electromagnetic and second electrostatic.

Electromagnetic Waves

If the wave is traveling with an oscillating magnetic field is called the electromagnetic wave. They are transverse by nature.

$$\vec{k} \perp \vec{E} \tag{1.25}$$

If EM wave propagate parallel to the external magnetic field

$$\vec{k} \| \vec{B}_0 \tag{1.26}$$

The propagation $k_\perp = 0$

and if EM wave propagate perpendicular to external magnetic field

$$\vec{k} \perp \vec{B}_0 \tag{1.27}$$

From above we know that electromagnetic waves can travel parallel or perpendicular to the external magnetic field. The propagating wave number can be parallel or perpendicular to the external magnetic field. But if we take the oblique propagation of an electromagnetic wave then we have a coupled wave containing both parallel and perpendicular parts of the wave, and this will be a new wave.

Electrostatic Waves

If the wave is traveling without oscillating magnetic field is called an electrostatic wave. Plane wave from the perspective of Faraday's Law of induction,

$$\vec{k} \times \vec{E} = \omega \vec{B} \tag{1.28}$$

These waves are purely longitudinal in nature

$$\vec{k} \| \vec{E} \tag{1.29}$$

1.4.3 Summary of Plasma Waves

Electrostatic

Electron Waves

$$\vec{B_0} = 0 \quad or \quad \vec{k} \| \vec{B_0} \qquad \omega^2 = \omega_p^2 + \frac{3}{2}k^2\upsilon_{th}^2 \qquad (Langmuir Waves)$$

$$\vec{k} \perp \vec{B_0} \qquad \omega^2 = \omega_p^2 + \omega_c^2 = \omega_h^2 \qquad (Upper Hybrid Oscillation)$$

Ions Waves

$$\vec{B_0} = 0 \quad or \quad \vec{k} \| \vec{B_0} \qquad \omega^2 = k^2\upsilon_s^2$$

$$= k^2 \frac{\gamma_e KT_e + \gamma_i KT_i}{M} \qquad (Acoustic Waves)$$

$$\vec{k} \perp \vec{B_0} \qquad \omega^2 = \Omega_c^2 + k^2\upsilon_s^2 \qquad (Electrostatic Ion Cyclotron wave)$$

$$\omega^2 = \omega_i^2 = \Omega_c \omega_c \qquad (Lower Hybrid Oscillation)$$

Electromagnetic

Electron Waves

$$\vec{B_0} = 0 \qquad \omega^2 = \omega_p^2 + k^2 c^2 \qquad (Light Waves) \tag{1.30}$$

$$\vec{k} \perp \vec{B}_0, \vec{E}_1 \| \vec{B}_0 \qquad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \qquad (O Waves) \tag{1.31}$$

$$\vec{k} \perp \vec{B}_0, \vec{E}_1 \perp \vec{B}_0 \qquad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2} \qquad (X Waves) \tag{1.32}$$

$$\vec{k} \| \vec{B}_0 \qquad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\frac{\omega_p^2}{\omega^2}}{1 - \frac{\omega_c^2}{\omega}} \qquad (R Waves) \tag{1.33}$$

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\frac{\omega_p^2}{\omega^2}}{1 + \frac{\omega_c^2}{\omega}} \qquad (L Waves) \tag{1.34}$$

Ions Waves

$$\vec{B_0} = 0 \qquad None$$

$$\vec{k} \| \vec{B}_0 \qquad \omega^2 = k^2 \upsilon_A^2 \qquad (Alfeven Waves)$$

$$\vec{k} \perp \vec{B}_0 \qquad \frac{\omega^2}{k^2} = c^2 \frac{\upsilon_s^2 + \upsilon_A^2}{c^2 + \upsilon_A^2} \qquad (Magnetosonic Waves)$$

1.5 Electrostatic Cyclotron Harmonic Waves

The wave propagation between the harmonic of cyclotron harmonic is called the cyclotron harmonic waves. These waves propagate perpendicular to the ambient magnetic field. The ion cyclotron harmonic waves have a wide range of application in both space and laboratory plasma. The ion cyclotron wave travel perpendicular to the ambient magnetic field is electrostatic in nature, they are also called **Bernstein waves**. [26].

1.5.1 A Brief History of Bernstein Waves

The Bernstein waves were discovered by **Ira B. Bernstein** in 1958, and these waves are named after him. The experimental confirmation of these waves was done by **Crawford in 1964**[5].

The Bernstein waves are purely Kinetic in nature, they have no counter fluid part. These waves are electrostatic in nature, but the electromagnetic effect can be induced by coupling of Z-mode of the hybrid band. The velocity distribution dispersion relation of Bernstein waves is so much sense to the particles. This detail helpful for information in satellite sounding measurements[12].

1.5.2 Conversion Modes

The mode conversion of waves in extraordinary or ordinary waves and Bernstein waves depends only the density gradients in the plasma edge.

High Field Side Launch

This type of Conversion is only possible with X-wave first harmonic. When X-wave approach to upper hybrid resonance(UHR). The X-mode converted into Bernstein waves. But for high dense plasma heating, this method is not applicable [34].

Direct X-B Conversion

The direct mode conversion from X-wave to Bernstein waves is as follow, the X-wave launch from the vacuum. The density scale length of the density gradient is the order of incident wave wavelength. The fast X-mode combined with the slow X-mode between the temporary region of R-mode and UHR and X-mode converted into the Bernstein mode at UHR [16].

OXB Conversion

In OXB conversion, the O-waves are launched and it makes an oblique angle. There is a coincidence between O-mode and X-mode for the optimal value of the refractive index at critical plasma density. And the group velocity and phase velocity of O-mode and X-mode are same. As a result, there is no absorption during power transmission. And in optic, the O-X modes conversion somehow matches with Brewster angle. The generated X-wave start propagating toward UHR. They coincide and generate Bernstein waves after applying hot and cold plasma approximation. The main condition for OXB conversion is that the cutoff density of O-wave must be less than plasma density [2].

1.6 Electromagnetic Cyclotron Harmonic Waves

Electromagnetic ion cyclotron waves are parallel propagating left-handed circularly polarized wave which propagates near the cyclotron frequency. However, for the heavy ions present in the inner magnetosphere, the wave spectrum shows more propagation bands near the cyclotron frequency of each ion i.e., for helium (He+) and oxygen (O+) ions, there exist two more propagation bands in addition to the proton cyclotron band.

1.7 Magnetosphere

The magnetosphere is the region around the planet made up by the magnetic field of the planet.

1.7.1 Earth Magnetosphere

The Earth magnetosphere is just like a bar magnet. But due to the solar wind, the shape of the Earth magnetic field is distorted and it looks like a spider shape [30].

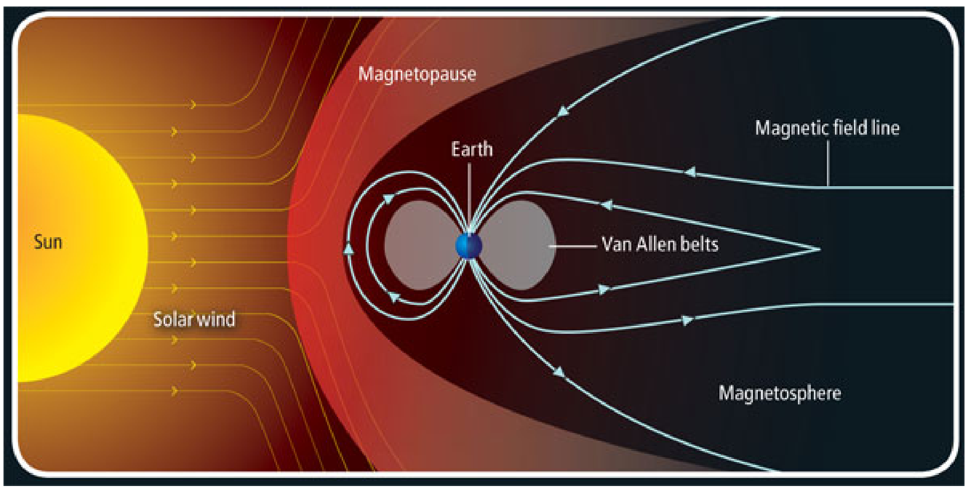

Fig. 1.4: Earth Magnetosphere under the influence of Solar wind

Due to the pressure of the solar wind on the magnetic field of the earth. The solar wind compressed the magnetic field lines in the day-side and stretched the lines on night-side.

Bow Shock

The solar wind contains high energy charged particles. The first protecting wall of earth magnetosphere is Bow Shock. Its shape is just like an arrow bow. The bow shock slows down the speed of solar wind from supersonic to sonic.

Magnetopause

The shape of the Magnetopause is just like an umbrella. As the speed of solar wind changes the magnetopause move forth and back as in response. The magnetopause is the region in which the earth magnetic field balanced the pressure of solar wind particles.

Van Allen Belts

Near to earth, the magnetic field lines are less disturbed and more like dipole shape. The charged particles get trapped inside these magnetic field lines and they spiral around these magnetic field lines.

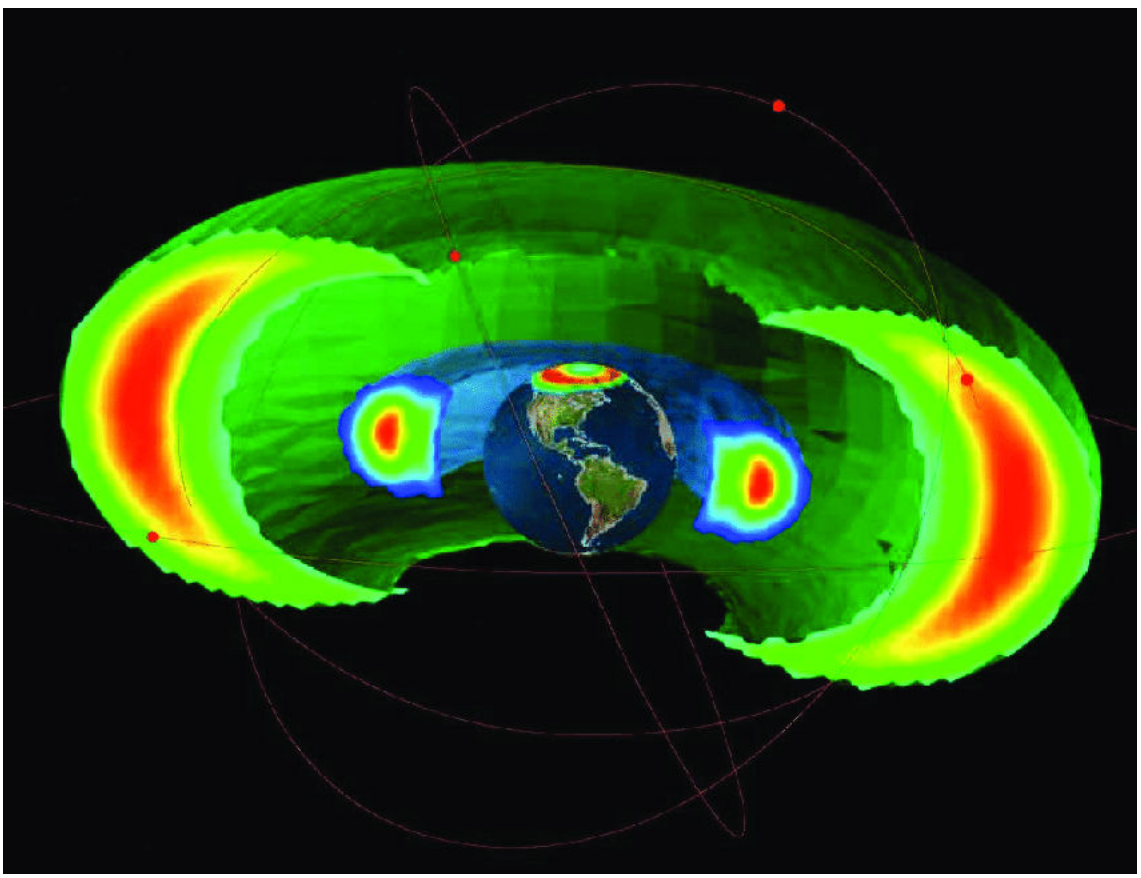

Fig. 1.5: Van Allen Belts

Inner Van Allen Belt(1000-8000 miles)

The inner Van Allen belt has maximum proton density. The inner Van Allen belt has high energy protons up to the energy level of hundred MeV. The ions in the inner belts absorbed the energy from the cosmic rays and follow the path of magnetic fields lines. The radiations belts are like dipole shape, so the charged particles get trapped inside them. These particles can rotate with the rotation of the earth.

Outer Van Allen Belt (12000-25000 miles)

Unlike inner Van Allen belt, the outer Van Allen belt contains protons of low energy up to the level of KeV to 1MeV. The ions and electrons from the solar wind diffuse from magnetopause to magneto-tail. Then these particles get drift toward the plasma sheet. After that, these plasma sheet particles inject to the outer Van Allen belt. Due to the charge difference ions and electrons they get deflect towards westward and eastward respectively [8] [27] [27].

1.8 Satellites

1.8.1 Van Allen Probes

The first name of Van Allen probes was RBSP(Radiation Belt Storm Probes). But NASA renames it in the honor of late Van Allen in November 2012. They are present in elliptical orbit from 600Km to 37000Km. The main goal of this mission was to resolve the scientific mysteries that how to charge particles become so high energized and the radiation belts conditions change with the changing condition on Sun. And how to charge particles accelerate and transport in the radiations belts. What kind of effects a solar storm did on the radiations belts and how these charge particles behave under such circumstances [17].

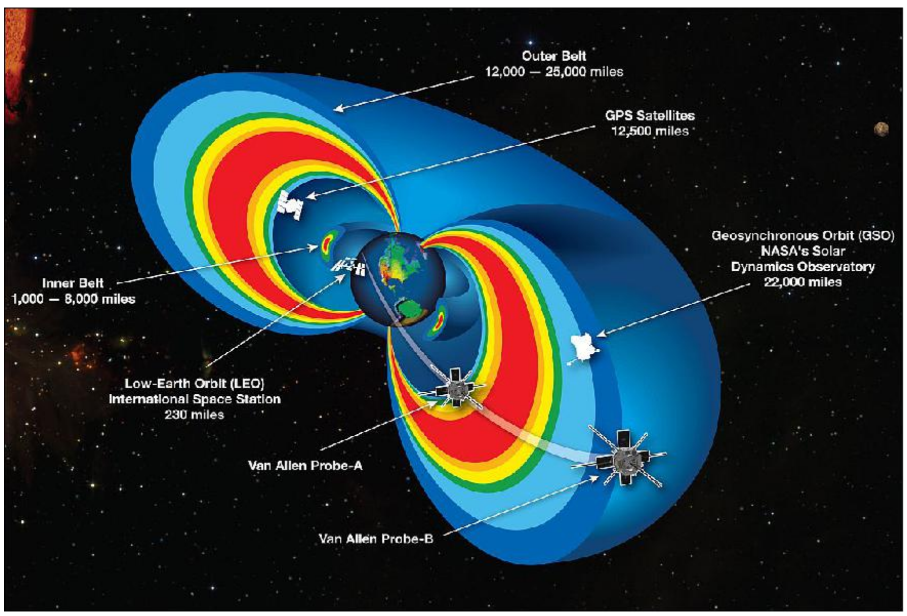

Fig. 1.6: Van Allen Probes

1.9 Space Weather

Space weather is created due to the activities on the sun. Sun is so much hot, it ejects hot plasma into space. This hot ejected plasma is called corona. These charged particles create electricity in the corona. The speed of solar wind can be reached up to million miles per hour.

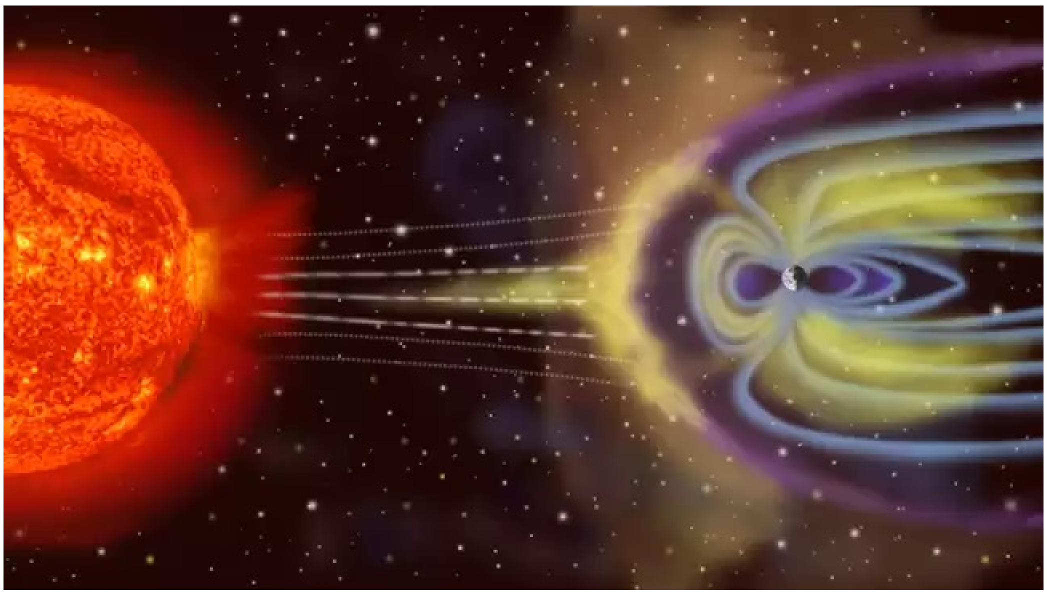

Fig. 1.7: Space Weather

Solar storms can cause serious problems to astronauts in space. Because during solar storms the density and energy of charged particles as well as radiation increase which can damage the astronaut DNA. which may lead to cancer.

People flying in the airline are also in danger during solar storms. These people especially flying on the north pole are exposed to high energy radiation. So that for their protection some time the route has to change of airline.

Most of the earth orbiting satellite are in the radiation belts region. As the density and energy increase during storms, these charged particles damage the equipment of satellite and some time they also damage the solar cells of the satellite. The electrons and ions particles create short-circuit in the satellite.

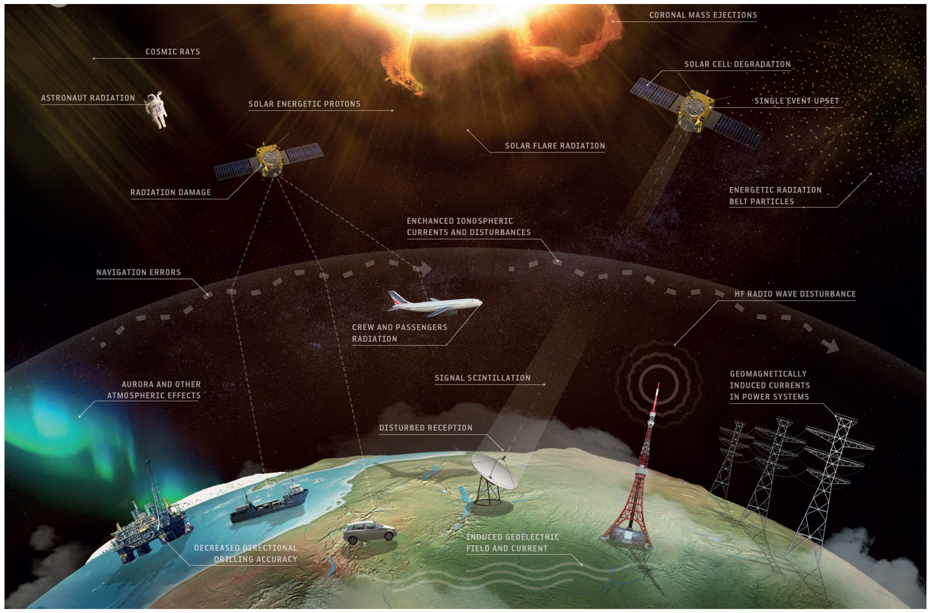

Fig. 1.8: Space Weather Effective Regions

Some times the solar storms create induce electric current due to charged particles. These induce current some times pass through the power transmission lines to produce major problems and become the cause of blackouts, this damage thousand of people. In OCT.30, 2003 Sweden faced power failure due to the solar storm.

Chapter 2

Cyclotron Harmonic Waves

2 Cyclotron Harmonic Waves

There are two types of electrostatic ion cyclotron harmonic waves(Bernstein waves), One which propagates perpendicular to the external magnetic field is called pure ion Bernstein waves and other one propagate not exactly perpendicular but close to 90^0 [7] [15]. These waves propagate between the cyclotron harmonic frequency have been observed several times in the magnetosphere of Earth. And these observed waves propagate exactly perpendicular to the ambient magnetic field. The frequency structure of these waves governed by ion gyro-frequency and harmonics of it [33] [18] [26].

The electrostatic low frequency waves propagating across the magnetic field was experiment observation by [6] [23]. And the waves propagating above the ion cyclotron frequency in plasma at high electron-ion temperature ration was observed by [14] [13].

In magnetosphere, these waves have been frequently observed within $2^0 - 3^0$ of geomagnetic equator from $L = 2 - 8$ at radial distance [31] [3] [32] [19].

Ion cyclotron harmonic waves (Ion Bernstein) plays an important role in theoretical as well as in experimental plasma. These waves have been studies in space and as well as in laboratory plasma [9] [11].

In this report, we use the dispersion relation of pure electrostatic cyclotron harmonic waves for the investigation of oxygen ion cyclotron harmonic waves. The dispersion relation is derived by using the Newberger Sum rule [25]. The ions cyclotron harmonic waves are takin in a uniform magnetic field and collisionless plasma. The electromagnetic effect is ignored in the dispersion relation. The electrostatic cyclotron doesn't include any relativistic effect.

2.1 Dispersion Relation of Cyclotron Harmonic Waves

The electrostatic Oxygen cyclotron harmonic waves are purely kinetic nature. The dispersion relation of electrostatic cyclotron harmonic waves depend on the equilibrium velocity distribution [38] [21] [20]. The plasma model we will use is uniform, immersed and collisionless in the uniform magnetic field, B_0. The electromagnetic effect is ignored here, so the model equation is Vlasov-Poisson system[12].

$$\frac{1}{\epsilon_0}\sum_j n_{0j}q_j\int f_j^0 dv = 0 \tag{2.1}$$

$$\frac{q_j}{m_j}(v \times B_0)\cdot\frac{\partial f_j^0}{\partial v} = 0 \tag{2.2}$$

By characteristics and plane wave assumption we get the dispersion relation of electrostatic cyclotron harmonic waves in magnetized plasma [12].

$$\epsilon(k,\omega) = 1-2\pi\sum_j\frac{\omega_{pj}^2}{k^2}\int_0^\infty dv_\perp v_\perp\int_L dv_{v\parallel}\sum_{n=-\infty}^{\infty}\left(\frac{n\omega_{cj}}{v_\perp}\frac{\partial f_j^0}{\partial v_\perp}+k_\parallel\frac{\partial f_j^0}{\partial v_\parallel}\right)\cdot\frac{j_n^2(k_\perp)v_\perp/\omega_{cj}}{k_\parallel v_\parallel+n\omega_{cj}-\omega} = 0 \tag{2.3}$$

From the above equation, $\omega_{pj} = (n_{0j}q_j^2/\epsilon_0 m_j)^{1/2}$ and $\omega_{cj} = q_jB_0/m_j$ are signed as Plasma frequency and gyrofrequency of jth component respectively. k is known as wave number. With respect of uniform magnetic field $\mathbf{B}_0$, the $k_\parallel$ and $k_\perp$ are parallel and perpendicular components of k respectively. The $v_\perp$ and $v_\parallel$ are the parallel and perpendicular components of velocity respectively. The wave frequency is represented as ω, where $\omega^2 = \omega_{pj}^2+\omega_{cj}^2$. And the term j_n is the Bessel function. The integral with subscript L represent the landau contour. Landau contour take the entire function of ω of dispersion relation except for countably infinite numbers of poles.

The above dispersion relation is more general, it takes propagation of waves in all direction. But we have concern only with the perpendicular propagation with respect to $textbfB_0$. That's why we take the parallel component of wave equal to zero ($k_{\|} = 0$).

$$\epsilon(k,\omega) = 1 + 2\pi \sum_j \frac{\omega_{pj}^2}{k_\perp^2} \int_0^\infty dv_\perp \int_{-\infty}^{\infty} dv_\| \frac{\partial f_j^0}{\partial v_\perp} \sum_{n=-\infty}^{\infty} \frac{n j_n^2(k_\perp v_\perp/\omega_{cj})}{\omega/\omega_{cj} - n} = 0 \qquad (2.4)$$

The above equation is called the dispersion relation of perpendicular propagation of cyclotron harmonic waves. In above dispersion relation the term f_j^0 is know as distribution function. we can replace this term with any distribution function such as Maxwellian and Kappa distribution function.

2.2 Maxwellian Distribution Function with Series Method

The Maxwellian distribution function is

$$f_j^0 = \frac{1}{v_{thj}^3}\pi^{-3/2}e^{-\frac{v_\perp^2}{v_{thj}^2}}e^{-\frac{v_\parallel^2}{v_{thj}^2}} \tag{2.5}$$

where,

$$v_{thj} = (\frac{2K_BT_j}{m_j})^{1/2}$$

from equation(2.4), we insert the value of f_j^0 as Maxwellian distribution function.

$$1 + 2\pi\sum_j \frac{\omega_{pj}^2}{k_\perp^2}\int_0^\infty dv_\perp \int_{-\infty}^\infty dv_\parallel \sum_n \frac{nj_n^2(k_\perp v_\perp/\omega_{cj})}{\omega/\omega_{cj} - n}(\frac{\partial(\frac{1}{v_{thj}^3}\pi^{-3/2}e^{-\frac{v_\perp^2}{v_{thj}^2}-\frac{v_\parallel^2}{v_{thj}^2}})}{\partial v_\perp}) = 0 \tag{2.6}$$

we solve the above equation in stepwise,

Taking the derivative of f_j^0 w.r.t "$v_\perp$"

$$\frac{\partial f_j^0}{\partial v_\perp} = -\frac{e^{-\frac{v_\perp^2}{v_{thj}^2}-\frac{v_\parallel^2}{v_{thj}^2}}}{\pi^{3/2}v_{thj}^5}v_\perp \tag{2.7}$$

$$1 + 2\pi\sum_j \frac{\omega_{pj}^2}{k_\perp^2}\int_0^\infty dv_\perp \int_{-\infty}^\infty dv_\parallel \sum_n \frac{nj_n^2(k_\perp v_\perp/\omega_{cj})}{\omega/\omega_{cj} - n}(-\frac{e^{-\frac{v_\perp^2}{v_{thj}^2}-\frac{v_\parallel^2}{v_{thj}^2}}}{\pi^{3/2}v_{thj}^5}v_\perp) = 0 \tag{2.8}$$

Solving the integral of "$v_\parallel$"

$$\int_{-\infty}^\infty e^{-\frac{v_\parallel^2}{v_{thj}^2}}dv_\parallel = v_{thj}\sqrt{\pi} \tag{2.9}$$

$$1 - 4\sum_j \frac{\omega_{pj}^2}{k_\perp^2 v_{thj}^4}\int_0^\infty e^{-\frac{v_\perp^2}{v_{thj}^2}}\sum_n \frac{nJ_n^2(k_\perp v_\perp/\omega_{cj})}{\omega/\omega_{cj} - n}dv_\perp = 0 \tag{2.10}$$

Solving the "$v_\perp$"

$$1 - 4\sum_j \frac{1}{k_\perp^2 \lambda_{Dj}^2} \frac{1}{v_{thj}^2} \sum_n \left(\frac{n v_{thj}^2 I_n(\frac{k_\perp^2 v_{thj}^2}{2\omega_{cj}^2})}{2(\frac{\omega}{\omega_{cj}} - n)}\right) e^{-\frac{k_\perp^2 v_{thj}^2}{2\omega_{cj}^2}} = 0 \tag{2.11}$$

where,

$$\lambda_{Dj} = \left(\frac{\epsilon_0 T_j}{n_{0j} q^2 j}\right)^{1/2}$$

$$1 - 2\sum_j \frac{1}{k_\perp^2 \lambda_{Dj}^2} \sum_n \left(\frac{n I_n(\frac{k_\perp^2 v_{thj}^2}{2\omega_{cj}^2})}{\frac{\omega}{\omega_{cj}} - n}\right) e^{-\frac{k_\perp^2 v_{thj}^2}{2\omega_{cj}^2}} = 0 \tag{2.12}$$

where,

$$\lambda_j = \frac{k_\perp^2 v_{thj}^2}{\omega_{cj}^2}$$

$$1 - 2\sum_j \sum_n I_n(\lambda_j/2)\left(\frac{n^2}{\frac{\omega^2}{\omega_{cj}^2} - n^2}\right) = 0 \tag{2.13}$$

In above relation the summation "j" is represented as sum-over species.

2.3 Newberger's Sum Rule

From Newberger's sum rule [25]

$$\sum_{n=-\infty}^{\infty} \frac{J_n(z)J_{n-m}(z)}{a-n} = \frac{(-1)^m \pi}{\sin(\pi a)} J_{m-a}(z)J_a(z) \tag{2.14}$$

$$\sum_{n=-\infty}^{\infty} \frac{J_n^2(z)}{n+a} = \frac{\pi}{\sin(a\pi)} J_a(z)J_{-a}(z) \tag{2.15}$$

$$\sum_{n=-\infty}^{\infty} \frac{nJ_n^2(z)}{n+a} = -\frac{a\pi}{\sin(a\pi)} J_a(z)J_{-a}(z) + 1 \tag{2.16}$$

$$\sum_{n=-\infty}^{\infty} \frac{n^2J_n^2(z)}{n+a} = \frac{a^2\pi}{\sin(a\pi)} J_a(z)J_{-a}(z) - a \tag{2.17}$$

$$\sum_{n=-\infty}^{\infty} \frac{n^3J_n^2(z)}{n+a} = -\frac{a^3\pi}{\sin(a\pi)} J_a(z)J_{-a}(z) + a^2 + \frac{z^2}{2} \tag{2.18}$$

"The advantage of sum rule is, that it give us exception from taking specific number of harmonic. It takes infinite number of harmonic and give more accurate results as compare to others."

2.4 Maxwellian Distribution with Newberger's Sum Rule

Using newberger's sum rule in equation(2.4)

$$1+2\pi\sum_j\frac{\omega_{pj}^2}{k_\perp^2}\int_0^\infty dv_\perp\int_{-\infty}^\infty dv_\parallel\frac{\partial f_j^0}{\partial v_\perp}\times(\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)}J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)=0 \tag{2.19}$$

From Maxwellian Distribution Function

$$f_j^0=\frac{1}{v_{thj}^3}\pi^{-3/2}e^{-\frac{v_\perp^2}{v_{thj}^2}}e^{-\frac{v_\parallel^2}{v_{thj}^2}} \tag{2.20}$$

where,

$$v_{thj}=(\frac{2K_BT_j}{m_j})^{1/2}$$

$$\frac{\partial f_j^0}{\partial v_\perp}=-\frac{e^{-\frac{v_\perp^2}{v_{thj}^2}}e^{-\frac{v_\parallel^2}{v_{thj}^2}}}{\pi^{3/2}v_{thj}^5}nv_\perp \tag{2.21}$$

$$\begin{aligned}&1-\frac{4}{\pi}\sum_j\frac{1}{v_{thj}^5}\frac{\omega_{pj}^2}{k_\perp^2}\int_0^\infty v_\perp e^{-\frac{v_\perp^2}{v_{thj}^2}}dv_\perp\int_{-\infty}^\infty e^{-\frac{v_\parallel^2}{v_{thj}^2}}\\&(\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)}J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)dv_\parallel=0\end{aligned} \tag{2.22}$$

Solving the integral of "$v_\parallel$"

$$\int_{-\infty}^\infty e^{-\frac{v_\parallel^2}{v_{thj}^2}}dv_\parallel=v_{thj}\sqrt{\pi} \tag{2.23}$$

$$1-\sum_j\frac{4}{k_\perp^2\lambda_{Dj}^2}\frac{1}{v_{thj}^2}\int_0^\infty v_\perp e^{-\frac{v_\perp^2}{v_{thj}^2}}(\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)}J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)dv_\perp=0 \tag{2.24}$$

where

$$\lambda_{Dj} = (\frac{\epsilon_0 T_j}{n_{0j} q_j^2})^{1/2}$$

Solving the integral of "$v_\perp$"

$$\begin{aligned}
&\int_0^\infty v_\perp e^{-\frac{v_\perp^2}{v_{thj}^2}} (\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)} J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}}) J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}}) - 1) dv_\perp \\
&= \frac{1}{2} v_{thj}^2 (-1 + 2F2[\{\frac{1}{2}, 1\}; \{1 - \frac{\omega}{\omega_{cj}}, 1 + \frac{\omega}{\omega_{cj}}\}; -\frac{k_\perp^2 v_{thj}^2}{\omega_{cj}}])
\end{aligned} \tag{2.25}$$

$$1 + \sum_j \frac{2}{k_\perp^2 \lambda_{Dj}^2} (1 - 2F2[\{\frac{1}{2}, 1\}; \{1 - \frac{\omega}{\omega_{cj}}, 1 + \frac{\omega}{\omega_{cj}}\}; -\lambda_j]) = 0 \tag{2.26}$$

where

$$\lambda_{Dj} = (\frac{\epsilon_0 T_j}{n_{0j} q_j^2})^{\frac{1}{2}}$$

and

$$\lambda_j = \frac{k_\perp^2 v_{thj}^2}{\omega_{cj}^2}$$

2.5 Comparison Between Newberger's Sum Rule and Series Method for Maxwellian Distribution

The dispersion relation results by both methods(Newberger's Sum Rule and Series Method) of cyclotron harmonic waves for Maxwellian distribution are plotted against different value of $\frac{\omega_p}{\omega_c}$. In series method the value of "n" is taken as 5 and 10.

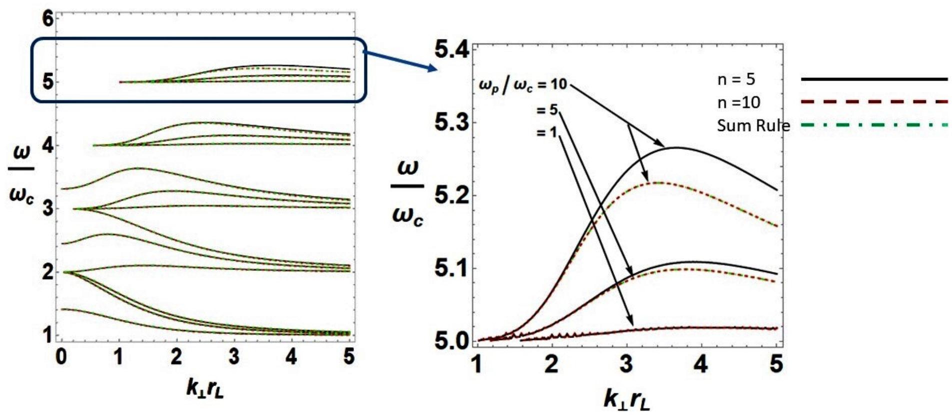

Fig. 2.1: Comparison Between Newberger's Sum Rule and Series Method for Maxwellian Distribution

From the above figure, the dispersion relation of Bernstein waves is plotted as, In Bessel series method the value of "n" is taken as 5(Black-Thick), and 10(Red-Dotted). Meanwhile In Newberger's sum rule method dispersion relation is shown as Green(dot-dashed). Both method give same results for low value of $\frac{\omega_p}{\omega_c}$. But the problem arises when we increase $\frac{\omega_p}{\omega_c}$ ratio, and Bessel series method gives different results for n=5 and n=10. Because as the value of $\frac{\omega_p}{\omega_c}$ increase the value of "n" also have to increase. But we don't know exactly, for what value of "n" the results are most accurate. This kind of problem can produce difficulties in the numerical solutions. So when Newberger's sum rule method solution is compared with the Bessel series method. It get fit for all values of $\frac{\omega_p}{\omega_c}$. Even when we go for a higher value of $\frac{\omega_p}{\omega_c} = 10$ the Newberger's sum rule methods plots get match exactly with all values of "n". Now from here to onward the Newberger,s sum rule method will be used for Other distribution functions.

2.6 Kappa Distribution Function

The plasma is composed on multiple species such as ions and electrons, In our cases we are taking three ions species and one electron species. The ions species consist on Oxygen, Helium, and Hydrogen.

Any classical and thermal equilibrium system can be explain by Maxwellian velocity distribution function. But in astrophysical plasma and space these distribution are quite rare. For understanding the non-equilibrium stationary states, Kappa distribution has well importance. Kappa distribution has unique importance and quite well in explaining the plasma for various location such as in solar wind, inner heliosphere and most important in planetary magnetosphere. Kappa has specific value in range of $\kappa\epsilon[\frac{3}{2},\infty]$. As the value of $\kappa \rightarrow \infty$ the system move toward thermal equilibrium and kappa distribution reduce to Maxwellian distribution, while for $\kappa \rightarrow \frac{3}{2}$ the system become anti-equilibrium. Kappa distribution cover-up high energy particles as compare to Maxwellian distribution.

2.6.1 Oxygen Cyclotron Harmonic Wave with Kappa Distribution using Newberger's Sum Rule

From equation(2.19)

$$1+2\pi\sum_j \frac{\omega_{pj}^2}{k_\perp^2}\int_0^\infty dv_\perp \int_{-\infty}^{\infty} dv_\parallel \frac{\partial f_j^0}{\partial v_\perp}\times(\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)}J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)=0 \tag{2.27}$$

Kappa distribution is

$$f_j^0(v) = A_{\kappa_j}(1+\frac{v^2}{\kappa_j\theta_j^2})^{(\kappa_j+1)} \tag{2.28}$$

$$f_j^0(v) = A_{\kappa_j}(1+\frac{v_\perp^2}{\kappa_j\theta_j^2}+\frac{v_\parallel^2}{\kappa_j\theta_j^2})^{-(\kappa_j+1)} \tag{2.29}$$

Where j represent different species such as electrons or ions. And κ is the spectral index, $\theta_j^2 = \frac{2(\kappa_j-\frac{3}{2})}{\kappa_j}v_{thj}^2$ represent the improved thermal speed and A_κ is normalization constant. The value of $\kappa > \frac{3}{2}$, otherwise the kinetic temperature will diverge. And for $\kappa \to \infty$ the Kappa distribution will reduce to Maxwellian distribution.

Taking derivative of f_j^0 with respect to $v_\perp$

$$\frac{\partial f_j^0(v)}{\partial v_\perp} = (\pi\kappa_j\theta_j^2)^{-\frac{3}{2}}\frac{\Gamma(\kappa_j+1)}{\Gamma(\kappa_j-\frac{1}{2})}(-\kappa_j-1)(1+\frac{v_\perp^2}{\kappa_j\theta_j^2}+\frac{v_\parallel^2}{\kappa_j\theta_j^2})^{-(\kappa_j+2)}\frac{2v_\perp}{\kappa_j\theta_j^2} \tag{2.30}$$

Putting the above value in equation(2.27)

$$\begin{gathered}1-4\pi\sum_j \frac{\omega_{pj}^2}{k_\perp^2}\frac{\kappa_j+1}{\kappa_j\theta_j^2}(\pi\kappa_j\theta_j^2)^{-\frac{3}{2}}\int_0^\infty dv_\perp\int_{-\infty}^{\infty}(1+\frac{v_\perp^2}{\kappa_j\theta_j^2}+\frac{v_\parallel^2}{\kappa_j\theta_j^2})^{-(\kappa_j+2)} \\ \frac{\Gamma(\kappa_j+1)}{\Gamma(\kappa_j-\frac{1}{2})}(\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)}J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)dv_\parallel = 0\end{gathered} \tag{2.31}$$

$$1-4\pi\sum_j \frac{\omega_{pj}^2}{k_\perp^2}\frac{(\kappa_j+1)\kappa_j}{2\kappa_j(\kappa_j-\frac{3}{2})v_{thj}^2}(\pi\kappa_j\theta_j^2)^{-\frac{3}{2}}\int_0^\infty dv_\perp\int_{-\infty}^\infty(1+\frac{v_\perp^2}{\kappa_j\theta_j^2}+\frac{v_\parallel^2}{\kappa_j\theta_j^2})^{-(\kappa_j+2)}$$
$$\frac{\Gamma(\kappa_j+1)}{\Gamma(\kappa_j-\frac{1}{2})}(\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)}J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)dv_\parallel=0 \tag{2.32}$$

Using the gamma function propertry

$$(n+1)\Gamma(n+1)=\Gamma(n+2)$$

equation become

$$1-2\pi\sum_j \frac{(\kappa_j-\frac{3}{2})}{(\kappa_j-\frac{3}{2})(\kappa_j-\frac{1}{2})}\frac{\Gamma(\kappa_j+2)}{\Gamma(\kappa_j-\frac{1}{2})}\int_{-\infty}^\infty(1+\frac{v_\perp^2}{\kappa_j\theta_j^2}+\frac{v_\parallel^2}{\kappa_j\theta_j^2})^{-(\kappa_j+2)}dv_\parallel\frac{(\pi\kappa_j\theta_j^2)^{-\frac{3}{2}}}{k_\perp^2\lambda_{\kappa_j}^2}$$
$$\int_0^\infty(\frac{\pi\omega}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}}\pi)}J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)dv_\perp=0 \tag{2.33}$$

Solving the Integration w.r.t $v_\parallel$

$$\int_{-\infty}^\infty(\kappa_j\theta^2 j)^{\kappa_j+2}(\kappa_j\theta^2 j+v_\perp^2+v_\parallel^2)^{-(\kappa_j+2)}dv_\parallel=\frac{\sqrt{\pi}(\kappa_j\theta_j^2)^{-(\kappa_j+2)}\Gamma(\kappa_j+\frac{3}{2})}{(\kappa_j\theta_j^2+v_\perp^2)^{\kappa_j+\frac{3}{2}}\Gamma(\kappa_j+2)} \tag{2.34}$$

Putting it in the equation

$$1-2\pi\sum_j \frac{(\pi\kappa_j\theta_j^2)\Gamma(\kappa_j+2)\Gamma(\kappa_j+\frac{3}{2})\sqrt{\pi}(\kappa_j\theta_j^2)^{(\kappa_j+2)}}{k_\perp^2\lambda_{\kappa_j}^2(\kappa_j-\frac{1}{2})\Gamma(\kappa_j-\frac{1}{2})\Gamma(\kappa_j-2)(\kappa_j^2\theta_{\kappa_j}^2+v_\perp^2)^{\kappa_j+\frac{3}{2}}}\frac{\omega\pi}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}})}$$
$$\int_0^\infty v_\perp(J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)dv_\perp=0 \tag{2.35}$$

Using Gamma function properties we get

$$0=1-2\sum_j\frac{(\kappa_j\theta_j^2)^{\kappa_j+\frac{1}{2}}}{k_\perp^2\lambda_{\kappa_j}^2}(\kappa_j+\frac{1}{2})\frac{\omega\pi}{\omega_{cj}\sin(\frac{\omega}{\omega_{cj}})}$$
$$\int_0^\infty\frac{v_\perp}{(\kappa_j\theta_j^2+v_\perp^2)^{\kappa_j+\frac{3}{2}}}(J_{(\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})J_{(-\frac{\omega}{\omega_{cj}})}(\frac{k_\perp v_\perp}{\omega_{cj}})-1)dv_\perp=0 \tag{2.36}$$

After solving the $v_\perp$ integration the equation we get

$$1-2\sum_j \frac{(\kappa_j\theta_j^2)^\kappa+\frac{1}{2}}{k_\perp^2\lambda_{\kappa_j}^2}(\kappa+\frac{1}{2})(\frac{1}{2}(\kappa_j\theta_j^2)^{-(\kappa_j+\frac{1}{2})}[\frac{1}{\frac{1}{2}+\kappa_j}$$
$$(-1+2F3[1,\frac{1}{2};\frac{1}{2}-\kappa_j,1+\frac{\omega}{\omega_{cj}},1-\frac{\omega}{\omega_{cj}};\frac{k_\perp^2\kappa_j\theta_j^2}{\omega_{cj}^2}])]+\pi^{\frac{1}{2}}(\frac{k_\perp^2}{\omega_{cj}^2})^{\frac{1}{2}+\kappa_j}(\frac{1}{\kappa_j\theta_j^2})^{\frac{1}{2}-\kappa_j}\kappa_j\theta_j^2$$
$$\frac{\Gamma[-\frac{1}{2}-\kappa_j]\Gamma[1+\kappa_j]}{\Gamma[\frac{3}{2}-\frac{\omega}{\omega_{cj}}+\kappa_j]\Gamma[\frac{3}{2}+\frac{\omega}{\omega_{cj}}+\kappa_j]}\frac{\omega}{\omega_{cj}}\csc(\pi\frac{\omega}{\omega_{cj}})2F2[1+\kappa;\frac{3}{2}-\frac{\omega}{\omega_{cj}}+\kappa,\frac{3}{2}+\frac{\omega}{\omega_{cj}}+\kappa;\frac{k_\perp^2\kappa_j\theta_j^2}{\omega_{cj}^2}])=0 \tag{2.37}$$

After simplification the final equation become

$$\begin{aligned}&1+2\sum_j \frac{1}{k_\perp^2\lambda_{\kappa_j}^2}((1-2F3[1,\frac{1}{2};\frac{1}{2}-\kappa_j,1+\frac{\omega}{\omega_{cj}},1-\frac{\omega}{\omega_{cj}};\frac{k_\perp^2\kappa_j\theta_j^2}{\omega_{cj}^2}])\\&+\pi^{\frac{1}{2}}(2\lambda_j^{'})^{\frac{1}{2}+\kappa_j}\frac{\omega}{\omega_{cj}}\csc(\pi\frac{\omega}{\omega_{cj}})\frac{\Gamma[\frac{1}{2}-\kappa_j]\Gamma[1+\kappa_j]}{\Gamma[\frac{3}{2}-\frac{\omega}{\omega_{cj}}+\kappa_j]\Gamma[\frac{3}{2}+\frac{\omega}{\omega_{cj}}+\kappa_j]}\\&2F2[1+\kappa;\frac{3}{2}-\frac{\omega}{\omega_{cj}}+\kappa,\frac{3}{2}+\frac{\omega}{\omega_{cj}}+\kappa;2\lambda_j^{'}])=0\end{aligned} \tag{2.38}$$

Here

$$\lambda_{\kappa_j}^2=\frac{\epsilon_0 T_j}{n_{0j}q_j^2}\frac{(\kappa_j-\frac{3}{2})}{(\kappa_j-\frac{1}{2})}$$

and

$$\lambda_j^{'}=\frac{(\kappa_j-\frac{3}{2})k_\perp^2 v_{thj}^2}{\omega_{cj}^2}$$

The equation(2.38) is called the dielectric function of electrostatic cyclotron harmonic waves(Berstein waves) in magnetized plasma.

Chapter 3

Results and Discussion

3 Results and Discussion

The dispersion relation is derived for a different distribution. In this report we are taking the system in which plasma is composed of four species such as, Oxygen, Helium, Hydrogen, and electrons as observed in the magnetosphere of the earth, reported by[22] and [36]. This report will only focus on the behavior of Oxygen species. Different parameters effect on Oxygen cyclotron harmonics waves will be monitored. We will take different value of $\frac{\omega_{piO}}{\omega_{ciO}}$ ration. And in kappa distribution, we will also observe the kappa indices effect on Oxygen cyclotron harmonic waves.

The low-hybrid frequency in case of ions can be found by [28]

$$1 = \Sigma_S \frac{\omega_{ps}^2}{\omega^2 - \omega_{cs}^2}$$

where "$S = e, H^+, O^+$"

3.1 Oxygen Cyclotron Harmonic Waves

The Oxygen cyclotron harmonic waves have been reported by [10], [37] and [36]. The parametric survey is carried out by Van Allen Probes. The Ion Bernstein instability has been reported by [22] for observation of [37]. But the ion Bernstein with the inclusion of heavy ions for kappa distribution is still needed to explain without any instability. The dispersion relation for both distribution functions(Maxwellian and Kappa) are normalized such as, $k_\perp r_L$ and $\frac{\omega}{\omega_{ciO}}$ are taken along X-axis and Y-axis respectively for different values of $\frac{\omega_p}{\omega_c} = 10$.

3.1.1 Event-1

There are two events has been reported by [37]. The first event had been observed by Van Allen probe B at $15:00UT$ during a magnetic storm on 1 November 2012.

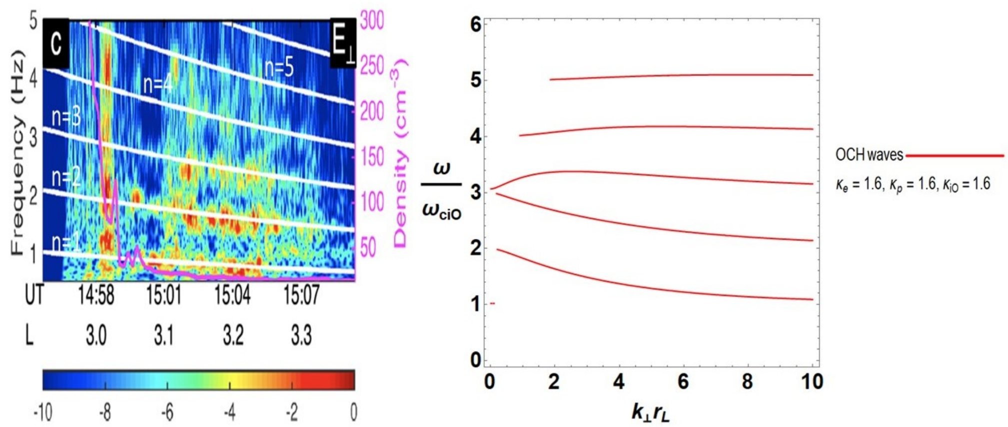

Fig. 3.1: (Event-1) Spectrogram and Dispersion curves of Oxygen Cyclotron Harmonic waves

Oxygen cyclotron harmonic waves in spectrogram are represented as white lines and in numerical results with kappa velocity distribution presented as plots of $\frac{\omega}{\omega_{ciO}}$ against $k_{\perp}r_L$, for $\frac{\omega_{piO}}{\omega_{ciO}} = 92$. The lower hybrid frequency is at 3.0. In numerical results the thin solid(Red) line denotes $\kappa_{iO} = 1.6$.Other parameters are $n_{iO} = 2.27$, $n_{iO} = 4.12$, $\frac{T_{iO}}{T_p} = 3.15$ and $\frac{T_{iO}}{T_p} = 12.23$.

3.1.2 Event-2

The second event had been observed by Van Allen probe B at $17:00UT$ during magnetic storm on 17 March 2013. During this event t

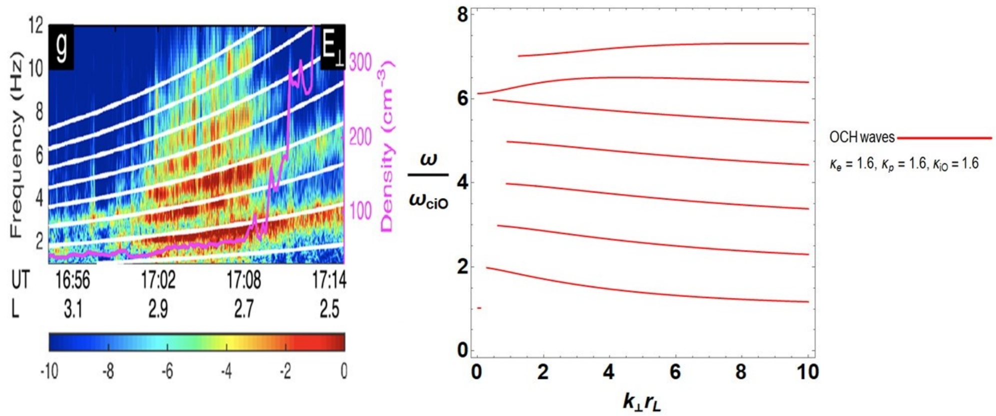

Fig. 3.2: (Event-2) Spectrogram and Dispersion curves of Oxygen Cyclotron Harmonic waves

Oxygen cyclotron harmonic waves in spectrogram are represented as white lines and in numerical results with a kappa velocity distribution presented as plots of $\frac{\omega}{\omega_{ciO}}$ against $k_{\perp}r_L$, for $\frac{\omega_{piO}}{\omega_{ciO}} = 77.9$. The lower hybrid frequency is at 6.1. In numerical results the thin solid(Red) line denotes $\kappa_{iO} = 1.6$.Other parameters are $n_{iO} = 3.39$, $n_{iO} = 1.26$, $\frac{T_{iO}}{T_p} = 0.58$, and $\frac{T_{iO}}{T_p} = 8.46$.

3.1.3 Kappa Variation Effect on Event-1

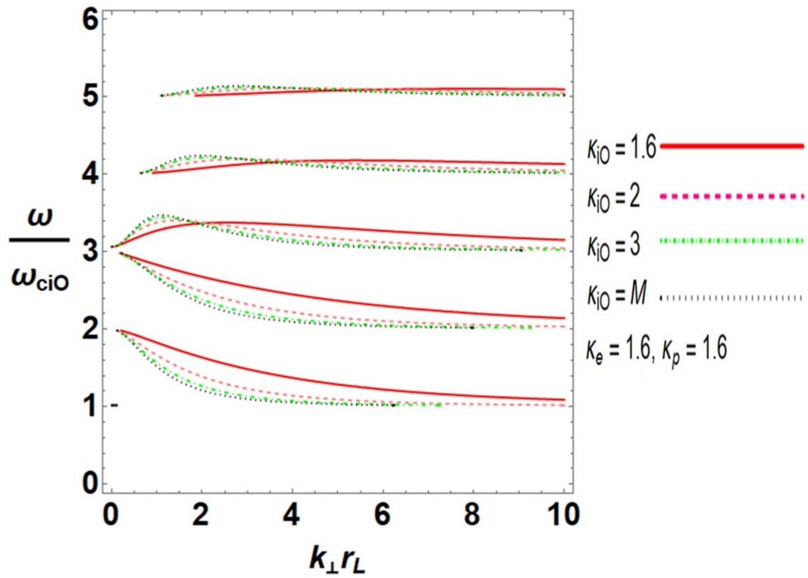

Fig. 3.3: (Event-1) Oxygen Cyclotron Harmonic waves in a plasma with a kappa velocity distribution presented as plots of $\frac{\omega}{\omega_{ciO}}$ against $k_\perp r_L$

The Numerical results of Oxygen Cyclotron Harmonic waves in a plasma with a kappa velocity distribution presented as plots of $\frac{\omega}{\omega_{ciO}}$ against $k_\perp r_L$, for $\frac{\omega_{piO}}{\omega_{ciO}} = 92$. The lower hybrid frequency is at 3.0. The Maxwellian curve is represented by the thick Dotted(Black) line, while the thin solid(Red) line denotes $\kappa_{iO} = 1.6$, dashes(Pink) indicate $\kappa_{iO} = 2$, dotted-dashed(Green) represent $\kappa_{iO} = 3$. The spectral index Kappa (κ_{iO}) variation has some significant impact on these Oxygen cyclotron harmonic waves in the inner magnetosphere. There are some significant differences in behavior between lower value ofκ_{iO} and Maxwellian. The quasi Maxwellian behavior is being observed for $\kappa_{iO} \geq 5$. The harmonics below the ω_{LH} the Oxygen cyclotron harmonic fall-off from upper to the lower cyclotron harmonic for a lower value of κ_{iO} toward higher wave-number. The Maxwellian(Black-Dotted) fall-off before the lower value of κ_{iO} towards lower cyclotron harmonic. Immediately below the lower-hybrid band, at high value of wave-number, there is an increase in frequency for the low value of κ_{iO} and this situation is reversed for the low value of wave number, But this effect can clearly be seen in the lower-hybrid and upper bands. In and above lower-hybrid bands for the low value of κ_{iO} there is a decrease in frequency.

3.1.4 Kappa Variation Effect on Event-2

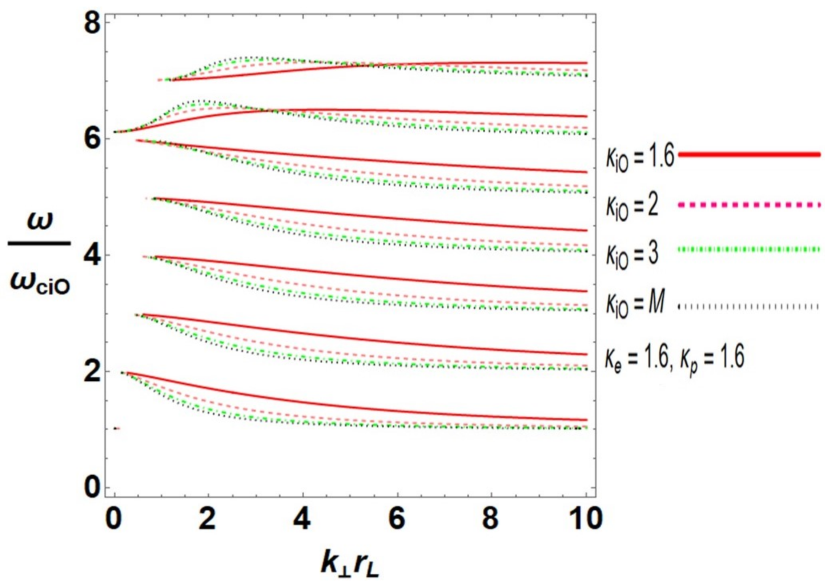

Fig. 3.4: (Event-1) Oxygen Cyclotron Harmonic waves in a plasma with a kappa velocity distribution presented as plots of $\frac{\omega}{\omega_{ciO}}$ against $k_\perp r_L$.

The Numerical results of Oxygen Cyclotron Harmonic waves in a plasma with a kappa velocity distribution presented as plots of $\frac{\omega}{\omega_{ciO}}$ against $k_\perp r_L$, for $\frac{\omega_{piO}}{\omega_{ciO}} = 77.9$. The lower hybrid frequency is at 6.1. The Maxwellian curve is represented by the thick Dotted(Black) line, while the thin solid(Red) line denotes $\kappa_{iO} = 1.6$, dashes(Pink) indicate $\kappa_{iO} = 2$, dotted-dashed(Green) represent $\kappa_{iO} = 3$. Like Event-1 there is a significant effect of spectral Index Kappa (κ_{iO}) effect has been observed. There are some significant differences in behavior between lower value ofκ_{iO} and Maxwellian. The quasi Maxwellian behavior is being observed for $\kappa_{iO} \geq 5$. The harmonics below the ω_{LH} the Oxygen cyclotron harmonic fall-off from upper to the lower cyclotron harmonic for a lower value of κ_{iO} toward higher wave-number. The Maxwellian(Black-Dotted) fall-off before the lower value of κ_{iO} towards lower cyclotron harmonic. Immediately below the lower-hybrid band, at high value of wave-number, there is an increase in frequency for the low value of κ_{iO} and this situation is reversed for the low value of wave number, But this effect can clearly be seen in the lower-hybrid and upper bands. In and above the lower-hybrid bands for low value of κ_{iO} there is a decrease in frequency[1][29][35]..

Like Event-1 the same effects of κ_{iO} has been observed in Event-2. The Only difference occurs in Event-1 and Event-2 is the low-hybrid band. The value of lower hybrid band in Event-1 is 3 and in Event-2 is 6.1.

3.2 Discussion

In this thesis, the dispersion relation of Oxygen cyclotron harmonic(Bernstein) waves has been investigated numerically for different distributions(Maxwellian and Kappa) by using the Vlasov Poisson model with multiple ions and electron species.

The dispersion relation expends separately for each species. The main focused species in this report is Oxygen. The dispersion relation is normalized for Oxygen ions. The most concentrated parameter for this report is the spectral index of kappa κ_{iO}. The spectral index of kappa for oxygen κ_{iO} species has a significant impact on harmonics. The dispersion curves obtained by both events are similar to [5][26]. These all curves are shifted to a higher value of $k_\perp r_L$(higher wave-number) for the low value of κ_{iO}. This observation is similar for all the frequency bands.

In and above the lower-hybrid bands the frequencyω of the wave goes to global maxima in each band. In the case of electron cyclotron harmonic waves these maxima are responsible for Q_n resonance in the magnetosphere[12]. The peak frequency of ω_{peak} increase as the value of κ_{iO} increase, while the wave number decrease for corresponding frequency.

The $\kappa_{iO} \geq 5$ yield the quasi Maxwellian behavior. And the low values of κ_{iO} express the hard ion tails. The frequency spans only in few parts of intra-harmonic space in the lower-hybrid band. But this span range decrease as moves in the further higher bands. The frequency bands above the LH band increase as the value of lower-hybrid frequency increase.

By analogy with the work of electron and ions cyclotron waves [38] [26], we can suggest that the dispersion curves obtained from oxygen cyclotron waves can be used for the diagnostic of enhancing levels of super-thermal ions presence.

No coupling has been reported in both events between the adjacent bands. [26] takes proton and electrons for ion cyclotron harmonic waves, that's why at a higher value of $\frac{\omega_{pi}}{\omega_{ci}}$ the coupling between adjacent band found. But in case of oxygen cyclotron harmonic waves the coupling vanish because there are two ions species, and an earlier study of cyclotron harmonic waves the coupling diminished as the number of thermal ions increase [26][9].

These Oxygen cyclotron harmonic waves are has great influence in the radiation belts of earth magnetosphere. The Oxygen cyclotron harmonics become the cause of accelerating the particles in the inner radiation belts. These charge particles and radiation are harm-full for the astronauts and space-crafts.

The "Alouette" ringing experiments suggests that the resonances could be used for measuring magnetic field accurately. Attenuation measurements could be used to determine collision frequencies, and in principle, it should be possible to determine electron density and temperature in magnetosphere region from transmission measurements between two antennas [5] and the possibility that ion cyclotron waves are the source of plasma heating in the magnetosphere.

The low hybrid frequency band showing in the 2nd figure is the second low-hybrid frequency band which observed due to the presence of second ion species. In this figure, the coupling between the band of oxygen cyclotron harmonic waves can be clearly seen. This coupling has been reported before by [26] and [9]. These coupling diminished as the thermal ions increase but for the low value of low-hybrid frequency not for the highest value of low-hybrid frequency. That's why for the lowest value of ω_{LH} the coupling has not been seen in the first lower hybrid band.

Chapter 5

References

References

1 I. B. Bernstein. Waves in a plasma in a magnetic field. *Physical Review*, **109**(1):10, 1958.

2 R. Bilato, F. Volpe, A. Köhn, R. Paccagnella, D. Farina, E. Poli, and M. Brambilla. Feasibility of electron bernstein wave coupling via oxb mode conversion in the rfx-mod reversed field pinch device. *Nuclear Fusion*, **49**(7):075020, 2009.

3 S. Boardsen, D. Gallagher, D. Gurnett, W. Peterson, and J. Green. Funnel-shaped, low-frequency equatorial waves. *Journal of Geophysical Research: Space Physics*, **97**(A10):14967–14976, 1992.

4 F. F. Chen. *Introduction to plasma physics and controlled fusion*, volume **1**. Springer, 1984.

5 F. W. Crawford et al. Cyclotron harmonic waves in warm plasmas. *Radio Sci. D*, **69**:789–805, 1965.

6 N. D'Angelo and R. Motley. Electrostatic oscillations near the ion cyclotron frequency. *Physics of Fluids (New York)*, **5**(MATT-107), 1962.

7 W. C. Demarcus. The constitution of jupiter and saturn. *The Astronomical Journal*, **63**:2, 1958.

8 K. Fox. Nasa's van allen probes spot an impenetrable barrier in space. *NASA Goddard Space Flight Center: Greenbelt, MD*, 2015.

9 R. Fredricks. Structure of generalized ion bernstein modes from the full electromagnetic dispersion relation. *Journal of Plasma Physics*, **2**(3):365–380, 1968.

10 H. Funsten, R. Skoug, A. Guthrie, E. MacDonald, J. Baldonado, R. Harper, K. Henderson, K. Kihara, J. Lake, B. Larsen, et al. Helium, oxygen, proton, and

electron (hope) mass spectrometer for the radiation belt storm probes mission. In *The Van Allen Probes Mission*, pages 423–484. Springer, 2013.

11 S. P. Gary, K. Liu, D. Winske, and R. E. Denton. Ion bernstein instability in the terrestrial magnetosphere: Linear dispersion theory. *Journal of Geophysical Research: Space Physics*, **115**(A12), 2010.

12 F. Henning, R. Mace, and S. Pillay. Electrostatic bernstein waves in plasmas whose electrons have a dual kappa distribution: Applications to the saturnian magnetosphere. *Journal of Geophysical Research: Space Physics*, **116**, number=A12, year=2011, publisher=Wiley Online Library.

13 A. Hirose, I. Alexeff, and W. Jones. Landau damping of ion acoustic waves in a uniform magnetic field. *The Physics of Fluids*, **13**(5):1290–1297, 1970.

14 A. Hirose, I. Alexeff, W. Jones, N. Krall, and D. Montgomery. Landau damping of electrostatic ion waves in a uniform magnetic field. *Physics Letters A*, **29**(1):31–32, 1969.

15 W. B. Hubbard and R. Smoluchowski. Structure of jupiter and saturn. *Space Science Reviews*, **14**(5):599–662, 1973.

16 H. Igami, Y. Yoshimura, S. Kubo, T. Shimozuma, H. Takahashi, H. Tanaka, K. Nagasaki, S. Inagaki, T. Mutoh, A. Komori, et al. Electron bernstein wave heating via the slow x–b mode conversion process with direct launching from the high field side in lhd. *Nuclear Fusion*, **49**(11):115005, 2009.

17 R. Kessel, N. Fox, and M. Weiss. The radiation belt storm probes (rbsp) and space weather. In *The Van Allen Probes Mission*, pages 531–543. Springer, 2012.

18 R. E. Kidder. Application of lasers to the production of high-temperature and high-pressure plasma. *Nuclear Fusion*, **8**(1):3, 1968.

19 Q. Ma, W. Li, R. M. Thorne, and V. Angelopoulos. Global distribution of equatorial magnetosonic waves observed by themis. *Geophysical Research Letters*, **40**(10):1895–1901, 2013.

20 R. Mace. A gordeyev integral for electrostatic waves in a magnetized plasma with a kappa velocity distribution. *Physics of Plasmas*, **10**(6):2181–2193, 2003.

21 R. Mace. Generalized electron bernstein modes in a plasma with a kappa velocity distribution. *Physics of Plasmas*, **11**(2):507–522, 2004.

22 K. Min, R. E. Denton, K. Liu, S. P. Gary, and H. E. Spence. Ion bernstein instability as a possible source for oxygen ion cyclotron harmonic waves. *Journal of Geophysical Research: Space Physics*, **122**(5):5449–5465, 2017.

23 R. W. Motley and N. D'angelo. Excitation of electrostatic plasma oscillations near the ion cyclotron frequency. *The Physics of Fluids*, **6**(2):296–299, 1963.

24 H. M. Mott-Smith. History of"plasmas". *Nature*, **233**:219, 1971.

25 B. S. Newberger. New sum rule for products of bessel functions with application to plasma physics. *Journal of Mathematical Physics*, **23**(7):1278–1281, 1982.

26 F. Nsengiyumva, R. L. Mace, and M. A. Hellberg. Ion bernstein waves in a plasma with a kappa velocity distribution. *Physics of Plasmas*, **20**(10):102107, 2013.

27 T. Phillips. Van allen probes discover a new radiation belt. science@ nasa. nasa, 2013.

28 R. I. Pinsker. Whistlers, helicons, and lower hybrid waves: The physics of radio frequency wave propagation and absorption for current drive via landau damping. *Physics of Plasmas*, **22**(9):090901, 2015.

29 S. Puri, F. Leuterer, and M. Tutter. Dispersion curves for the generalized bernstein modes. *Journal of Plasma Physics*, **9**(1):89–100, 1973.

30 J. A. Ratcliffe et al. *An introduction to ionosphere and magnetosphere*. CUP Archive, 1972.

31 C. T. Russell, R. E. Holzer, and E. J. Smith. Ogo 3 observations of elf noise in the magnetosphere: 2. the nature of the equatorial noise. *Journal of Geophysical Research*, **75**(4):755–768, 1970.

32 O. Santolík, J. Pickett, D. Gurnett, M. Maksimovic, and N. Cornilleau-Wehrlin. Spatiotemporal variability and propagation of equatorial noise observed by cluster. *Journal of Geophysical Research: Space Physics*, **107**(A12):SMP–43, 2002.

33 J. Schmitt. Dispersion and cyclotron damping of pure ion bernstein waves. *Physical Review Letters*, **31**(16):982, 1973.

34 R. Sund and J. Scharer. Mode conversion of high-field-side-launched fast waves at the second harmonic of minority hydrogen in advanced tokamak reactors. In *AIP Conference Proceedings*, volume **694**, pages 170–173. AIP, 2003.

35 J. Tataronis and F. Crawford. Cyclotron harmonic wave propagation and instabilities: I. perpendicular propagation. *Journal of Plasma Physics*, **4**(2):231–248, 1970.

36 M. Usanova, N. Ahmadi, D. Malaspina, R. Ergun, K. Trattner, Q. Reece, T. Leonard, S. Fuselier, R. Torbert, C. Russell, et al. Mms observations of harmonic electromagnetic ion cyclotron waves. *Geophysical Research Letters*, **45**(17):8764–8772, 2018.

37 M. Usanova, D. Malaspina, A. Jaynes, R. Bruder, I. Mann, J. Wygant, and R. Ergun. Van allen probes observations of oxygen cyclotron harmonic waves in the inner magnetosphere. *Geophysical Research Letters*, **43**(17):8827–8834, 2016.

38 A. F. Viñas, R. L. Mace, and R. F. Benson. Dispersion characteristics for plasma resonances of maxwellian and kappa distribution plasmas and their comparisons to the image/rpi observations. *Journal of Geophysical Research: Space Physics*, **110**(A6), 2005.